P.E. Civil Exam Review: Engineering Cost Analysis

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ASCE | KNOWLEDGE & LEARNING

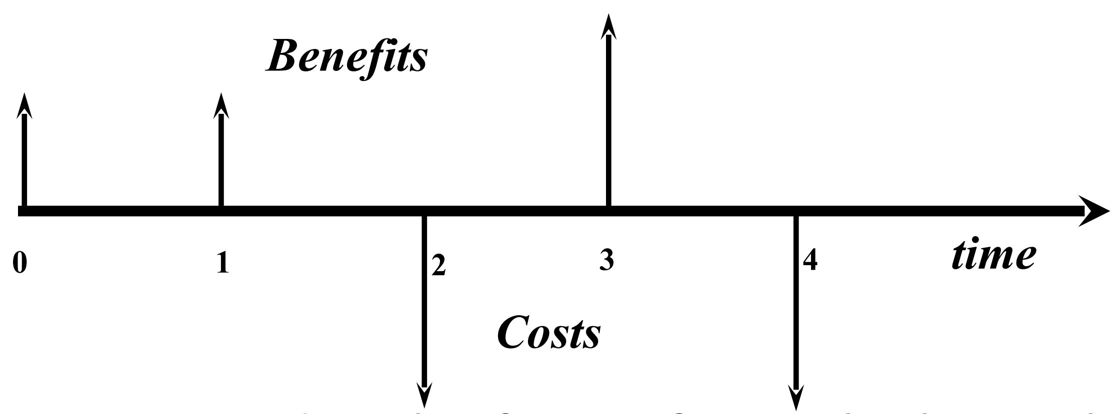
Definition

Engineering Economics involves the systematic evaluation of the economic merits of proposed solutions to engineering problems. To be economically acceptable, solutions to engineering problems must demonstrate a positive balance of long term benefits over long term costs, and they must also:

- promote the well-being of an organization
- embody creative technology and ideas
- permit identification and scrutiny of their outcomes
- translate profitability to bottom line through valid and acceptable measure of merit.

Cash Flow Diagrams

- Benefits expressed as “up” arrows
- Costs expressed as “down” arrows
- *End-of-year convention* is generally used
- Length of arrow represents magnitude (approx.)
- Can be diagrammed from borrower or lender perspective.



- Can also expressed in the form of a *Cash Flow Table*

Principles of Engineering Economy

1. Develop the alternatives
2. Focus on the differences
3. Use a consistent viewpoint
4. Use a common unit of measure
5. Consider all relevant criteria
6. Make uncertainty explicit
7. Revisit your decisions

Origins of Interest

- Rental charge for the use of money
- Interest existed in Babylon as early as 2000 B.C.
- Well-established international banks in 575 B.C.
  - Home offices in Babylon
  - Charged 6-25%
- Outlawed in Middle Ages on Scriptural grounds
  - "Usury" mentioned in Exodus 22: 21-27
- Reasons For Charging Interest
  - risk
  - administration
  - inflation
  - opportunity cost

Compound Interest

- When interest is charged on the remaining principal to be paid plus any accumulated interest.

Period	**Amount Owed at Beginning of Period**	**Interest Charged During Period**	**Amount Owed At End of Period**
1	P	iP	$P(1+i)$
2	$P(1+i)$	$iP(1+i)$	$P(1+i)^2$
.	.	.	.
		$iP(1+i)^{N-1}$	$P(1+i)^N$

$$F_N = P\,(1+i)^N$$

Example

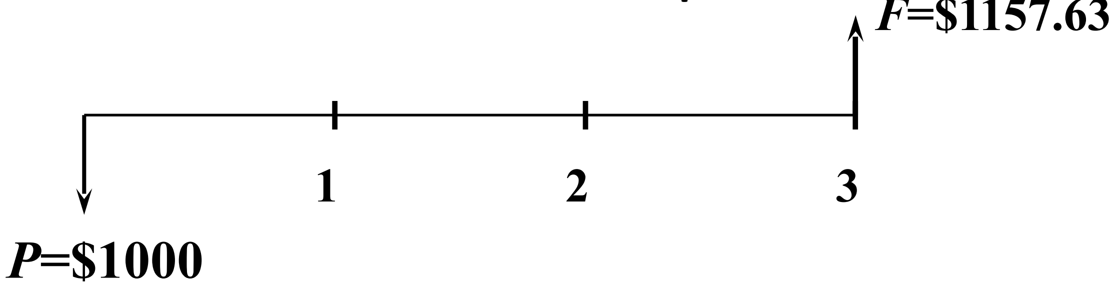
- Borrow $1000 for 3 periods at *i*=5% compounded each period.

Period	(1) Amount Owed at Beginning of Period	(2)= (1)*0.05 Interest Owed at End of Period	(3)=(1)+(2) Amount Owed at End of Period
1	$1000.00	$50.00	$1050.00
2	$1050.00	$52.50	$1102.50
3	$1102.50	$55.13	$1157.63

$$F_3 = 1000\,(1+.05)^3 = \$1157.63$$

The Concept of Equivalence

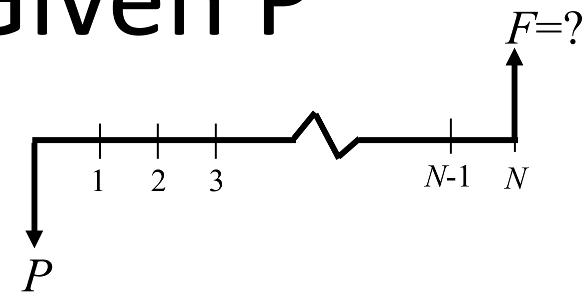
- Equal cash flows that occur at different points in time do not have equal value
- Which would you prefer:
  - $1000 today or $1000 in 3 years?
- If the interest rate is 5%, compounded annually



  - $1000 today is ***equivalent*** to $1157.63 in 3 years

Finding F Given P

$$F = P(1+i)^N$$



- $(1+i)^N$ *single payment compound amount factor*
- denoted (*F/P, i, N*)

A firm borrows $1000 for 8 years at i=10%. How much must be repaid in a lump sum at the end of the eighth year?

