P.E. Civil Exam Review: Hydraulics

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ASCE | KNOWLEDGE & LEARNING

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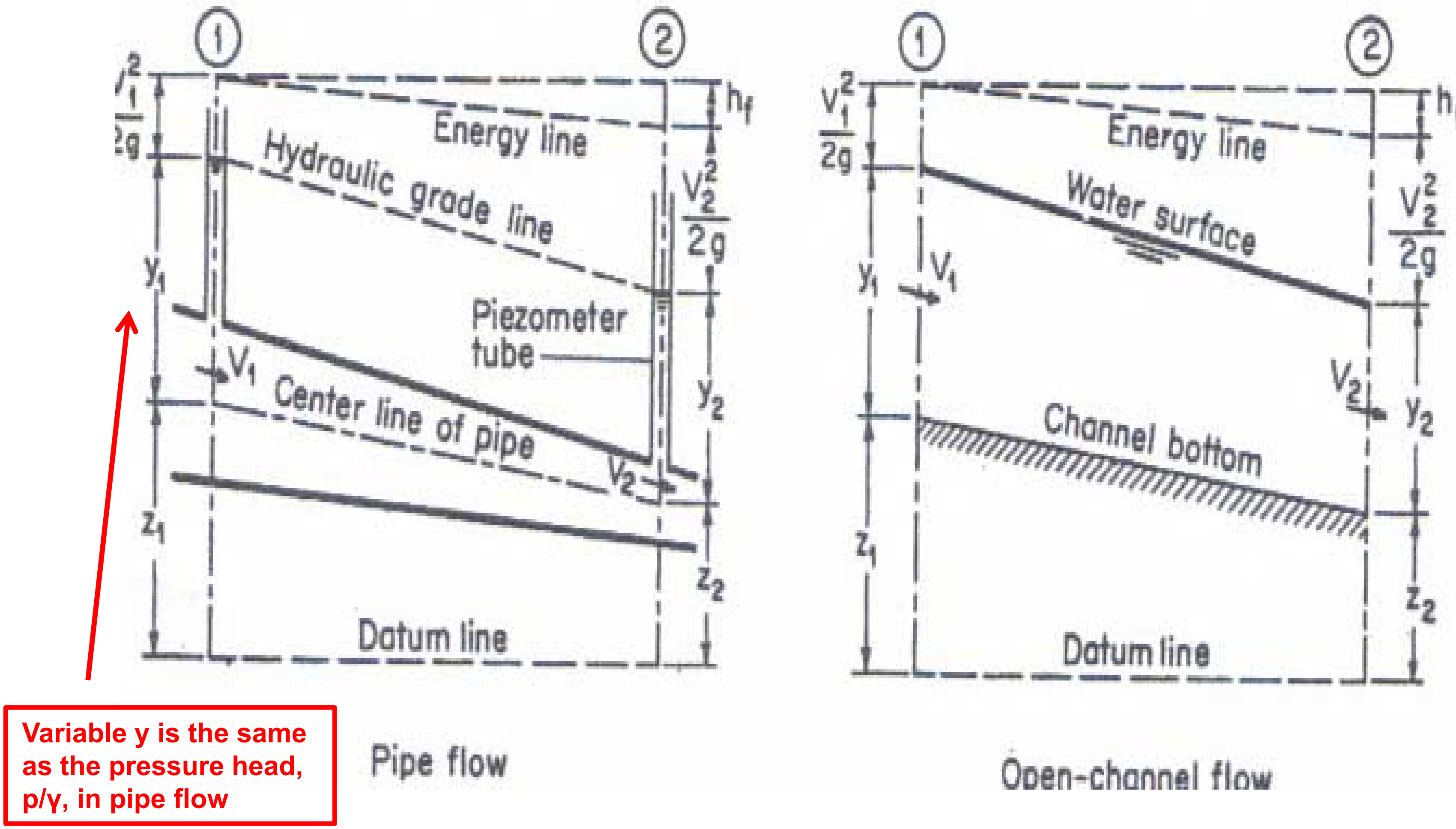
Topics:

Table: US and SI Units

Fundamental	Variable	US Units	SI Units
	Mass, m	--------	Kilogram (kg)
	Force, F	pound (lb_f) (1 lb_f = 1 slug x 1 ft/s^2)	---------
	Length, L	feet (ft)	meter (m)
	Time, t	second(s)	second(s)
Derived	Mass, m	slug	--------
	Force, F	---------	newton (N) (1 N = 1 kg x 1 m/s^2)
	Density, ρ	$slugs/ft^3$	kg/m^3
	Specific Weight, Y	lb_f/ft^3	N/m^3
	Pressure, p	lb_f/ft^2 (psf)	N/m^2 or Pascal (Pa)
	Pressure Head, h = (p/γ)	feet (ft)	meter (m)
	Energy or Work, E	lb_f-ft	N-m or Joule(J)
	Power, P = E/t	lb_f-ft/s Note: I HP=550lb-ft/s	N-m/s or J/s (Watt, W)
	Dynamic Viscosity, μ	lb_f- s/ft^2	$N\text{-}s/m^2$
	Kinematic Viscosity, η = μ/ρ	ft^2/s	m^2/s

Comparison Between Full-Pipe Flow and Open Channel Flow

(Chow 1959)



Commonly Used Cross-Sections

Full Pipe Flow:

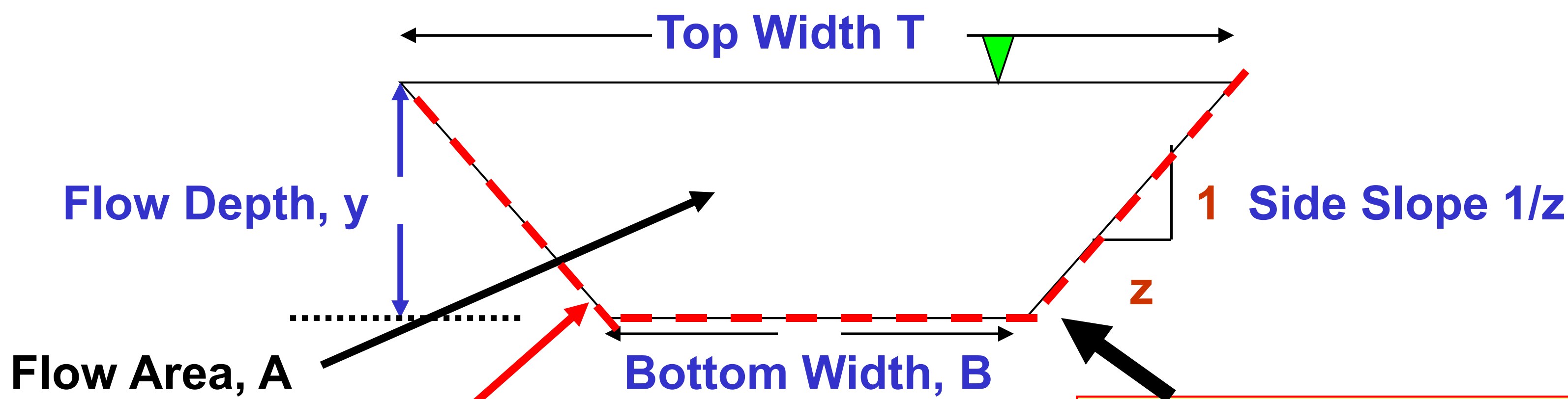
1. Circular – Most widely used section:
   - Water supply networks (pressurized/full flow).
   - Urban storm sewer systems (pressurized when surcharged. Open channel flow otherwise).

Open Channels:

1. Trapezoidal
2. Rectangular (trapezoidal with side slopes vertical)
3. Triangular (zero bottom width).

Cross Sectional Properties – Trapezoidal Channel Section

Widely Used Hydraulic Variables:



- Wetted Perimeter, P (shown in dashed line)
- Hydraulic Radius, R = A/P
- Hydraulic Depth, D = A/T:

Special cases:

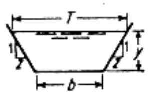
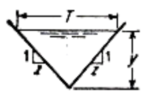
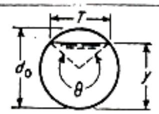
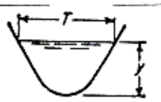
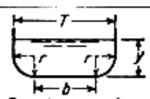
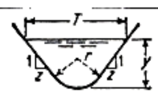
1) Rectangular Section:
   A = By; P = B + 2y;
   R = By/(B+2y);
   D = A/T = By/B = y

2) Wide Rectangular Section:
   $B \geq 20y$;
   Hydraulic Radius, R = y

Important Note:

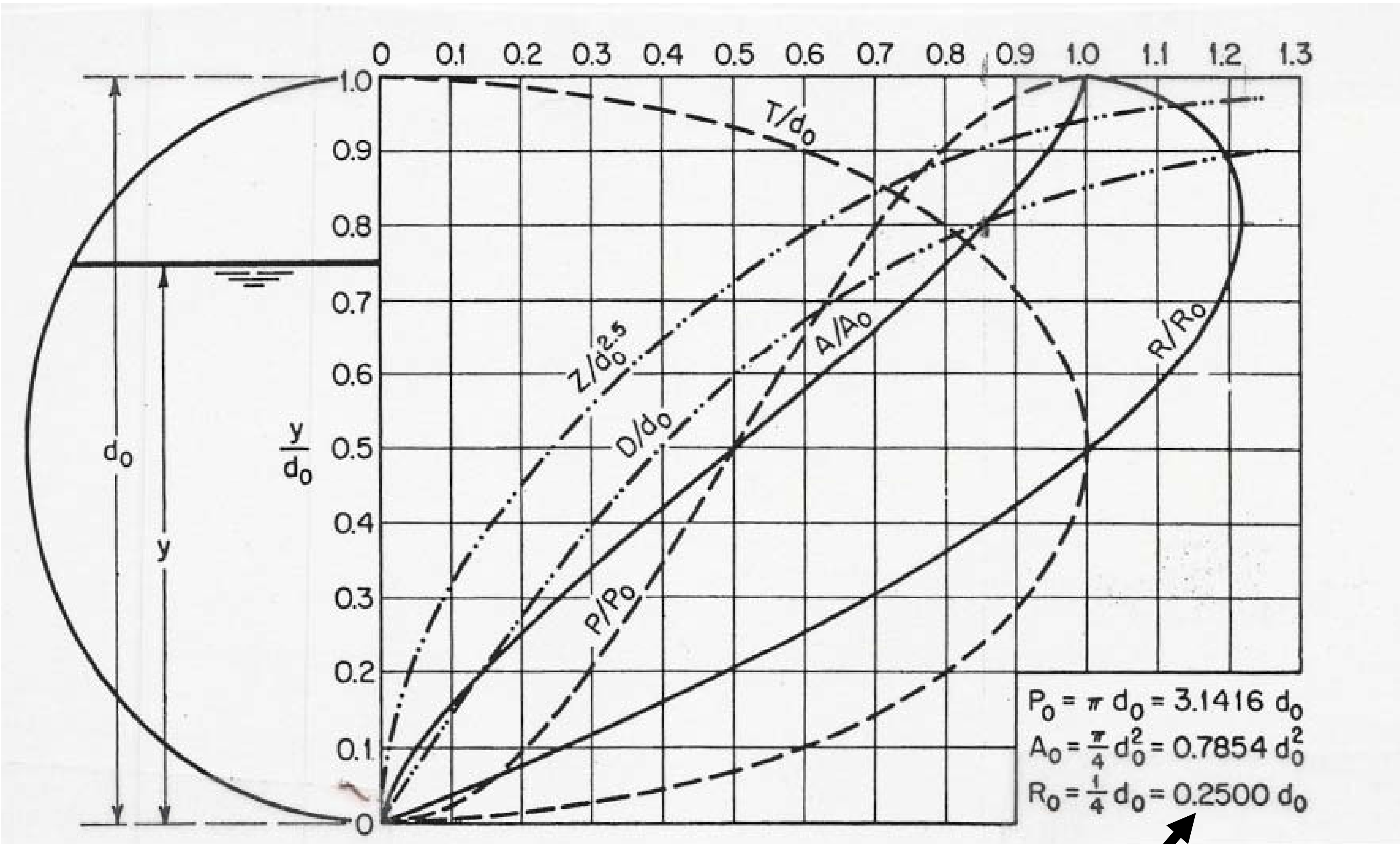
- **Most channels require a formulae to compute cross-sectional properties (Slide 7).**
- **Circular sections require the use of a special nomograph (Slide 8)**