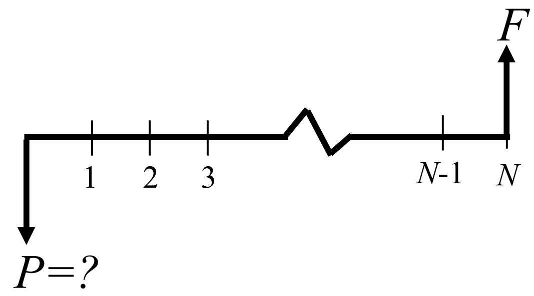
$F = P\ (F/P,\ 10\%,\ 8)$

$= \$1000\ (2.1436)$

$= \$2{,}143.60$

Finding P Given F

$$P = F(1+i)^{-N}$$



- $(1+i)^{-N}$ *single payment present worth factor*
- denoted (P/*F, i, N*)
- Tabled in the back of the book

A firm wants to have $2143.60 8 years from now. What amount should be deposited now at *i*=10%.

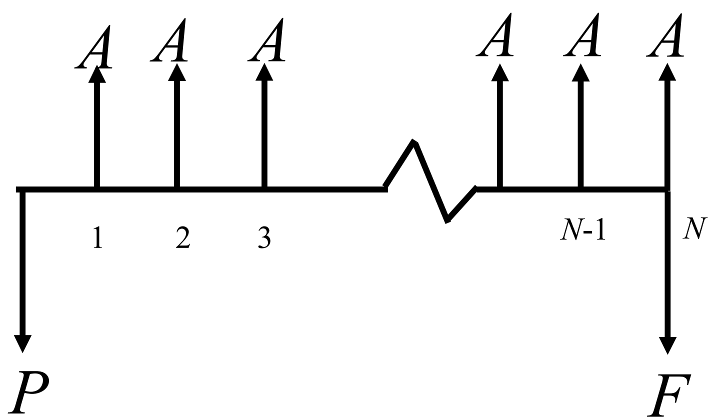
$P = F\,(P/F,\ 10\%,\ 8)$

$= \$2{,}143.60\ (0.4665)$

$= \$1000.00$

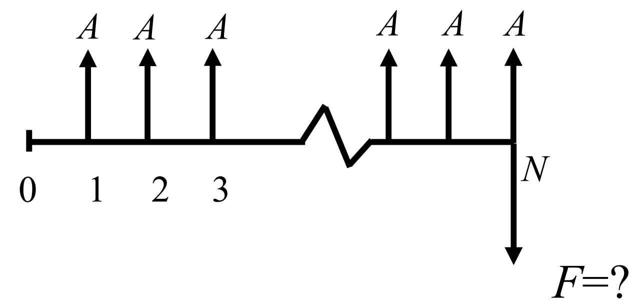
Uniform Series

- Referred to as ***Annuities***, denoted *A*
- *A* occurs *at the end of* periods 1 through *N*
- *P* occurs *one period before* first *A*
- *F* occurs *at the same* time as the last *A*



Uniform Series: Finding F Given A

$$F = A\left[\frac{(1+i)^N - 1}{i}\right]$$



- $\left[\frac{(1+i)^N - 1}{i}\right]$ *uniform series compound amount factor*
- denoted (*F/A, i, N*)

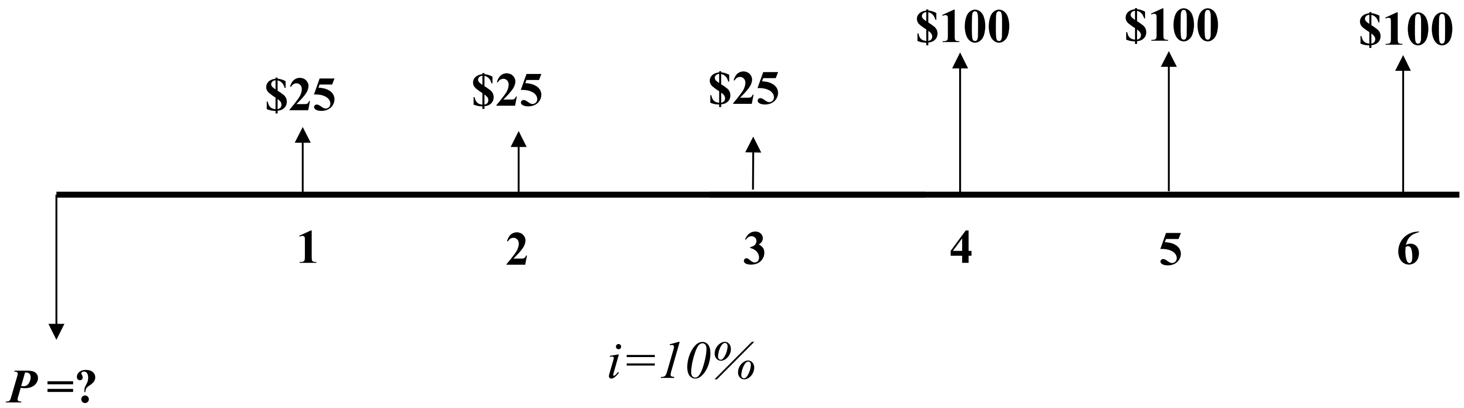
If eight annual deposits of $187.45 are placed into an account earning *i*=10%, how much would accumulate immediately after the last deposit?