Table: Geometric Elements of Channel Sections (Chow, 1959)

Section	Area A	Wetted perimeter P	Hydraulic radius R	Top width T	Hydraulic depth D	Section factor Z
 Rectangle	by	$b + 2y$	$\frac{by}{b + 2y}$	b	y	$by^{1.5}$
 Trapezoid	$(b + zy)y$	$b + 2y\sqrt{1 + z^2}$	$\frac{(b + zy)y}{b + 2y\sqrt{1 + z^2}}$	$b + 2zy$	$\frac{(b + zy)y}{b + 2zy}$	$\frac{[(b + zy)y]^{1.5}}{\sqrt{b + 2zy}}$
 Triangle	zy^2	$2y\sqrt{1 + z^2}$	$\frac{zy}{2\sqrt{1 + z^2}}$	$2zy$	$\frac{1}{2}y$	$\frac{\sqrt{2}}{2} zy^{2.5}$
 Circle	$\frac{1}{8}(\theta - \sin\theta)d_0^2$	$\frac{1}{2}\theta d_0$	$\frac{1}{4}\left(1 - \frac{\sin\theta}{\theta}\right) d_0$	$(\sin \frac{1}{2}\theta)d_0$ or $2\sqrt{y(d_0 - y)}$	$\frac{1}{8}\left(\frac{\theta - \sin\theta}{\sin\frac{1}{2}\theta}\right) d_0$	$\frac{\sqrt{2}}{32} \frac{(\theta - \sin\theta)^{1.5}}{(\sin\frac{1}{2}\theta)^{0.5}} d_0^{2.5}$
 Parabola	$\frac{2}{3}Ty$	$T + \frac{8}{3}\frac{y^2}{T}$ *	$\frac{2T^2y}{3T^2 + 8y^2}$ *	$\frac{3}{2}\frac{A}{y}$	$\frac{2}{3}y$	$\frac{2}{9}\sqrt{6}\, Ty^{1.5}$
 Round-cornered rectangle ($y > r$)	$\left(\frac{\pi}{2} - 2\right) r^2 + (b + 2r)y$	$(\pi - 2)r + b + 2y$	$\frac{(\pi/2 - 2)r^2 + (b + 2r)y}{(\pi - 2)r + b + 2y}$	$b + 2r$	$\frac{(\pi/2 - 2)r^2}{b + 2r} + y$	$\frac{[(\pi/2 - 2)r^2 + (b + 2r)y]^{1.5}}{\sqrt{b + 2r}}$
 Round-bottomed triangle	$\frac{T^2}{4z} - \frac{r^2}{z}(1 - z\cot^{-1} z)$	$\frac{T}{z}\sqrt{1 + z^2} - \frac{2r}{z}(1 - z\cot^{-1} z)$	$\frac{A}{P}$	$2[z(y - r) + r\sqrt{1 + z^2}]$	$\frac{A}{T}$	$A\sqrt{\frac{A}{T}}$

* Satisfactory approximation for the interval $0 < x \leq 1$, where $x = 4y/T$. When $x > 1$, use the exact expression $P = (T/2)[\sqrt{1 + x^2} + 1/x \ln(x + \sqrt{1 + x^2})]$.

Nomograph: Cross Sectional Properties of a Circular Section (Chow, 1959)



Cross-Sectional Properties for Full Circular Pipe Flow

Problem 1(a): Computation of Cross Sectional Elements in Channels

a) Trapezoidal Channel Section:

Given: A trapezoidal channel (z = 2, B = 15 ft) carries a flow at a depth of y = 5 feet.

Compute using the Geometric Elements Table from Slide 7:

1) Flow area A in sq. ft :	a) 100	b) 170	c) 125	d) 75
2) Wetted Perimeter, P in ft :	a) 55.1	b) 37.4	c) 25.5	d) 44.2
3) Hydraulics Radius, R in ft :	a) 2.7	b) 2.8	c) 2.0	d) 3.4
4) Top width, T in ft:	a) 35	b) 55	c) 20	d) 15

Problem 1(b): Computation of Cross Sectional Elements in Channels

b) Circular Channel Section:

Given: A circular culvert (diameter, d_0 = 4 ft.) carries a flow at a depth of y = 3 feet.

Note: $A_0 = 3.14 \times 4^2/4 = 12.6\ ft^2$; $P_0 = 3.14 \times 4.0 = 12.6$ ft (full pipe flow); $R_0 = A_0/P_0 = 1.0$

Compute using the nomograph from Slide 8:

1) Flow area A in sq. ft: a) 12.6 b) 10.3 c) 8.5 d) 15.2

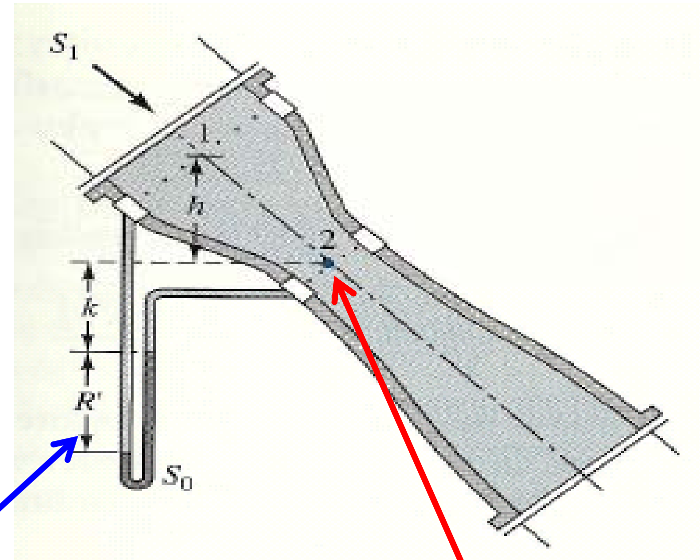
2) Wetted Perimeter, P in ft: a) 12.6 b) 9.5 c) 8.5 d) 6.3

3) Hydraulic Radius, R in ft: a) 1.0 b) 0.9 c) 1.5 d) 1.2

FLOW MEASUREMENT

Pipe Flow:

- **Venturi meter**



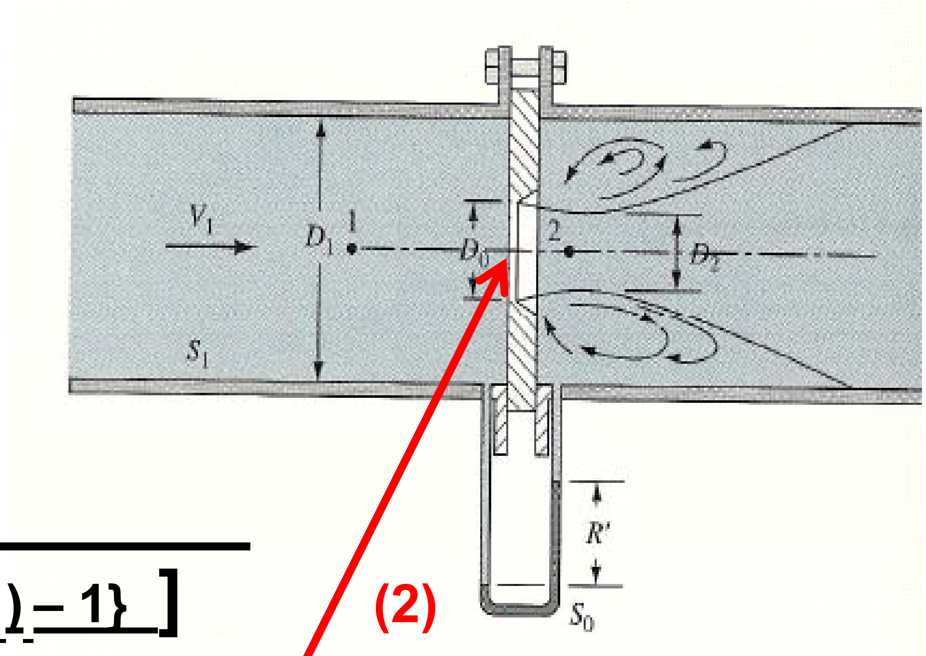
Discharge Equation:

$$Q = C_d A_2 \sqrt{\frac{[2g\, R' \{(S_0 / S_1) - 1\}]}{[1 - (D_2/D_1)^4]}} \quad (1)$$

- Q = flow rate in cfs (or m^3/s)
- R' = gage difference between 1 & 2(pressure drop) in ft (or m)
- A_2 = Area of venturi throat
- D_1 , D_2 = Pipe diameters at 1 and 2 in ft (or m)
- S_0, S_1 = Specific gravity of fluids in manometer and pipe, respectively
  (Note: For water SG = 1 and specific weight, γ = 62.4 lb/ft³)
- C_d = Discharge Coefficient (typical range 0.95-0.99)

Note: The flow equation above is only dependent on the gage difference R' and not on the orientation (horizontal , vertical or inclined) of the venturi meter. Equation is independent of h.