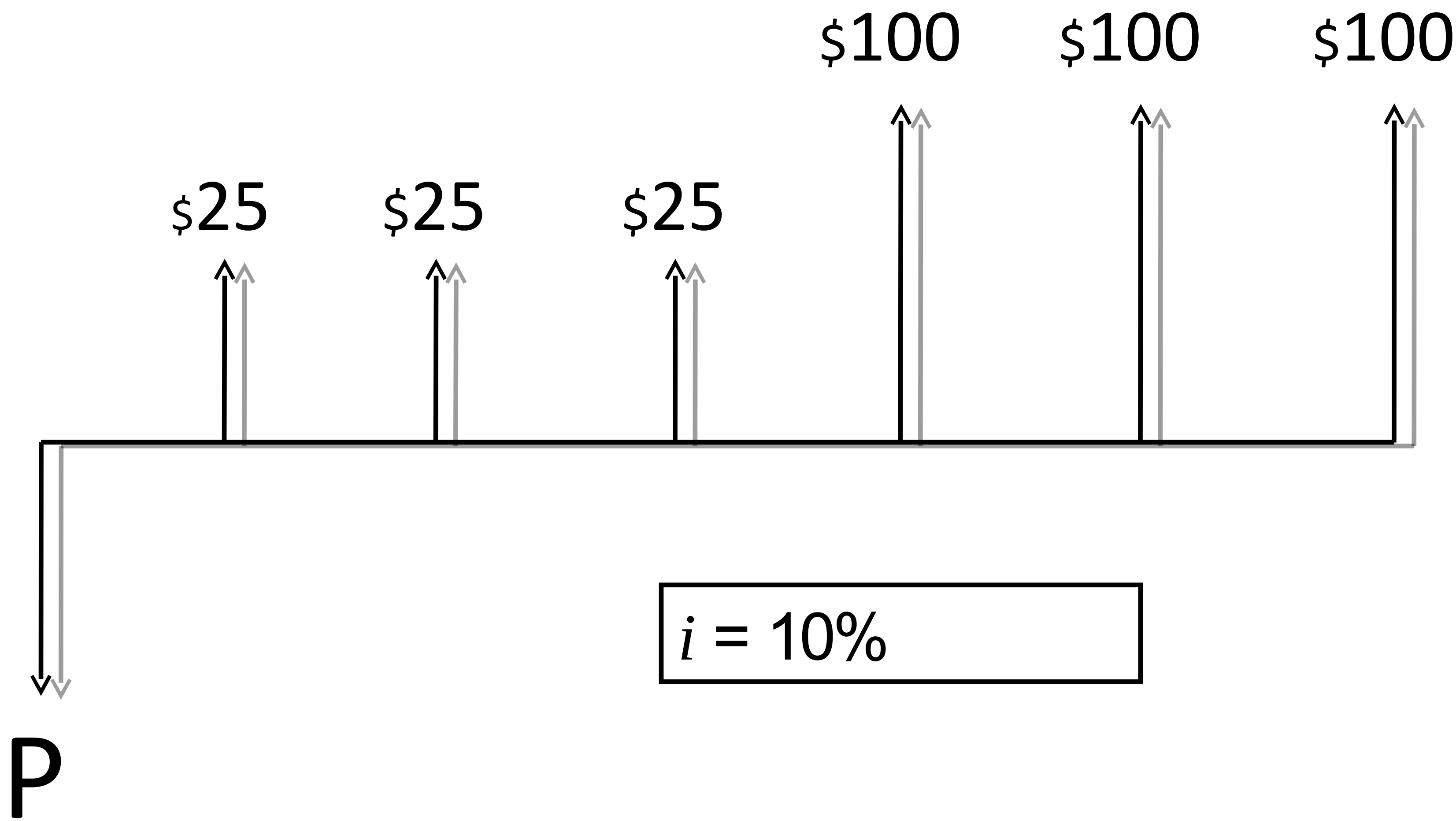
$F = A\ (F/A,\ 10\%,\ 8)$

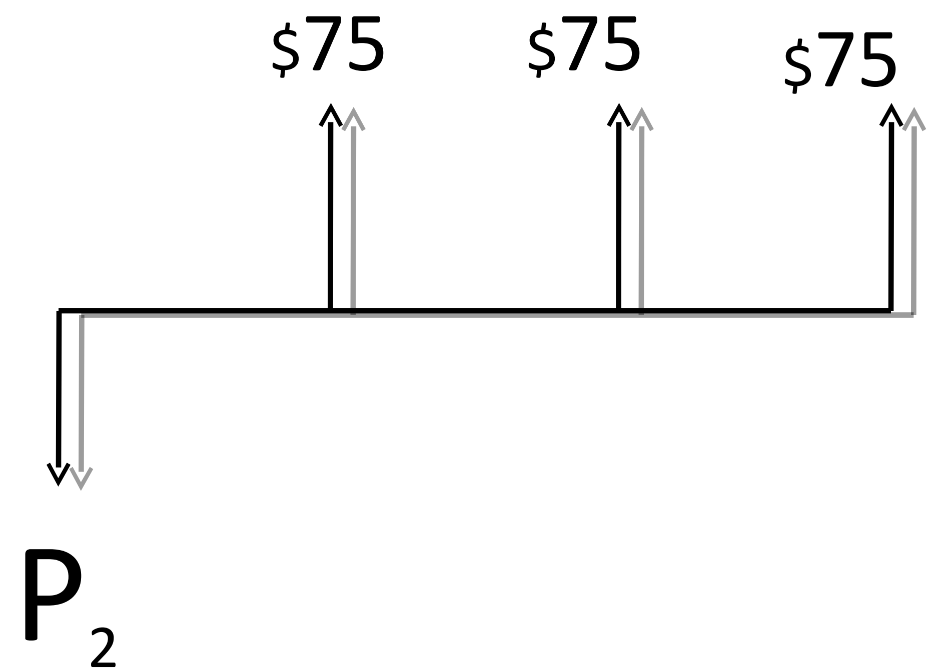
$= \$187.45(11.4359)$

$= \$2{,}143.60$

Class Exercise




$100
$100
$100
$25
$25
$25
i = 10%
P



P = P1 + P3

P1= 25 (4.355) = 108.87

P2= 75(2.487) = 186.52

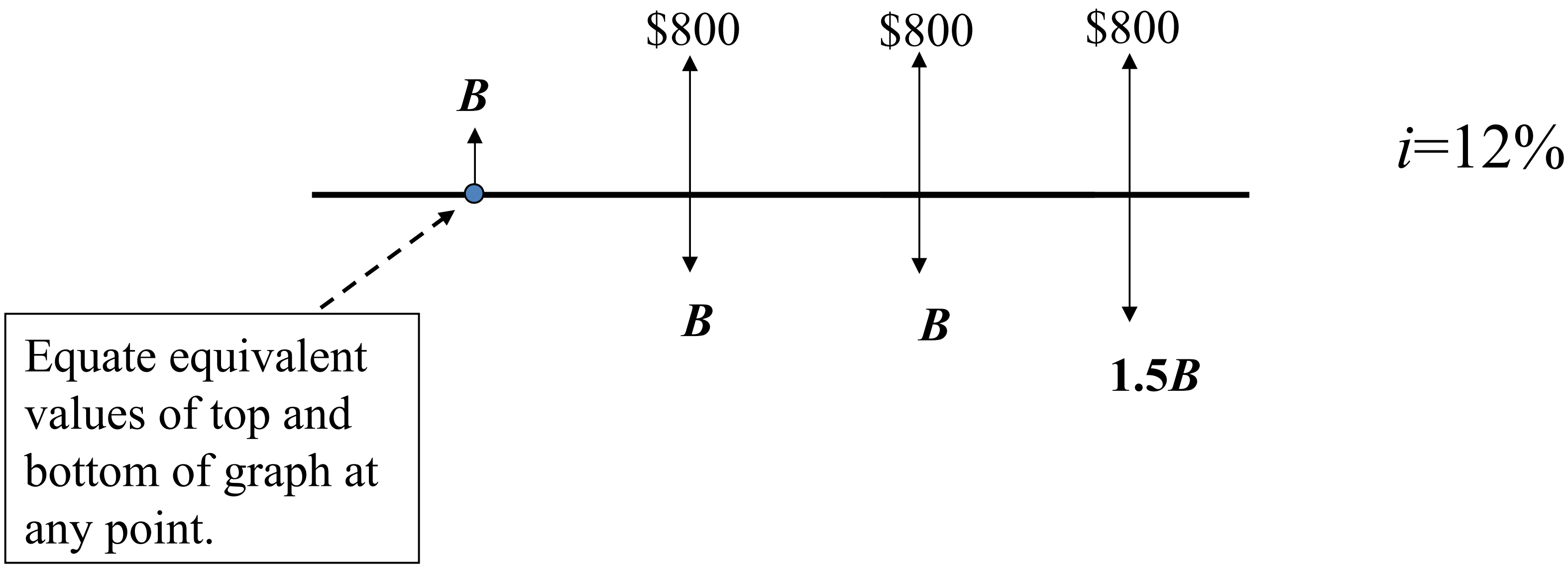
P3= 186.52 (0.7513) = 140.13

$$P = P_1 + P_3$$

P = 108.87 + 140.13 = <u>$ 249.00</u>

Solving For Variables

- Find equivalent value of costs and benefits at the same point in time.



$$B + 800\overbrace{(2.402)}^{(P/A,12\%,3)} = B\overbrace{(2.402)}^{(P/A,12\%,3)} + 0.5B\overbrace{(.7118)}^{(P/F,12\%,3)}$$

$$B = 1093.12$$

Solving for N

Given *P, F, i*:

$$N = \frac{\ln(F / P)}{\ln(1 + i)}$$

- :

P =1000 F =2000 i =10%

$$N = \frac{\ln(2/1)}{\ln(1+0.1)} = 7.27\,yrs$$

(can also use tables)

Given *P, A, i*:

$$N = \frac{\ln\left[A/(A - Pi)\right]}{\ln(1 + i)}$$

Example:

P = 6600 A=1000 i =10%

$$N = \frac{\ln\left[1000/(1000 - 6600(0.1)\right]}{\ln(1+0.1)}$$

$$= 11.32\,yrs$$

(can also use tables)

Solving for *i*

Given *P, F, n*:

$$i = \left(\frac{F}{P}\right)^{\frac{1}{N}} - 1$$

Example:

P=1000 F=2000 N=8

$$i = 2^{\frac{1}{8}} - 1$$

$$i = 9.05\%$$

Also note that:

2000 = 1000(F/P, i, 8)

(F/P, i,8) = 2

$i \sim 9\%$ (from the tables)

Class Exercises

$P=1000,\ F=3500,\ N=6,\ i=?$

$P=1000,\ A=50,\ i=3\%,\ N=?$

Nominal and Effective Interest Rates

- Often the time between compounding is less than one year.
- Generally express this using:
  - r nominal annual rate per year
  - m number of compounding periods per year
  - r/m interest rate per period

$$i_{eff} = \left(1 + \frac{r}{m}\right)^{m} - 1$$

Example

- Suppose you deposit $1000 into an account paying 6% per year compounded semi-annually
- r=6% m=2
- 0.06/2 = 3% per 6 month period

Period	EOY	Value
0	0	$1,000.00
1	0.5	$1,030.00
2	1	$1,060.90

$$i_{eff} = \left(1 + \frac{0.06}{2}\right)^2 - 1 = 6.09\%$$

The Effect of Increasing m when r=6%

• m=1	annual	i_{eff} = 6%
• m=2	semi-annual	i_{eff} = 6.09%
• m=4	quarterly	i_{eff} = 6.136%
• m=12	monthly	i_{eff} = 6.1678%
• m=52	weekly	i_{eff} = 6.17998%
• m=365	daily	i_{eff} = 6.18313%

Application of Money-Time Relationships

Topics of Interest

- Illustrate several basic methods
- Briefly describe some of the assumptions and interrelationships
  - Finding *minimum attractive rate of return* (MARR)
  - *Present Worth* method
  - *Future Worth* method
  - *Annual Worth* method
  - *Internal Rate of Return* method

Determining the MARR

- Minimum attractive rate of return (MARR)
- The value of *i* used in our calculations
- Depends upon many factors
  - amount of money available (capital rationing)
  - quantity of good projects (essential vs. elective)
  - the amount of perceived risk
  - cost of administering projects
  - the type of organization

The Present Worth (PW) Method

- Equivalent worth of cash flows at time zero.
- If *PW*>0 the project is acceptable

  i = effective rate per period (MARR)

  F_k = Future cash flow at end of period k

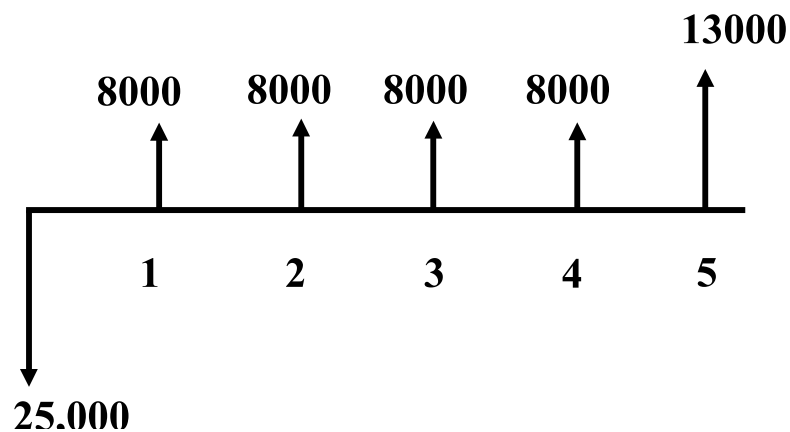
  N = number of periods in the planning horizon

$$PW = F_0 + F_1(1+i)^{-1} + F_2(1+i)^{-2} + \ldots + F_N(1+i)^{-N}$$

$$PW = \sum_{k=0}^{N} F_k (1+i)^{-k}$$

Example of the PW Method

- An investment of \$25000 is expected to save \$8000 per year for 5 years. The system will be sold at the end of five years for \$5000. Assume the *MARR*=20%



$$PW = -\$25,000 + \$8000\overbrace{(2.9906)}^{(P/A,20\%,5)} + \$5000\overbrace{(0.4019)}^{(P/F,20\%,5)}$$

$$PW = \$934.29 \quad accept \quad or \quad reject \quad project?$$

The Future Worth (FW) Method

- Equivalent worth of cash flows <u>at time *N*</u>.
- If *FW* >0 the project is acceptable
- Same as *PW* method multiplied by (*F/P*, *i*, *N*)

$$FW = F_0(1+i)^N + F_1(1+i)^{N-1} + F_2(1+i)^{N-2} + \ldots + F_N$$

$$PW = \sum_{k=0}^{N} F_k (1+i)^{N-k}$$

<u>Example:</u> *FW*= \$934.29(2.48832)=\$2324.80

- Can also use (Bank Balance Approach)
- Compare against "do nothing"