Orifice Meter



(2)

Discharge Equation:

$$Q = C_d A_0 \sqrt{\frac{[\,2gR'\{(S_0/S_1) - 1\}\,]}{[1 - (D_2/D_1)^4]}}$$

where,

- Q = flow rate in cfs (or m^3/s)
- A_0 = Area of orifice opening in ft^2 (or m^2)
- D_1 , D_2 = Pipe diameters at 1 and 2 in ft (or m)
- S_0, S_1 = Specific gravity of fluids in manometer and pipe, respectively (Note: For water SG = 1 and specific weight, γ = 62.4 lb/ft^3)
- C_d = Discharge Coefficient (typical range 0.60-0.82)
- R' = Pressure Drop (gage difference) in ft (or m)

Example 1: Venturimeter

Problem:

Determine the flow of water (specific gravity, S_1 =1) through a 24 inch pipe diameter (D_1) using a venturi meter with a 6 inch throat diameter (D_2). The gage difference, R' in the manometer is 11.8 inches of Mercury (Hg). Assume a coefficient of discharge C_d = 0.95 and specific gravity of mercury, S_0 = 13.6

Flow Equation:

$$Q = C_d A_2 \sqrt{\frac{[2g R' \{(S_0/S_1) - 1\}]}{[1 - (D_2/D_1)^4]}}$$

Note: Pressure head drop
$\Delta h = \{h + (p_1 - p_2)/\gamma\} = [R' \{(S_0/S_1) - 1\}]$

h = height and p is pressure difference between points 1 and 2 (see figure in Slide 11)

Solution:

$A_2 = 3.14(6/12)^2/4 = 0.196\ ft^2$

Pressure head drop $\Delta h = [R' \{(S_0/S_1) - 1\}]$

$= (11.8/12) \times \{(13.6/1.0) - 1\} =$ 12.39 ft of water

(or pressure drop = 12.39 x 62.4 = 773.1 psf)

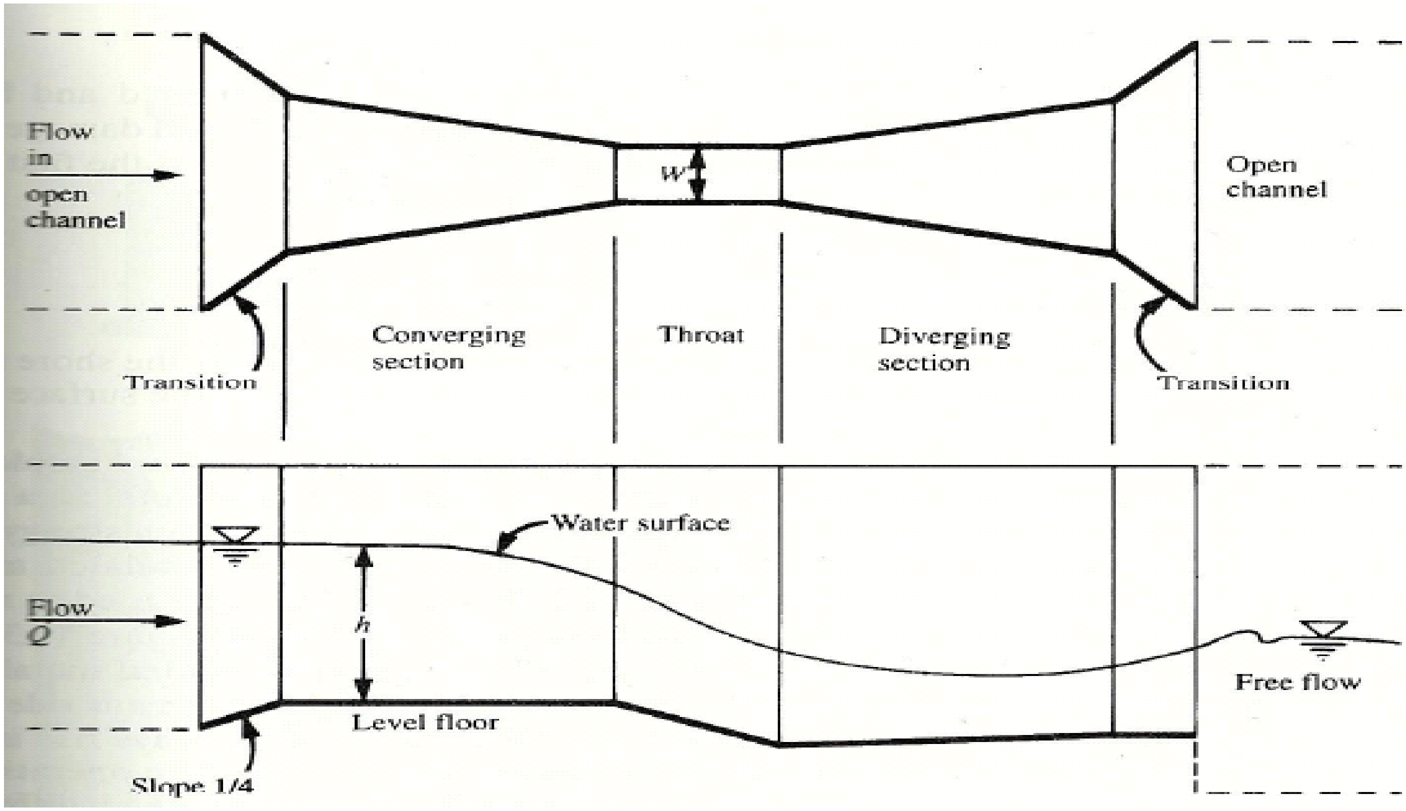
Flow rate $Q = 0.95 \times 0.196 \times \{(2 \times 32.2 \times 12.39)/(1 - (6/24)^4\}^{0.5}$

= 5.27 cfs.

Note: 12.39 ft is = $R'\{(S_0/S_1)-1\}$

Open Channel Flow Measurement:

Parshall Flume (source: Viessman et al, 2005)



Flow Equation : $Q = 4Wh^{1.522\,W^{0.026}}$ **(3)**

where Q = flow in cfs, W = throat width in feet and h = upper head with respect to the bottom of the flume in feet.

Example 2: Parshall Flume

Problem:

Calculate the wastewater flow through a Parshall flume with a throat width W = 5 feet and a free flowing upper head, h = 1.5 feet

Solution:

Flow $Q = 4Wh^{1.522\ w^{0.026}}$

$= 4 \times 5 \times 1.5^{1.522 \times 5^{0.026}}$

$= 38.1$ **cfs = 24.6 MGD**

FLUID DYNAMICS

a) Law of Conservation of Mass - Equation of Continuity

$\sum$ Inflow Mass = $\sum$ Outflow Mass.

- **For steady flow of incompressible fluid (density constant) in a full pipe or channel flow:**

$$Q = V_1 A_1 = V_2 A_2 \qquad (4)$$

where,

Q = volumetric flow rate (cfs or m^3/s)
V = average velocity at a flow section (ft/s or m/s)
A = fluid flow area at the flow section (ft^2 or m^2)

Example 3: Application of Equation of Continuity: Full Pipe Flow

Given: Section 1: $V_1 = 3.0$ ft/s and $d_1 = 2.0$ ft;
Section 2: $d_2 = 3.0$ ft.

Find the discharge and velocity at section 2.

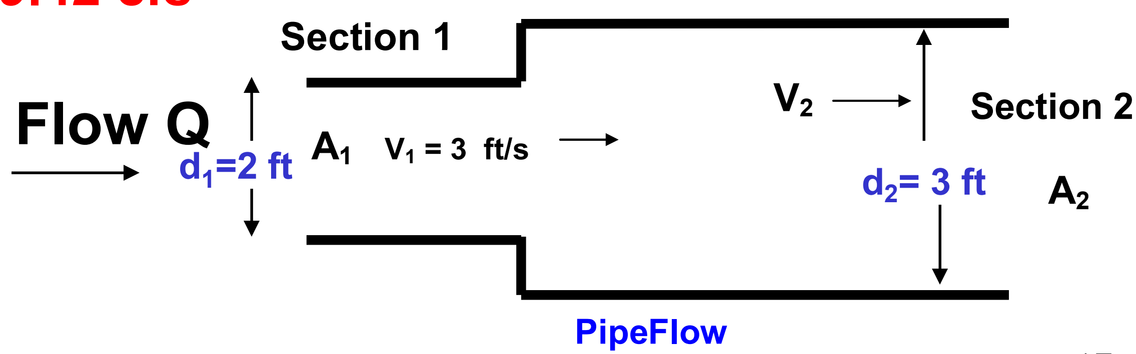
$$Q = V_1 \times A_1 = V_2 \times A_2$$

$$Q = 3.0\,(3.14(2)^2/4) = 9.42 \text{ ft}^3\text{/s or cfs}$$

and

$$V_2 = Q/A_2 = 9.42/(3.14 \times 3^2/4) = 1.33 \text{ ft/s}$$

$$Q_2 = Q = 9.42 \text{ cfs}$$

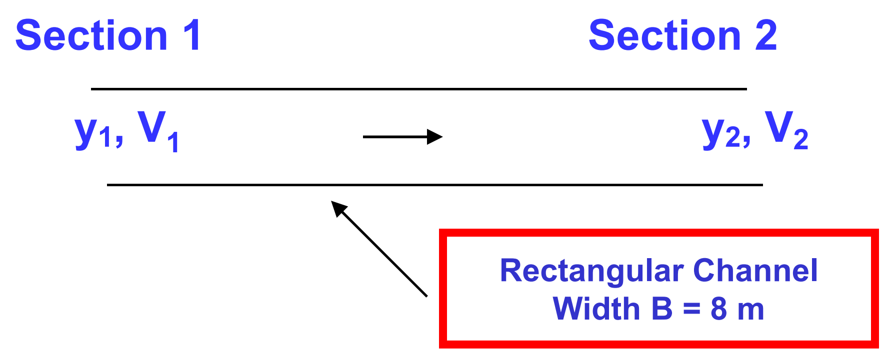


Problem 2: Application of Continuity – Open Channel

Given: An 8-m wide rectangular channel carries a flow under the following conditions:

Section 1: Velocity V_1= 4 m/s; depth, flow depth, y_1 = 4m.

Section 2: Flow depth, y_2 = 3.2m.



Compute:

1. The discharge per unit width q = Q/B (m^3/s/m):

   a) 20 b) 15 c) 12 d) 16

2. The velocity V_2 (in m/s) at section 2:

   a) 6 b) 5 c) 4 d) 8

b) Law of Conservation of Energy for Steady Flow of an Incompressible Fluid

Bernoulli Equation (or Energy Equation): Full Pipe Flow

Total energy head at any point is:

$$H = p/\gamma + z + V^2/2g - h_L + h_p = \text{constant} \quad (5)$$

where,
- H = total energy or head
- p/γ = pressure head
- z = potential energy or elevation head
- $V^2/2g$ = kinetic energy or velocity head
- h_L = sum of frictional (h_f) and minor head losses (h_m)
- h_p = energy added or subtracted from the fluid. (positive for a pump or negative for a turbine).

All quantities expressed as head in feet or meters.

b) Law of Conservation of Energy (Cont..)

Bernoulli Equation (or Energy Equation):

Open Channel Flow

- At a cross-section:

$$H = y + z + \alpha V^2/2g \qquad (6)$$

H = Total energy head (ft or m);

y = Flow depth (ft or m);

z = Elevation above datum (ft or m);

V = Average Velocity (ft/s or m/s);

α = Kinetic Energy Correction factor (normal range 1.0-1.3).