Annual Worth (AW) Method

- Can find PW then multiply by (*A/P*, *i*, *N*)
- If *AW*>0 project is acceptable
- Expressed many different ways

$\underline{R}$ = equivalent uniform annual revenues (savings)

$\underline{E}$ = equivalent uniform annual expenses (costs)

CR = capital recovery

(annualized cost of investment, *I*, minus salvage value, *S*)

$$AW = \underline{R} - \underline{E} - CR \quad where,$$

$$CR = I(A/P,i,N) - S(A/F,i,n)$$

AW Applied to Previous Example

$$AW = \overbrace{\$8000}^{\underline{R}-\underline{E}} - \overbrace{[\$25000\overbrace{(0.3344)}^{(A/P,20,5)} - \$5000\overbrace{(0.1344)}^{(A/F,20,5)}]}^{CR}$$

$$AW = \$312.40$$

$$Note: \ Same \ \ result \ \ achieved \ \ by:$$

$$AW = \$934.29\overbrace{(0.3344)}^{(A/P,20,5)} = \$312.40$$

Annual Worth (AW) Method

- expressed on annual basis
- Can find PW then multiply by (*A/P*, *i*, *N*)
- If *AW*>0 project is acceptable
- $\underline{R}$ = equivalent uniform annual revenues (savings)

$\underline{E}$ = equivalent uniform annual expenses (costs)

CR = capital recovery

(annualized cost of investment, *I*, minus salvage value, *S*)

$$AW = \underline{R} - \underline{E} - CR \quad where,$$

$$CR = I(A/P, i, N) - S(A/F, i, n)$$

PW *Method*

EOY	A	B	C	D
0	-390	-920	-660	0
1-10	69	167	133.5	0

$$PW_A = -390 + 69\overbrace{(6.1446)}^{P/A,10\%,10} = 33.977$$

$$PW_B = -920 + 167\overbrace{(6.1446)}^{P/A,10\%,10} = 106.148$$

$$PW_C = -660 + 133.5\overbrace{(6.1446)}^{P/A,10\%,10} = 160.304$$ *C is the Best*

$$PW_D = 0$$

FW *Method*

EOY	A	B	C	D
0	-390	-920	-660	0
1-10	69	167	133.5	0

$$FW_A = -390\overbrace{(2.5937)}^{F/P,10\%,10} + 69\overbrace{(15.937)}^{F/A,10,10} = 88.138$$

$$FW_B = -920\overbrace{(2.5937)}^{F/P,10\%,10} + 167\overbrace{(15.937)}^{F/A,10,10} = 275.342$$

$$FW_C = -660\overbrace{(2.5937)}^{F/P,10\%,10} + 133.5\overbrace{(15.937)}^{F/A,10,10} = 415.801 \quad C \text{ is the Best}$$

$$FW_D = 0$$

Incremental Analysis Procedure

1. Rank investments in increasing order of initial investment cost. Least cost alternative is "current best".

2. Analyze the increment between "current best" and next alternative on the list. If no more alternatives, STOP.

3. If IRR of the increment > MARR then the higher cost alternative becomes "current best". Go to 2.

4. Else, reject higher cost alternative, keep "current best", and go to 2.

Back to Previous Example

EOY	A	B	C	D
0	-390	-920	-660	0
1-10	69	167	133.5	0

Ranked by initial investment: D-A-C-B

D is current best.

EOY	A-D	C-A	B-C
0	-390	-270	-260
1-10	69	64.5	33.5
IRR	***11.99%***	***20.05%***	***4.90%***
	A is Current Best	C is Current Best	C is Preferred Alt.

Class Exercise

	A	B	C
Initial Investment	$100	$300	$25
Annual Benefits	$20	$70	$60

N=10   MARR=10%

a) *Use NPW and select the best alternative.*

b) *Use incremental IRR and select the best alternative.*

Benefit- Cost Analysis

	A	B	C
Annual Benefit ($)	375,000	460,000	500,000
Annual Cost ($)	150,000	200,000	250,000
B/C ratio	2.5	2.3	2.0

$B - A$

$$\frac{B}{C} = \frac{460{,}000 - 375{,}000}{200{,}000 - 150{,}000} = \frac{85{,}000}{50{,}000} = 1.7$$

$C - B$

$$\frac{B}{C} = \frac{500{,}000 - 460{,}000}{250{,}000 - 200{,}000} = \frac{40{,}000}{50{,}000} = 0.8$$

User Benefits

U_n = Net User Benefit

U_p = Total Annual Cost for Present Facility

U_f = Total Annual Cost for Future Facility

$$U_n = U_p - U_f$$

Owner's Costs

$\boldsymbol{C_f}$ = Equivalent Capital Cost of Future Facility

$\boldsymbol{C_p}$ = Equivalent Capital Worth of Existing Facility

Cn = Net Capital Cost of Replacing Present Facility with Future Facility

$$C_n = C_f - C_p$$

Owner's Costs

$\boldsymbol{M_f}$ = Equivalent O&M of the Future (proposed)

$\boldsymbol{M_p}$ = Equivalent O&M of the Present (existing)

$\boldsymbol{M_n}$ = Net O&M cost of proposed facility over existing facility

$$M_n = M_f - M_p$$

Conventional B/C

$\boldsymbol{B_n = U_n}$ = Net Annual Benefits (savings in costs)

$\boldsymbol{C_n = M_n}$ = Costs consist of the annual equivalent costs to the owner of the facility, including capital costs and maintenance. The numerator consists of all of the user's benefits; the denominator is the sum of all the owner's costs.

Conventional B/C

$$Conventional\ B/C = \frac{Net\ Savings\ to\ Users}{Owner's\ Net\ Capital\ Cost + Owner's\ O\&M\ Cost}$$

$$= \frac{U_n}{C_n + M_n} = \frac{B_n}{C_n + M_n}$$

	EXISTING ROUTE	PROPOSED ROUTE A	PROPOSED ROUTE B
Construction Cost ($)	0	100,000	100,000
Annual Equivalent to Construction Cost *A = $100,000 (A/P, 8%, 20)* 0.1019	0	$10,190	$10,190
Estimated User's Cost ($/yr)	200,000	165,000	195,000
O&M Costs ($/yr)	250,000	270,000	240,000
Total Annual Cost ($)	450,000	445,190	445,190
Annual Savings over Existing Route ($/yr)	0	4,810	4,810

- Route A compared to Existing

$$\frac{U_p - U_f}{(C_f - C_p) + (M_f - M_p)} = \frac{200{,}000 - 165{,}000}{(10{,}190 - 0) + (270{,}000 - 250{,}000)}$$

$$= 1.16$$

- Route B compared to Existing

$$\frac{U_p - U_f}{(C_f - C_p) + (M_f - M_p)} = \frac{200{,}000 - 195{,}000}{(10{,}190 - 0) + (240{,}000 - 250{,}000)}$$

$$= 26.32$$

Find the rate of return of the investment in route A or route B from the previous example:

- For both cases:

$$\left(U_p - U_f\right) = \left(M_f - M_p\right) = C_f - C_p$$

$15,000 = $100,000 (*A/P, i, 20*)

0.150 = (*A/P, i, 20*)

By Interpolation:

i = 14%	0.1510 = (A/P, 15%, 20)
i = ?	0.1500 = (A/P, i, 20)
i = 13%	0.1424 = (A/P, 12%, 20)

$$i = 13 + \left(\frac{0.0076}{0.0086}\right) = 13.9\%$$

Cash Flow Example

Use i = 1%. Alternative X, which involves $400,000 benefit occurring 60 years from now, has an annual benefit equivalent to

B = $400,000 (A/F, 1%, 60) = $4,880 / yr

0.0122

Assume an expenditure of $100,000 required right now to produce this $400,000 future benefit. This annual equivalent cost is

C = $100,000 (A/P, 1%, 60) = $2,220 / yr

0.0222

Therefore the B/C may be calculated as

$$Alternative\ X: \frac{B}{C} = \frac{\$4{,}880}{\$2{,}220} = \mathbf{2.20}$$

Cash Flow Example (continued)

Assume now a competing alternative project Y involves

benefit of \$140,000 occurring immediately = \$140,000

cost of \$100,000 occurring immediately = \$100,000

$$Alternative\ Y: \frac{B}{C} = \frac{\$140{,}000(\frac{A}{P}, 1\%, 60)}{\$100{,}000\ (\frac{A}{P}, 1\%, 60)} = \mathbf{1.40}$$

For a more realistic value of *i*, use *i* = 7%.

$$Alternative\ X: \frac{B}{C} = \frac{400{,}000\,(\frac{A}{F}, 7\%, 60)}{100{,}000\ (\frac{A}{P}, 7\%, 60)} = \mathbf{0.67}$$

$$Alternative\ Y: \frac{B}{C} = \frac{140{,}000(\frac{A}{P}, 7\%, 60)}{100{,}000\ (\frac{A}{P}, 7\%, 60)} = \mathbf{1.40}$$

A new bridge is proposed to replace and existing bridge. The data below show current and projected costs. Assume i = 6% and n = 25

	OLD BRIDGE	NEW BRIDGE
User trips per year	2,000,000	8,000,000
User Cost ($/user trip)	0.50	0.25
Annual user cost ($/yr)	1,000,000	2,000,000
Maintenance ($/yr)	200,000	200,000
Capital Cost ($) [P]	0	5,000,000
Annual Equivalent to Capital Cost A = P(A/P, 6%, 25) = $5,000,000 (0.0782)		$391,000/yr

$$B/C = \frac{U_p - U_f}{\left(C_f - C_p\right) + \left(M_f - M_p\right)}$$

$$= \frac{(2{,}000{,}000)(\$0.50) - (8{,}000{,}000)(\$0.25)}{(\$391{,}000 - \$0) + (\$200{,}000 - \$200{,}000)}$$

$$= -2.56$$

Problem 1 - Engineering Economics

A person, now 22, expects to retire in 40 years at age 62. He anticipates that a lump sum retirement fund of $400,000 will see him nicely through his sunset years. How much should be paid annually at the end of each year for the next 40 years to accumulate a $400,000 retirement fund if interest accrues at 6% compounded on the amount paid in?

$$A = F\ (A/F,\ i,\ n)$$
$$A = \$400{,}000\ (A/F,\ 6\%,\ 40)$$
$$A = \$400{,}000\ (0.0065)$$
$$A = \$2{,}600\ /\ \text{yr}$$

Problem 2 - Cost Analysis

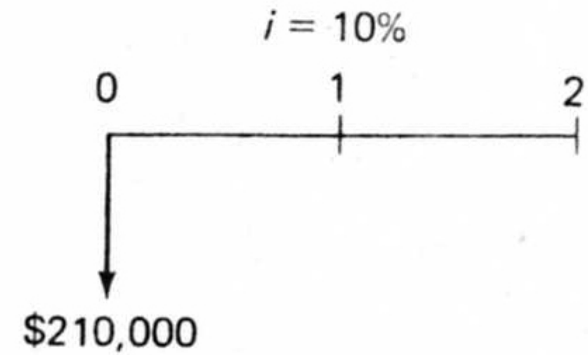
A subdivision developer asks your opinion on whether to construct roads all at once or in stages. He finds he can put in the base course and pavement complete now for $210,000. As an alternative, the county engineer will permit him to install only the base course now (cost estimated at $120,000), with the paving installed two years from now (cost estimated at $100,000). The developer lends and borrows at 10% (so use *i* = 10% for this example). Which do you recommend as the more economical alternative?

The present worth (cost in this case) of each alternative may be determined, and the alternative with the lowest cost selected.

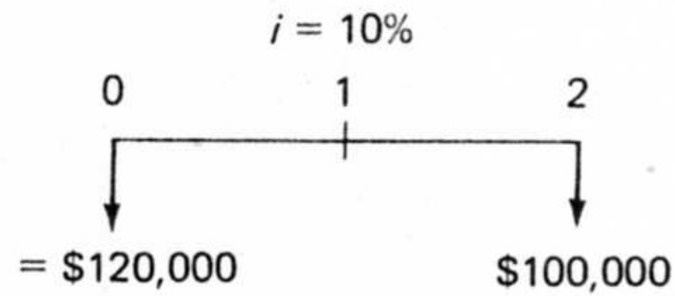
present cost of future pavement cost, $P_3 = F(P/F, i, n)$
P_3 = $100,000(*P/F*, 10%, 2) = -82,640
present cost of base course = -120,000
total present cost of alternative B = -$202,640

Problem 2 (continued) - Cost Analysis

Alternative A, base and pavement now @ $210,000



Alternative B, base now @ $120,000, pavement 2 years later @ $100,000



Problem 2 (continued)- Cost Analysis

In other words, the developer can either allocate $210,000 (alternative A) for the base and pavement installation all at once now, or allocate $202,640 (alternative B) for stage construction. If stage construction is selected (alternative B), $120,000 is spent now for the base, while $82,640 is invested at 10% to accumulate to $100,000 in two years, enough to pay for the pavement installation at that time.