- Between two cross-sections (1-upstream and 2-downstream):

$$H_1 - h_L = H_2 \qquad (7)$$

h_L = Total Head Loss

Head Expressed as Energy

Rate of energy (fluid power) of any term in the Bernoulli Equation:

Power, P **= γ Q h** **(8)** ←

Units
- SI- N.m/s or J/s or W
- US: lb-ft/s
- Horse Power, HP

Example:

pressure head, p/γ = 50 m (490 KPa)
discharge Q = 0.6 m^3/s

Water power, P = 9806x0.6x50.0
= 294,180 N.m/s or J/s or W
= 294,180/776 = 379 HP

Note: **1 Horse Power, HP = 550 lb-ft/s or 776 N.m/s**

Example 4: Calculation of Head Terms - Bernoulli Equation

Given: Flow in a 24 inch pipe Q = 20.0 cfs.
Pressure, p = 60 psi (gage)
Elevation z = 500 feet.

Calculate the total head H expressed in feet, lbf.ft/s and horse power.

1. Pressure Head, p/γ = (60.0x144)/62.4 = <u>138.5 feet.</u>
2. Elevation Head, z = <u>500 feet</u>
3. Average Velocity, V = Q/A = 20.0 / $(3.14(24/12)^2/4)$ = 6.4 ft/s
4. Velocity Head, $V^2/2g = (6.4)^2/(2\text{x}32.2)$ = <u>0.64 feet</u>
5. Total Head, H = $p/\gamma + z + V^2/2g$ = 138.5 + 500.0 + 0.64 = <u>639.14 feet</u>
6. Total Rate of Energy or Power = γ Q H = 62.4(20.0)(639.14)

   = 797,646.72 lbf.ft/s = (797,646.72/550) = <u>1450 HP.</u>

Grade Lines
Full Pipe and Open Channel Flow

These lines can be plotted with respect to the centerline of a pipe or channel bottom that is located at height z above an arbitrary datum

a) Hydraulic Grade Line (HGL)

- **Obtained by plotting the peizometric head:**

$$(p/\gamma + z) \qquad (9)$$

b) Energy Grade Line (EGL)

- **Obtained by plotting the total energy head:**

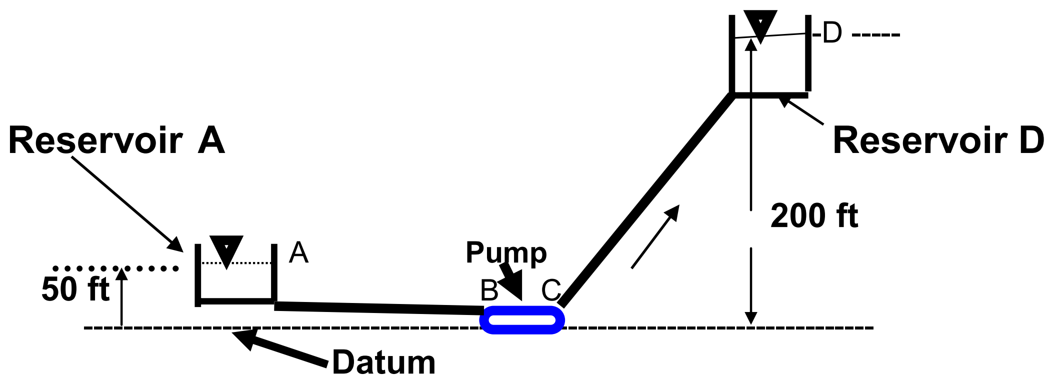
$$H = p/\gamma + z + V^2/2g \qquad (10)$$

Note: For Open Channel Flow pressure head, p/γ = *flow depth, y*

Example 5: Application of Bernoulli Equation for Full Pipe Flow: Pumping Between Reservoirs (Streeter, 1979)

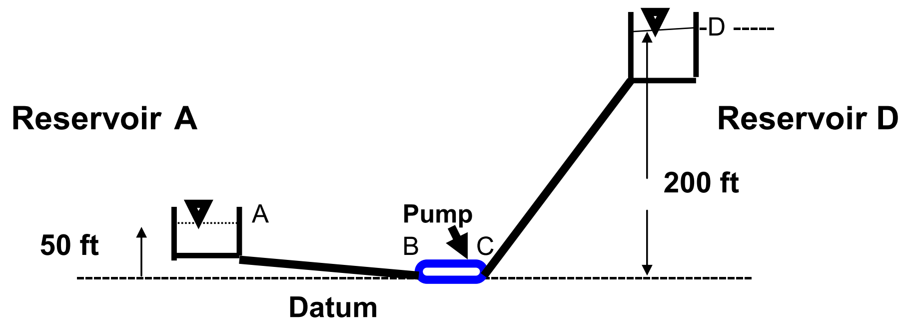
A pump BC delivers 5.62 cfs. of oil (specific gravity = 0.762) from Reservoir A to Reservoir D. Loss of energy from A to B is 8.25ft or lb-ft/s / lb/s) and from C to D is 21.75 or ft (lb-ft/s / lb/s).

Determine the head delivered by the pump to the water system and its horse power. Plot the EGL.



Pumping Oil Between Reservoir A to D.

Example 5: Bernoulli Equation for Full Pipe Flow: Pumping Between Reservoirs (cont.)



Pumping Oil Between Reservoir A to D.

Solution

Bernoulli Equation from A to D (with the datum at BC) with gage pressures at A and D equal to zero (free water surface in the reservoirs) gives:

$$(p_A/\gamma + V_A^2/2g + z_A) + H_{pump} - H_{loss} = (p_D/\gamma + V_D^2/2g + z_D) \qquad (11)$$

or

$$(0 + \text{negl.} + 50.0) + H_{pump} - (8.25+21.75) = (0 + \text{negl.} + 200)$$

- **Pump Head = H_{pump} = 180 ft. (or lb.ft/s/lb/s)**
- **Pump Power = $\gamma\, Q\, H_{pump}$ = (0.762x62.4)x (5.62)x(180) = 48,100.366 lb-ft/s**
- **Pump Horse Power (or HP) = 48,100.366/550 = 87.46 HP**

Example 5: Computation of EGL (cont.)

•EGL at A = 50.0 feet above datum at BC.

•Energy loss from A to B is 8.25 ft.

•EGL at B (before pump) = 41.75 ft.

•Pump adds 180 ft. of energy

•EGL at C = 221.75 ft.

•Loss of energy from C to D is 21.75 ft.

•EGL at D = 221.75 – 21.75 = 200.0 ft above datum at BC.

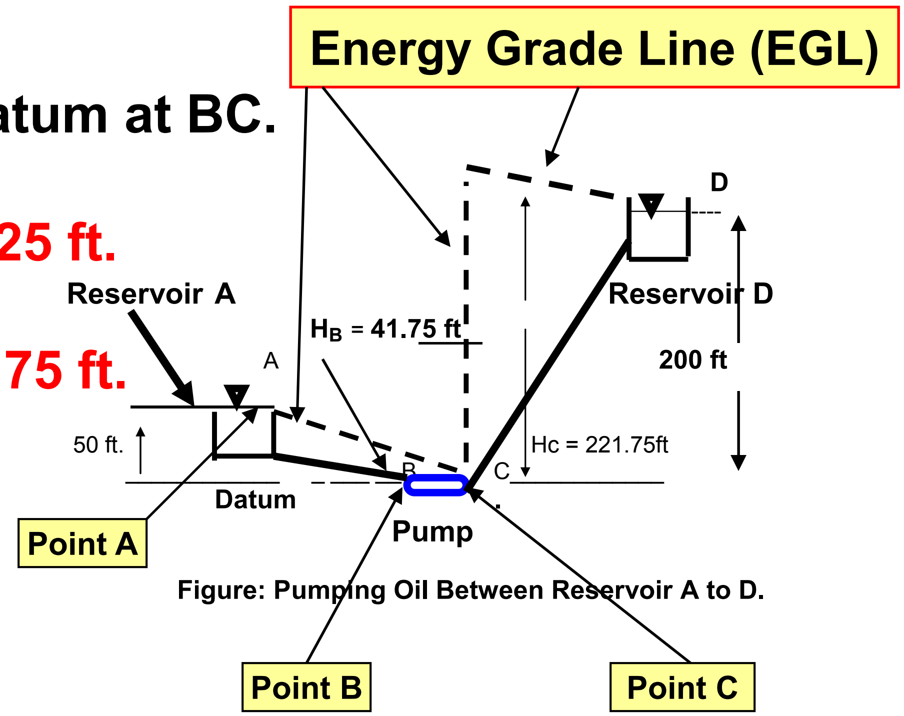
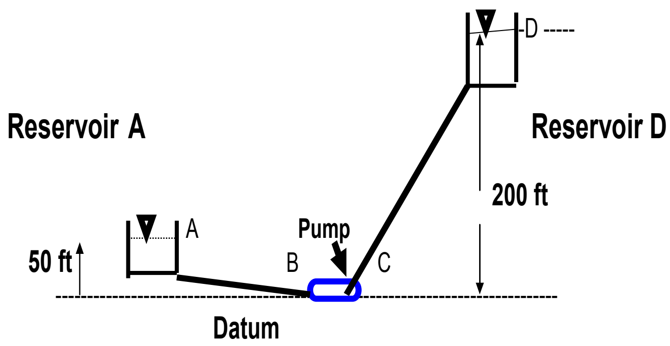


Figure: Pumping Oil Between Reservoir A to D.

Problem 3: Pumping Problem

Given:

A pump BC delivers 5.62 cfs. of oil (specific gravity = 0.762) from Reservoir A to Reservoir D. Loss of energy from A to B is 8.25ft or lb-ft/s / lb/s) and from C to D is 21.75 or ft (lb-ft/s / lb/s).



Pumping Oil Between Reservoir A to D.

Determine:

1) Head delivered by the pump in feet <u>if all losses are neglected</u> is:

a) 180 b) 165 c) 150 d) 205

2) The horse power of the pump is <u>if all losses are neglected</u> is:

a) 87.5 b) 60.5 c) 50.0 d) 72.9

3) The pressure at the intake point B of the pump in psi is:

a) 25.5 b) 13.8 c) 30.5 d) 45.6

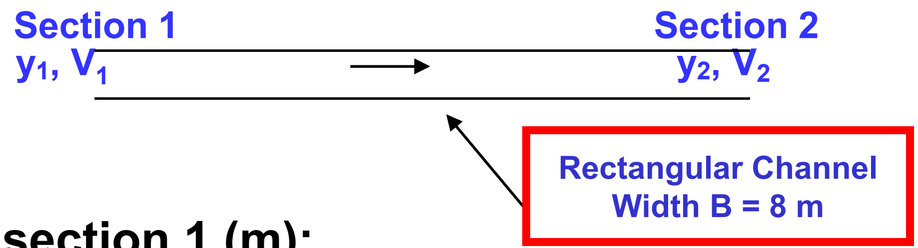
4) The pressure at the delivery end C of the pump in psi is:

a) 85.5 b) 60.5 c) 50.5 d) 73.2

Problem 4: Application of Bernoulli Equation: Open Channel

Given: An 8-m wide rectangular channel carries a flow under the following conditions:

Section 1: Velocity V_1= 4 m/s; depth, flow depth, y_1 = 4m, elev. z_1 =100m.
Section 2: Flow depth, y_2 = 3.2m; elev. z_2 = 100.1m
Assume K.E. correction factor α = 1.