Conclusion

The developer saves ($210,000 - $202,640) = $7,360 by utilizing stage construction, if all goes according to plan.

Problem 3 - Engineering Economics

A contractor buys a $10,000 truck for nothing down and $2,600 per year for five years. What is the interest rate she is paying?

$A = P\,(A/P, i, n)$, 2,600 = $10,000 ($A/P, i$, 5) ,0.2600 = ($A/P, i$, 5)

The equation with these data included appears as

$$A/P = 0.26 = \frac{i\,(1+i)^5}{(1+i)^5 - 1}$$

Since the i appears three times in the equation, it is usually easier to interpolate from the tables

i	*A/P*	*A/P*
10%	0.2638	
i		0.2600
9%	0.2571	0.2571
Subtract	0.0067	0.0029

Then interpolate

$$i = 9\% + \frac{0.0029}{0.0067} \times 1\% = 9.43 = 9.4\%$$

Problem 4 – Cost Comparison

A county engineer has a choice of paving with either Type A pavement or Type B pavement. Type A pavement has a life expectancy of 10 years, after which part of the material can be salvaged and reused. Type B pavement only lasts five years but is much less expensive. Which is the better alternative? Two sequential five year installations of Type B are compared to one 10 year life of Type A.

		Pavement cost per mile
	Type A	Type B
Cost new	$20,000	$5,000
Annual maintenance	1,000	2,000
Estimated life	10 years	5 years
Salvage value at end of life	2,500	0
i	6%	6%

Problem 4 (continued) - Engineering Economics Cost Comparison

Find *NPW* of each

Type A	Present worth
Cost new	- $20,000
Annual maintenance *P* = -1,000 x (*P/A*, 6%, 10) =	- 7,360
Less salvage *P* = 2,500 x (*P/F*, 6%, 10) =	<u>+ 1,396</u>
	- $25,964 NPW Type A
Type B	**Present worth**
Cost new first application	- $5,000
Second application *P* = -5,000 x (*P/F*, 6%, 5) =	- 3,636
Annual maintenance *P* = -2,000 x (*P/A*, 6%, 10) =	<u>- 14,720</u>
	- $23,356 NPW Type B

Problem 4 (continued) -

In this example the life of Type A pavement is ten years while the life of Type B pavement is five years. In comparing the equivalent annual costs in typical textbook problems, frequently no adjustment is made for escalating replacement costs at the end of the shorter life. This implies that the replacement cost is identical to the first cost.

Type A

A_1 = annualized purchase cost = - 20,000 (*A/P*, 6%, 10) = -$2,718

A_2 = annualized maintenance cost = - 1,000

A_3 = annualized salvage = + 2,500 (*A/F*, 6%, 10)

= + 190

Net annual worth of Type A = $A_1 + A_2 + A_3$ = -$3,528

Type B

A_1 = annualized purchase cost = - 5,000 (*A/P*, 6%, 5) = -$1,187

A_2 = annualized maintenance cost = - 2,000

net annual worth of Type B = $A_1 + A_2$ = -$3,187

The answer is to choose Type B because it has the lowest net annual worth of costs.

Problem 5 - Engineering Economics

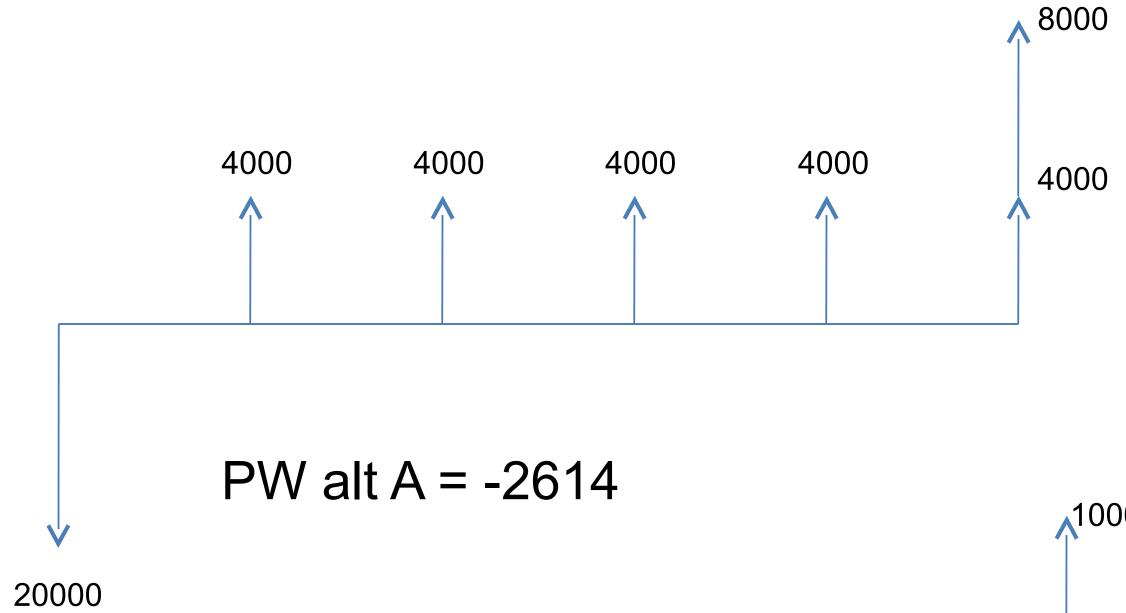
An equipment rental firm has $30,000 to invest and is considering the addition of a backhoe to its rental inventory. If the firm uses a 15% return on investment which (if either) of the following alternatives should be selected?

	Alternative A	Alternative B
First cost	- $20,000	- $30,000
Salvage value, 5 years	+ 8,000	+ 10,000
Annual maintenance	- 5,000	- 6,000
Annual rental income	+ 9,000	+ 14,000

Solve by present worth analysis.

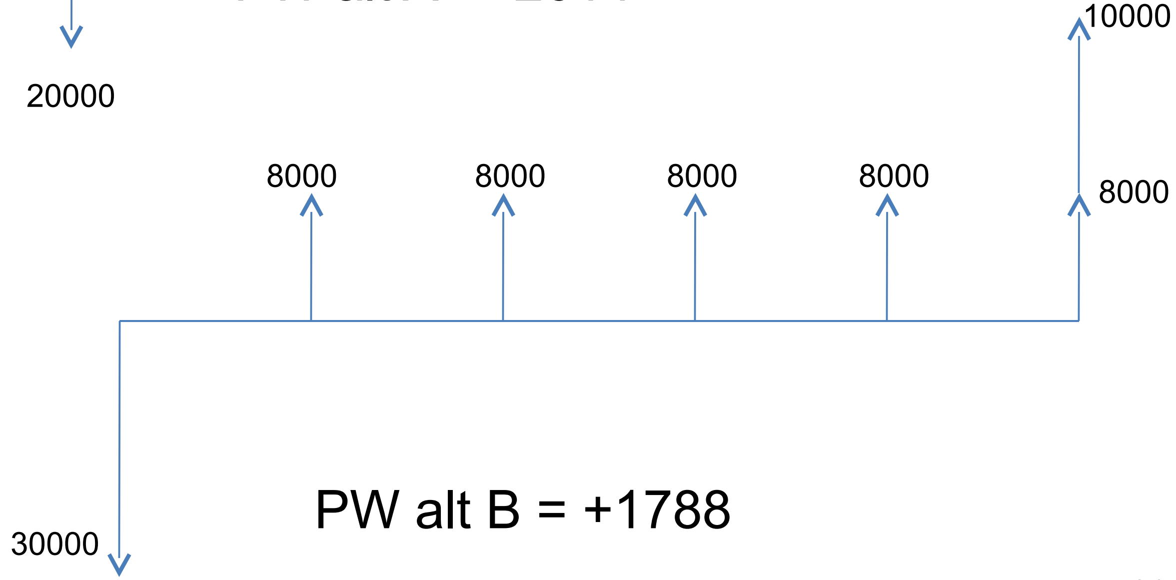
Problem 5 (continued) - Engineering Economics

Alternative A Cash Flow



PW alt A = -2614

Alternative B Cash Flow



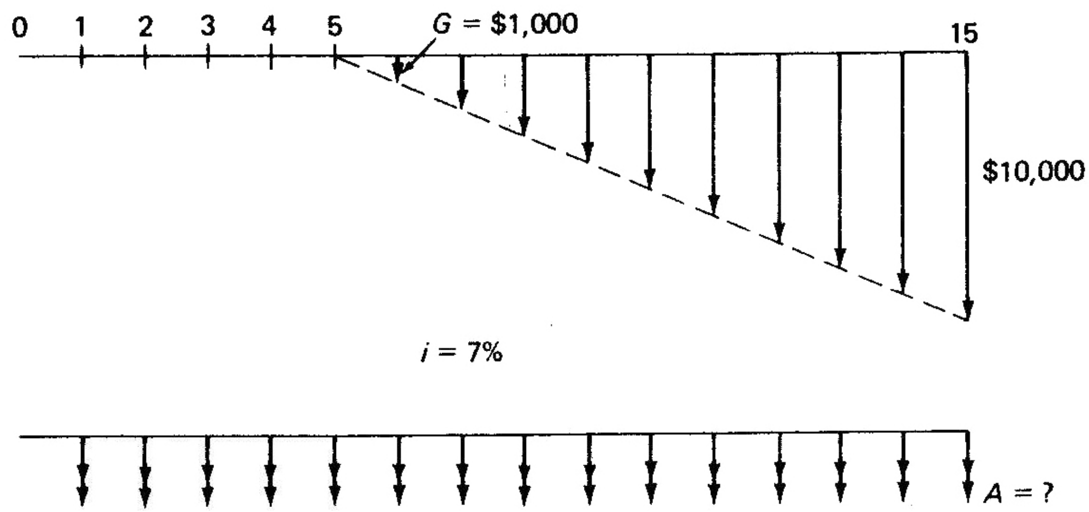
PW alt B = +1788

Problem 6 - Engineering Economics

An airport runway is expected to incur no maintenance costs for the first five years of its life. In year six maintenance should cost $1,000, in year seven maintenance should cost $2,000, and each year thereafter until resurfacing the runway is expected to increase in maintenance costs by $1,000 per year.

If resurfacing is expected after 15 years of service, what equivalent uniform annual maintenance cost is incurred if *i* = 7 percent?

A = 1,000 (F/G, 7%, 11) (A/F, 7%, 15)
= $2,719 per year equivalent annual cost



Problem 7 - Alternative Cost Analysis

Three different artificial turfs are available for covering the playing field in a college football stadium. The costs associated with each are tabulated as follows (assume *i* = 15%).

	Turf King	Turf Ease	Turf Magic
Cost new (installed) ($)	+ 540,000	+ 608,000	+ 467,000
Annual maintenance cost ($)	- 2,300	- 1,600	- 2,500
Expected life (yr)	12	15	10
Salvage value ($)	- 54,000	- 57,000	- 40,000

Since different lives are involved the lowest common multiple of 10, 12, and 15 would be 60. Therefore, to simplify the calculations, use the *AW* technique.

NAW (King) = -540,000 (*A/P*, 15%, 12) – 2,300
+ 54,000(*A/F*, 15%, 12) = -$100,070

NAW (Ease) = -608,000(*A/P*, 15%, 15) - 1,600
+ 57,000(*A/F*, 15%, 15) = -$104,370

NAW (Magic)=-467,000(*A/P*, 15%, 10) – 2,500
+ 40,000(*A/F*, 15%, 10) = -$93,600

Problem 8 - Alternative Cost Analysis

Two roofs are under consideration for a building needed for 20 years. Their anticipated costs and lives are:

	Roof C	Roof D
Cost new ($)	50,000	25,000
Replacement cost ($)	-	Rise 10% / yr
Life of roof (yr)	20	10
Salvage value @ 20 yr ($)	0	0
Interest rate (%)	12	12

Roof C

A1 = -$50,000(A/P, 12%, 20)
= -$ 6,695 / yr

Roof D

A2 = -$25,000(A/P, 12%, 20)
= -$ 3,348 / yr

A3 = -$25,000(F/P, 10%, 10) (P/F, 12%, 10) (A/P, 12%, 20)= -$2,796 /yr

Net annual worth roof D = A2 + A3 = -$6, 144 / yr

Problem 9 - Benefit Cost Analysis

A firm is considering three alternatives as part of a production program. They are

	A	B	C
Installed cost ($)	10,000	20,000	15,000
Uniform annual benefit ($)	1,265	1,890	1,530
Useful life (yr)	10	20	30

Assuming a minimum attractive rate of return of 6%, which alternative, if any, would you select, based on *B/C* ratio method?