Compute:

1. Total energy head, H_1 at section 1 (m):

   a) 50.5   b) 120.2   c) 104.8   d) 110.5

2. Total energy head, H_2 at section 2 (in m):

   a) 66.5   b) 104.8   c) 118   d) 104.6

→ 3. Energy Loss, h_l between sections 1 and 2 (m):

   a) 0.5   b) 0.2   c) 1.2

PIPE HYDRAULICS

FRICTION AND MINOR LOSSES

- For real fluid flow the **total head loss term, h_L** must be specified.
- Total head loss includes **two types** of losses:

  1. Friction loss, h_f, and
  2. Minor Loss, h_m.

Total Head Loss, $h_L = (h_f + h_m)$. **(12)**

Generalized Form of Frictional Formulas

- **All frictional equations can be written in general form as:**

$$h_f = K Q^x \qquad (13)$$

- **This form of the head loss equation is very convenient if K and x can be considered constant.**

- **K and x are defined as follows for each of the equations.**

Generalized Form of Frictional Formulas – Darcy-Weisbach

1) Darcy-Weisbach (full pipe flow): $h_f = K Q^x$

- US Units with L = feet; D =feet; Q = cfs:

$$K = f L / (39.69 D^5) \qquad (14a)$$

$$x = 2.0 \qquad (14b)$$

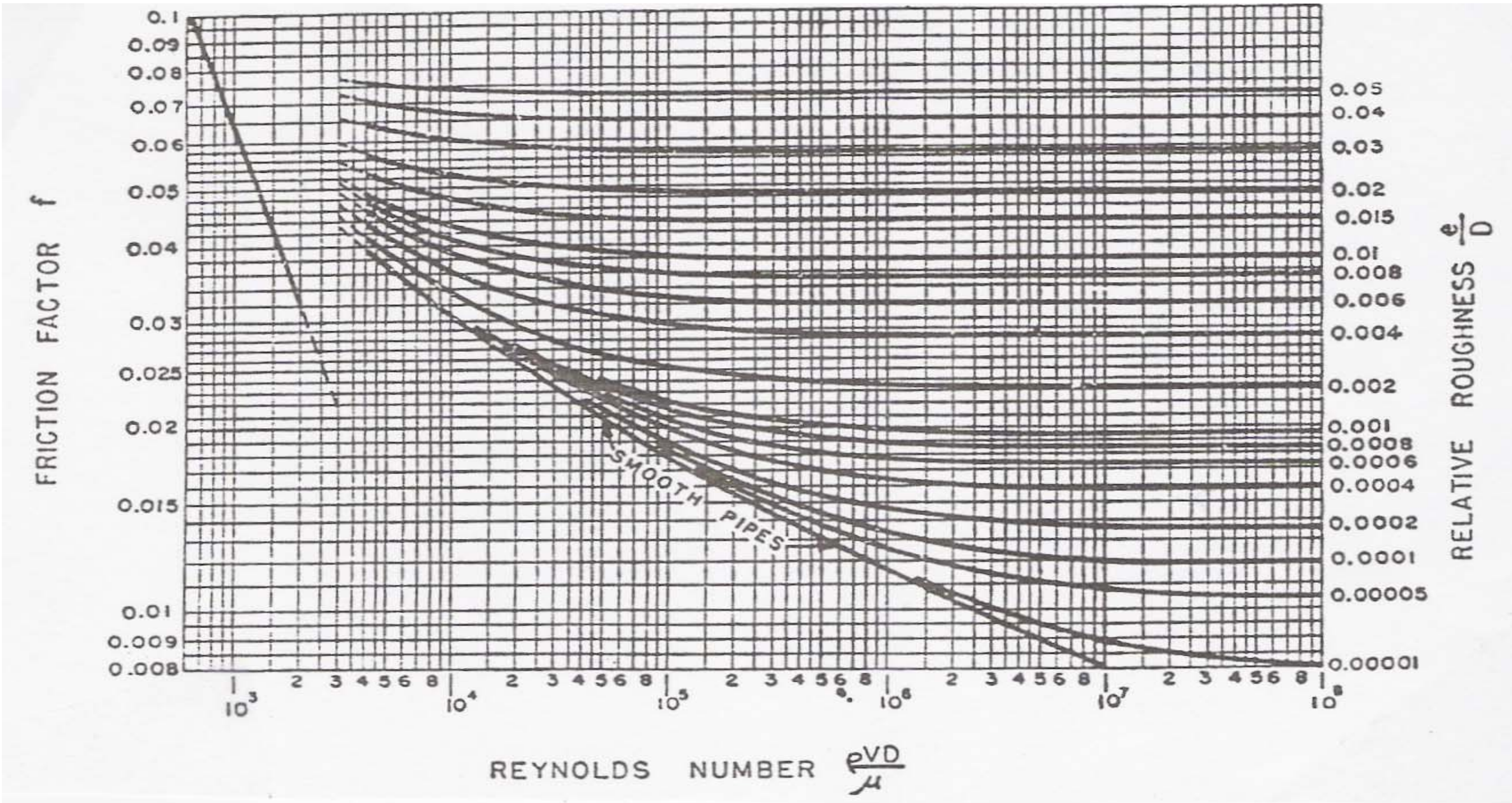
f is pipe friction factor obtained from Stanton-Moody's diagram

- SI Units with L = meters; D = meters; Q = m^3/s:

$$K = f L / (12.09 D^5) \qquad (14c)$$

$$x = 2.0 \qquad (14d)$$

Figure: Stanton/Moody Diagram for Friction Factors in Circular Pipes (Streeter, 1979)



Generalized Form of Frictional Formulas - Hazen-Williams

2) Hazen-Williams (full pipe flow): $h_f = K Q^x$

- U.S. Units with L = feet; D =feet; Q = cfs:

$$K = (4.73\ L) / (C^{1.85}\ D^{4.865}) \quad (15a)$$

$$x = 1.85 \quad (15b)$$

C is Hazen's pipe roughness coefficient

- S.I. Units with L = meters; D =meters; Q = m^3/s:

$$K = (10.70\ L) / (C^{1.85}\ D^{4.865}) \quad (15c)$$

$$x = 1.85 \quad (15d)$$

Generalized Form of Frictional Formulas - Hazen-Williams (Cont.)

Notes: Hazen Williams Equation

- Next slide gives typical values of C.
- The higher the C value the newer the pipe.
- A smooth PVC pipe can have C value as high as 150.
- An old pipe with significant encrustation will have a C value 60 or lower.
- Hazen's C can be assumed constant for a range of velocities between 6-9 ft/s.
- A +5% or - 5% adjustment to C is recommended for velocities outside 6-9 ft/s (positive adjustment for lower velocities).

Table: Typical Hazen Williams Pipe Roughness Coefficient, C.

Pipe Description	Roughness Coefficient, C
Extremely smooth, straight pipes; asbestos-cement	140
Very smooth pipes; concrete; new cast iron	130
Wood stave; new welded steel	120
Vitrified clay; new riveted steel	110
Cast iron after years of use	100
Riveted steel after years of use	95
Old pipes in bad condition	60 - 80

Minor Losses, h_m

• Minor losses associated with fittings such as valves, bends etc.

• Accounts for 10-20% of total head loss (some cases quite significant).

Note: K' depends on the fitting minor loss coefficient k_m and pipe size

• General form of minor loss equation:

$$h_m = K_m V^2/2g = K_m Q^2/(2gA^2) = K' Q^2 \qquad (16)$$

minor loss coefficient.

Minor Losses, h_m (Cont.)

Table: Head Loss Coefficients K_m for Various Fittings

Fitting	K_m
Globe Valve (fully open)	10.0
Angle Valve (fully open)	5.0
Swing Check Valve (fully open)	2.5
Gate Valve (fully open)	0.2
Close Return Bend	2.2
Standard Tee	1.8
Standard Elbow	0.9
Medium Sweep Elbow	0.8
Long Sweep Elbow	0.6
Square Entrance	0.5
Rounded Entrance	0.01-0.05
Re-Entrant	0.8-1.0

Problem 5: Friction Loss and Minor Losses and Coefficients

- A 12 inch pipe is 1500 feet long and has a Hazen's roughness coefficient C= 120 and carries water at a flow rate, Q = 5 cfs.
- Following fittings are installed: a) 1- Globe valve; b) 2-Standard elbows; and c) 1-Gate valve.

Answer the following:

1) The Hazens loss coefficient, $K = (4.73\ L) / (C^{1.85}\ D^{4.865})$ is:

a) 10.5 b) 110.2 c) 1.0 d) 3.5

2) The combined minor loss coefficient, $K' = \sum K_m/(2gA^2)$ is:

a) 2.5 b) 15.2 c) 25.6 d) 0.30

3) The Hazens head loss due to friction, $h_f = K\ Q^{1.85}$, in feet is:

a) 19.6 b) 60.2 c) 120.0 d) 35.5

4) The total minor head loss, $h_m = K_m\ Q^2/(2gA^2)$ in feet is:

a) 29.6 b) 3.2 c) 7.5 d) 17.5

Analysis and design of Full Pipe Flow Systems

- **Requires seven hydraulic variables:**

  1. **Discharge, Q**
  2. **Pipe length, L**
  3. **Pipe size or diameter, D**
  4. **Head loss due to friction, h_f and minor loss, h_m**
  5. **Pipe roughness, ε, or relative roughness, ε/D**
  6. **Fluid density, ρ**
  7. **Dynamic viscosity, μ, (or kinematic viscosity, $\nu = \mu/\rho$).**

- **Three types of simple pipe flow problems:**

  Type 1: Given Q, L, D, ε, μ, ρ : solve for h_f
  Type 2: Given h_f L, D, ε, μ, ρ : solve for Q
  Type 3: Given $h_{f\,,}$ L, Q, ε, μ, ρ : solve for D

Analysis and design of Full Pipe Flow Systems

Pipes in series, parallel, branched and networks

- **Many water utilities employ pipes in series, parallel or branched or a combination of these.**
- **Analyzing these systems typically involves the use of the following two equations:**
  1. **Equation of continuity (law of conservation of mass)**
  2. **Head loss equation (law of conservation of energy)**

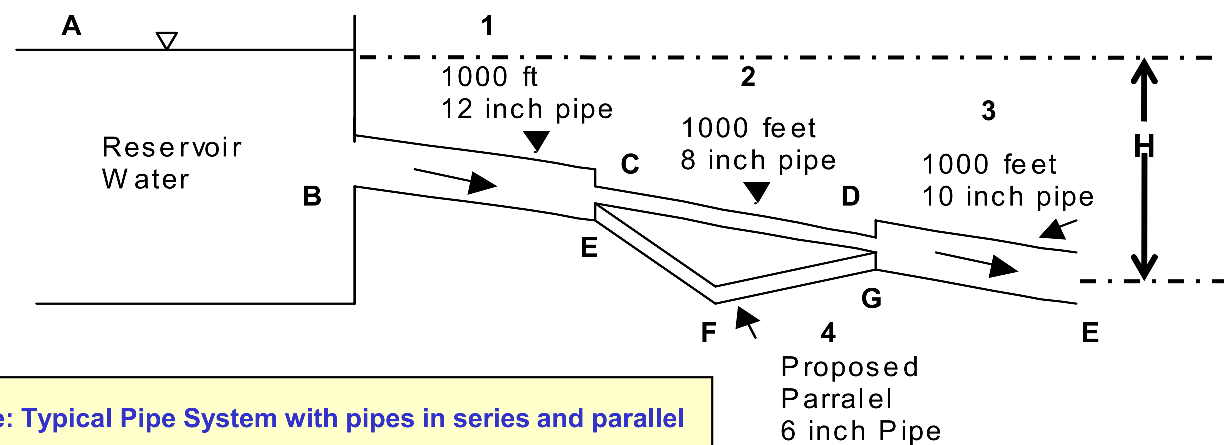


Figure: Typical Pipe System with pipes in series and parallel

1) Pipes in Series and Parallel : Equivalent K

Pipes in Series:

For pipes in series an equivalent K_e can be determined by summing the individual K vales of each pipe.

$$h_e = h_1 + h_2 + h_3 \quad \text{(Energy Equation)} \quad (17)$$

$$Q_e = Q_1 = Q_2 = Q_3 = Q \quad \text{(Continuity Equation)} \quad (18)$$

$$K_e = \Sigma K_i = (K_1 + K_2 + K_3) \quad (19)$$

Pipes in Parallel:

An *equivalent K_e can be determined by summing the reciprocals of the individual K values of each pipe.*

$$h_e = h_2 = h_4 \quad \text{(Energy Equation)} \quad (20)$$

$$Q_e = Q_2 + Q_4 = Q \quad \text{(Continuity Equation)} \quad (21)$$

$$(1/K_e)^{1/x} = \Sigma(1/K_1)^{1/x} = (1/K_2)^{1/x} + (1/K_4)^{1/x} \quad (22)$$

Example 6: Equivalent Pipe for Pipes in Series and Parallel

For the pipe system problem shown in the Figure below answer the following questions. Use Hazen- Williams method assuming all pipes have a Hazen C = 120.

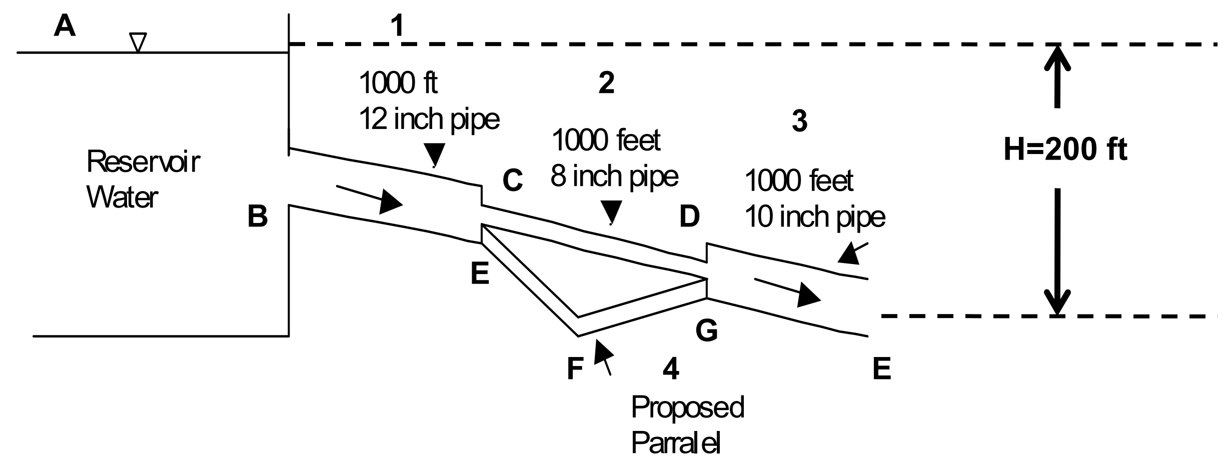
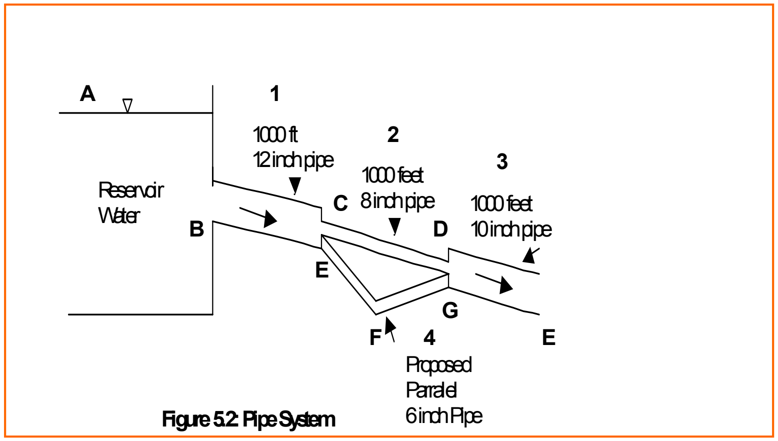


Figure 5.2: Pipe System

1. **Find the discharge Q for H = 200 feet assuming the system is composed of Pipes 1, 2 and 3 in series only.**
2. **For the same system find the discharge when the parallel pipe 4 is also included.**

Example 6: Equivalent Pipe for Pipes in Series (cont.)

Solution to part (1)



Step 1: Find Hazen K for each pipe:

$K_1 = (4.73 \times 1000) / \{120^{1.85} \times (12/12)^{4.87}\} = 0.67$

$K_2 = (4.73 \times 1000) / \{120^{1.85} \times (8/12)^{4.87}\} = 4.852$

$K_3 = (4.73 \times 1000) / \{120^{1.85} \times (10/12)^{4.87}\} = 1.638$

$K_4 = (4.73 \times 1000) / \{120^{1.85} \times (6/12)^{4.87}\} = 19.697$

Step 2: Find equivalent K (Slide 41 – Equation 19) for the three pipe in series 1, 2 and 3:

$$K_e = \Sigma K_i = (K_1 + K_2 + K_3) = 0.674 + 4.852 + 1.638 = 7.164$$

Step 3: Determine the discharge using head loss equation:

$$h_e = K_e Q^{1.85}$$

$$200 = 7.164\, Q^{1.85}$$

solving for Q, $Q = (200/7.164)^{(1/1.85)}$

$$Q = (27.917)^{(0.54)} = 6.05 \text{ cfs}$$

Example 6 : Equivalent Pipe for Pipes in Parallel (cont.)

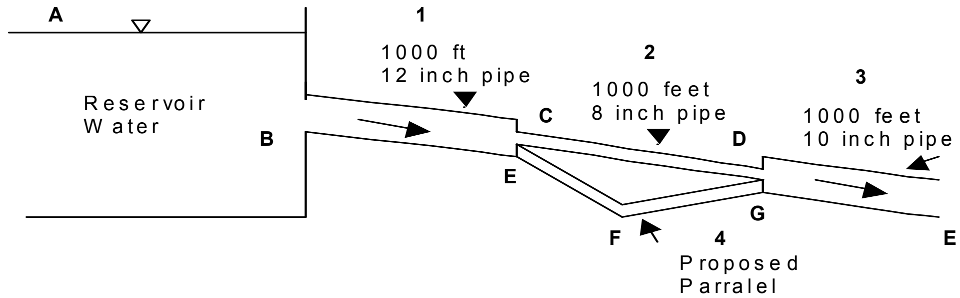


Figure 5.2: Pipe System

Solution to part (2):

Step 1: First Combine the two parallel pipes 2 and 4 (see figure above) into one equivalent pipe. From Slide 41 – Equation 22:

$$(1/K_e)^{1/1.85} = (1/4.852)^{1/1.85} + (1/19.697)^{1/1.85}$$
$$= 0.426 + 0.20 = 0.626$$
$$K_e = \{1/0.626\}^{1.85} = 2.379$$

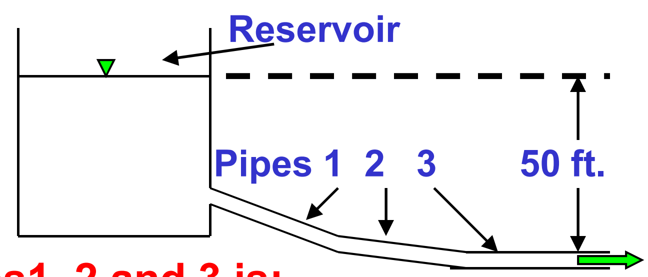
Step 2: Using pipe 1, the equivalent pipe from step 2 and pipe 3 in series solve for discharge as in part (1)

$$K_e = \Sigma K_i = (K_1 + K_2 + K_3) = 0.674 + 2.379 + 1.638 = 4.691$$
$$h_e = K_e Q^{1.85}$$
$$200 = 4.691\, Q^{1.85}$$
$$Q = 7.60 \text{ cfs.}$$

Problem 6: Pipes in Series

Problem Statement:

Water discharges from a reservoir into the pipe system.
Pipes 1, 2 and 3 are in series
Hazen K_1 = 1.5; K_2 = 4.0; K_3 =1.7;
Total length of all three pipes is 3000 feet.
Answer the following:



1) The equivalent K_e for a single pipe to replace pipes1, 2 and 3 is:

a) 5.5 b) 7.2 c) 8.5 d) 4.0

2) If the equivalent pipe has the same total length L_e = 3000 feet and is made of PVC (Hazen C = 150), the diameter, d_e, of the equivalent pipe in inches is:

a) 6.5 b) 12.5 c) 8.5 d) 9.8

3) The discharge Q from the reservoir in cfs is:

a) 5.5 b) 3.2 c) 8.6 d) 2.9

2) Branched Pipe System

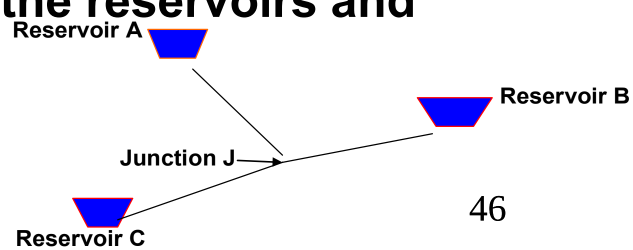
The following hydraulic conditions must considered:

- **The hydraulic head, h, is equal to the elevation z above the datum if the water level in the reservoirs, A, B and C have a free surface (the gage pressure, p, is zero).**
- **The friction equation, $h_f = KQ^x$, must be satisfied in each pipe.**
- **<u>Continuity equation</u> for flow at junction J must be met in two possible ways based on the hydraulic grade ($p/\gamma + z$) at junction J.**

1) Flow from Res. 1 can flow into Res. 2 and 3: $Q_1 = Q_2 + Q_3$

2) Flow from Res. 1 and 2 can flow into Res. 3: $Q_1 + Q_2 = Q_3$

- **Junction J is an internal point in the pipe and the pressure is unknown**
- **The change in velocity head ($V^2/2g$) between the reservoirs and junction, J, is small and can be neglected.**



Branched Pipe System

Example 7: Branched Pipe System

Given:

Reservoir A: $p_1/\gamma + z_1 = z_1 = 30$ m;

Reservoir B: $p_2/\gamma + z_2 = z_2 = 18$ m;

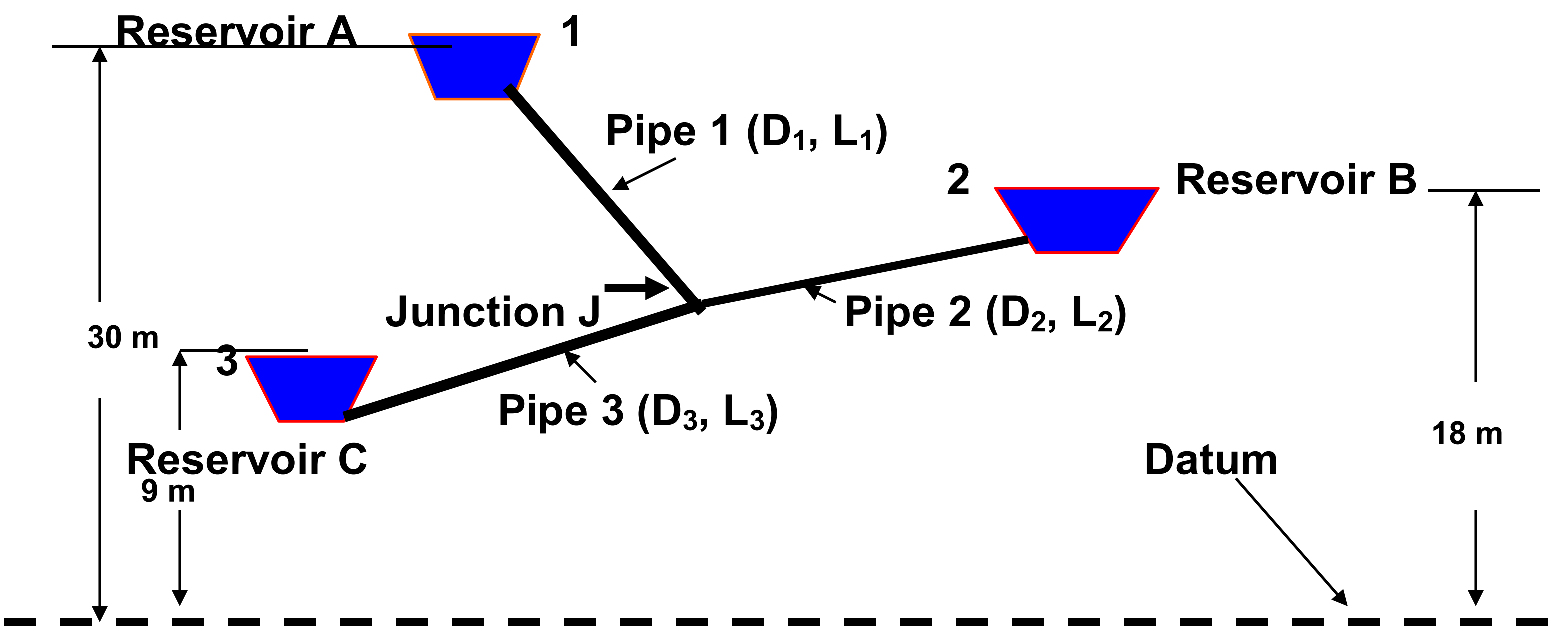
Reservoir C: $p_3/\gamma + z_3 = z_3 = 9$ m;

Pipe Diameters: $D_1 = 1.0$ m ; $D_2 = 0.45$ m; $D_3 = 0.60$

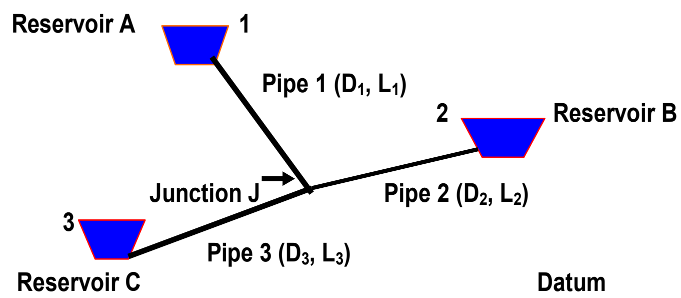
Pipe Lengths: $L_1 = 3000$ m; $L_2 = 600$ m; $L_3 = 1000$m

Hazen C for all pipes C = 130 (new cast iron pipe)

Note: Gage pressures p_1, p_2 and p_3 at the top of Reservoirs A, B and C are zero



Example 7 : Branched Pipe System (cont.)



Solution:

Step 1:

Calculate Hazen's K values for each pipe (refer to Eq. 15c, Slide 33):

$$K_1 = 10.7\ L_1/(C^{1.85}D_1^{4.87}) = (10.7\text{x } 3000)/(130^{1.85} \text{ x } 1.0^{4.87}) = 3.942$$

$$K_2 = (10.7\text{x } 600)/(130^{1.85} \text{ x } 0.45^{4.87}) = 30.552$$

$$K_3 = (10.7\text{x } 1000)/(130^{1.85} \text{ x } 0.6^{4.87}) = 15.812$$

Step 2:

Assume h_j for junction J = 20 m

Head loss in pipe 1 = $h_{f1} = (p_1/\gamma + z_1) - (p_J/\gamma + z_J) = 30 - 20 = 10$ m (flow A to J)

Head loss in pipe 2 = $h_{f2} = 20 - 18 = 2$ m (flow J to B)

Head loss in pipe 3 = $h_{f3} = 20 - 9 = 11$ m (flow J to C)

Example 7: Branched Pipe System (cont.)

Step 3:

Compute Discharges into Junction J

Discharge from reservoir A into junction J (Note: HG at 1 > HG at J):

$$Q_1 = (h_{f1}/K_1)^{1/1.85} = (10/3.942)^{1/1.85} = 1.654 \text{ m}^3\text{/s (inflow)}$$

$$Q_2 = (2/30.552)^{1/1.85} = 0.229 \text{ m}^3\text{/s (outflow)}$$

$$Q_3 = (11/15.812)^{1/1.85} = 0.821 \text{ m}^3\text{/s (outflow)}$$

Step 4:

Calculate the deficit as:

$$\delta_1 = Q_1 - (Q_2+Q_3) = (1.654) - (0.229 + 0.821) = 0.604 \text{ m}^3\text{/s}$$

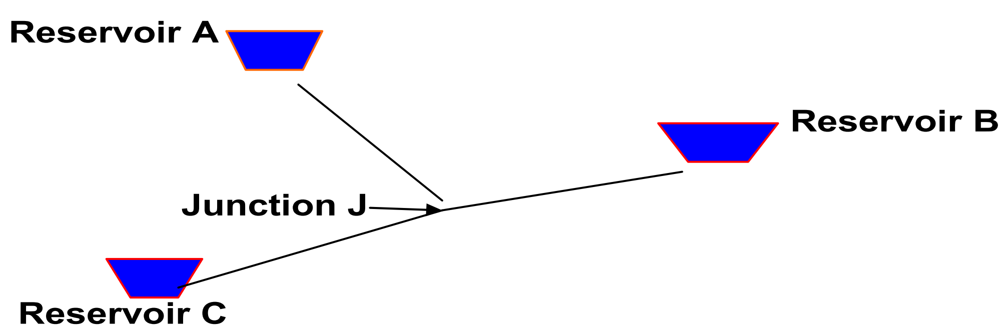


Figure 5.3: Branched Pipe System

Example 7 : Branched Pipe System (cont.)

Step 5: **Repeat Steps 2-4 using another hydraulic head for junction J.**

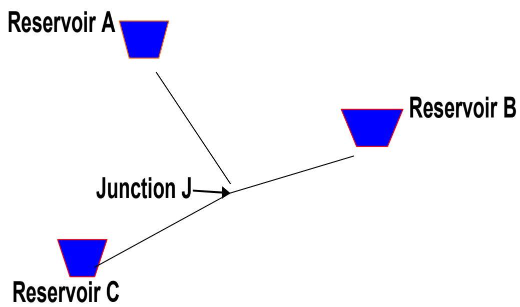


Figure 5.3: Branched Pipe System

- **Assume the HG of J = $z_J + p_J/\gamma$ = 15 m**

Head loss in pipe 1: h_{f1}= 30 – 15 = 15m; (flow A to J)

Head loss in pipe 2: h_{f2} = 18 – 15 = 3 m; (flow B to J)

Head loss in pipe 3: h_{f3} = 15 – 9 = 6 m; (flow J to C)

- **Compute Discharges into junction J (Note: HG at 1 > HG at J)**

$Q_1 = (h_{f1}/K_1)^{1/1.85} = (15/3.942)^{1/1.85} = 2.059\ m^3/s$ (inflow)
$Q_2 = (3/30.552)^{1/1.85} = 0.285\ m^3/s$ (inflow)
$Q_3 = (6/15.812)^{1/1.85} = 0.592\ m^3/s$ outflow)

- **Calculate the deficit :**

$\delta_2 = (Q_1 + Q_2) - Q_3) = (2.059 + 0.285) - 0.592 = 1.752\ m^3/s$

Example 7 : Branched Pipe System (cont.)

Step 6:

Plot flow deficit at junction J, δ versus Hydraulic Grade at J, $p_j/\gamma + z_j$

Answer:

From the graph the flow deficit to Junction J is zero at $p_j/\gamma + z_j = 23$ m.

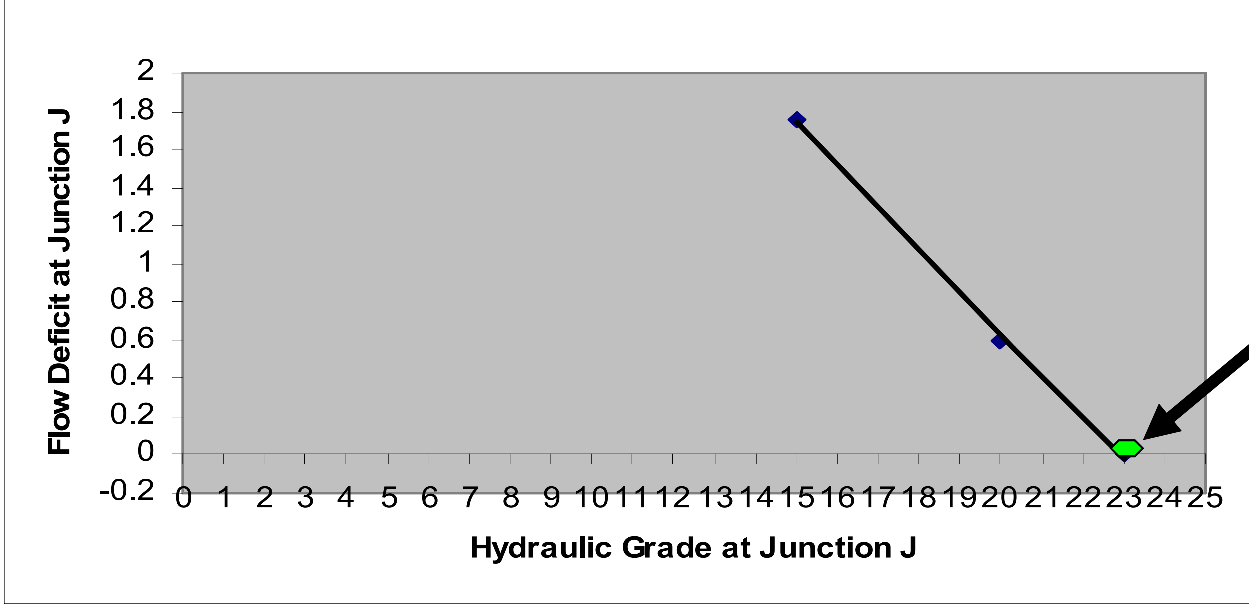


Figure 5.5: Flow Deficit Versus Hydraulic Grade at Junction J

Final Solution
$h_j = 23$ m at $\delta = 0$

3) Looped Pipe Systems: Hardy Cross Method

This method involves:

1. **Assuming an initial distribution of flows in each pipe satisfying continuity at each node;**
2. **Determining the head losses in each pipe;**
3. **Making successive corrections, δ , to the flows in each pipe until the total head loss around a loop is zero.**

$$\delta = - \{ (\Sigma K Q^{x}) / (\Sigma K x Q^{x-1} \} = - \Sigma h_f / (x \Sigma h_f / Q) \qquad (23)$$

Note: **For Hazen Williams equation x =1.85; $h_f = \sum KQ^x$**

Example 8: Pipe Network Analysis Using Hardy Cross Method

For the welded steel pipe network determine the discharges in each pipe. What are the pressures at each node assuming that all nodes are at the same elevation, z. The pressure head at A is 50 feet.

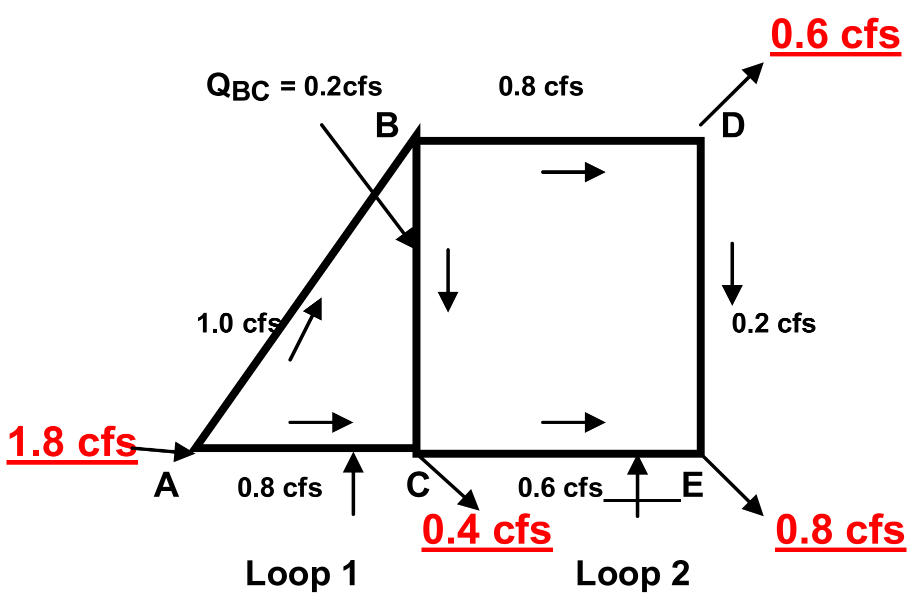


Figure : Pipe Network with Two Loops

Example 8: Pipe Network Analysis Using Hardy Cross Method (Cont.)

Table: Hardy Cross Method – Iteration 1

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Loop	Pipeline	K	Q_a(cfs)	$h_f = KQ_a^{1.85}$ (ft)	$\lvert h_f/Q_a \rvert$	Corrected, Q_a(cfs)
1	AB	14.25	+1.0	+14.25	14.25	+0.9
	BC	7.12	+0.2	+0.36	1.80	+0.185
	AC	14.25	-0.8	-9.43	11.79	-0.9
				$\sum = +5.18$	$\sum = 27.84$	
2	BD	14.25	+0.8	+9.43	11.79	+0.715
	DE	7.12	+0.2	+0.36	1.80	+0.115
	CE	14.25	-0.6	-5.54	9.23	-0.685
	BC	7.12	*-0.2*	-0.36	1.80	-0.185
				$\sum = +3.89$	$\sum = 24.62$	

Example 8: Pipe Network Analysis Using Hardy Cross Method (cont.)

Calculate loop flow corrections using the sums from Columns 5 and 6 of Slide 54: Flow Corrections after Iteration 1

δ_1 **= - 5.18 / (1.85x27.84) = -0.10**

δ_2 **= - 3.89 / (1.85x24.62) = -0.085 cfs**

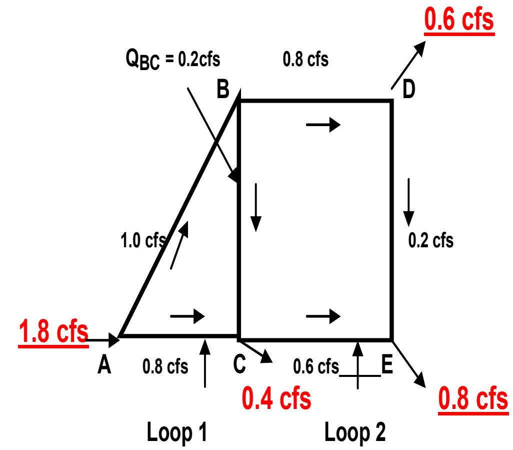


Figure : Pipe Network with Two Loops

adjusted Q_{AB} = +1.00 - (0.10) = 0.9 cfs

adjusted Q_{BC} = - (0.2-*0.10*) – *0.085* = - *0.185 cfs*

Correction with respect to Loop 1

Final Correction with respect to Loop 2

Example 8 : Pipe Network Analysis Using Hardy Cross Method (cont.)

Table: Hardy Cross Method – Iteration 2

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Loop	Pipeline	K	Q_a(cfs)	$h_f = KQ^{1.85}$ (ft)	$\lvert h_f/Q_a \rvert$	Corrected Q_a(cfs)
1	AB	14.25	+0.9	+11.73	13.03	+0.9
	BC	7.12	+0.185	+0.31	1.68	+0.185
	AC	14.25	-0.9	-11.73	13.03	-0.9
				$\sum = +0.31$	$\sum = 27.74$	
2	BD	14.25	+0.715	+7.66	10.71	+0.715
	DE	7.12	+0.115	+0.13	1.13	+0.115
	CE	14.25	-0.685	-7.08	10 .34	-0.685
	BC	7.12	-0.185	-0.185	1.68	-0.185
				$\sum = +0.40$	$\sum = +0.40$	

Example 8 : Pipe Network Analysis Using Hardy Cross Method (cont.)

Flow Corrections after Iteration 2:

δ_1 = - (0.31) / (1.85)(27.74) = - 0.006 (negligible)

δ_2 = - (0.40) / (1.85)(23.86) = - 0.009 (negligible)

The final corrected flows are shown in Column 7 of Slide 56.

Table: Final Flows and Pressure Heads

(1)	(2)	(3)	(4)	(5) Pressure Head (feet)
Pipeline	Flow Q_a (cfs)	Head Loss $h_f = KQ^{1.85}$ (feet)	Nodes	
AB	+0.900	+11.7	A	Given 50.0
BC	+0.185	+0.3	B	50 - h_{AB} = 38.3
AC	-0.900	-11.7	C	50 – h_{AC} = 38.3
BD	+0.715	+7.7	D	h_B - h_{BD} = 30.6
DE	+0.115	+0.1	E	h_C - h_{CE} = 31.2
CE	-0.685	-7.1		

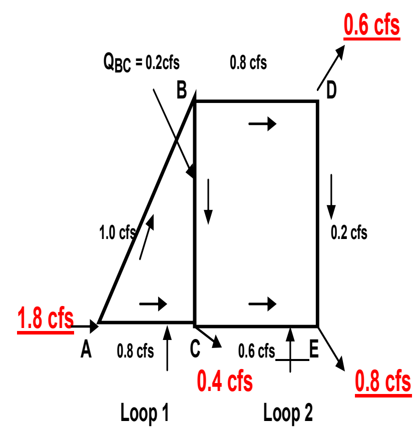


Figure : Pipe Network with Two Loops

Open Channel Hydraulics

Froude Number, F_r

Widely used as an indicator of the state of flow in Open Channels.

$F_r < 1$ subcritical flow

$F_r > 1$ supercritical flow

$F_r = 1$ critical flow in a channel

$$F_r = V / (gD)^{0.5} = Q/\{A(gD)^{0.5}\} = Q/(gA^3/T)^{0.5} \qquad (24)$$

where, V is the average velocity,
Q is the discharge, and
D is the hydraulic depth = A/T.

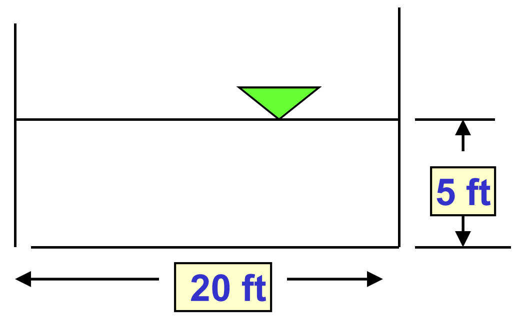
Example 9: Froude Number, F_r : Rectangular Channel Special Case

Froude Number, $F_r = V/(gD)^{0.5}$

$= V/(gy)^{0.5} = (Q/By)/(gy)^{0.5}$

$= (q/y)/(gy)^{0.5}$

$$F_r = q/(gy^3)^{0.5}$$



Given: Q = 400 cfs; B = 20 ft; y = 5 ft; q = 400/20 = 20 cfs/ft;

Find: $F_r = 20/(32.2(5)^3)^{0.5} = 0.32 < 1$ (subcritical flow)

Critical Flow In Open Channel

- Two methods to calculate critical depth, y_c:

  - Analytical method where a trial and error procedure can be used to solve for y_c
    (see example in Appendix)

  - Graphical method: Use a Design Chart (Slide 62)
    (easier to use than the Analytical method)

Graphical method is illustrated in the following example.

Computation of Critical Depth

•**For Critical Flow, Froude Number:**

$$F_r = V/\{gD/\alpha\}^{0.5} = Q/\{A(g(A/T)/\alpha)^{0.5}\} = Q/\{(gA^3/T)/\alpha\}^{0.5} = 1 \qquad (25)$$

•**Separating Q from the other variables, section factor in critical flow:**

$$Z_c = (A^3/T)^{0.5} = Q/(g/\alpha)^{0.5} \qquad (26)$$

Given Q, Eq. 26 can be used to calculate the section factor Z_c and critical depth, y_c, since A and T are functions of flow depth (analytical method or use Design Chart – Slide 62)

Special Case: **For a rectangular channel of unit width Eq. 22 reduces to:**

$$Fr^2 = q^2/\{(g/\alpha)y_c^3\} = 1 \qquad (27)$$

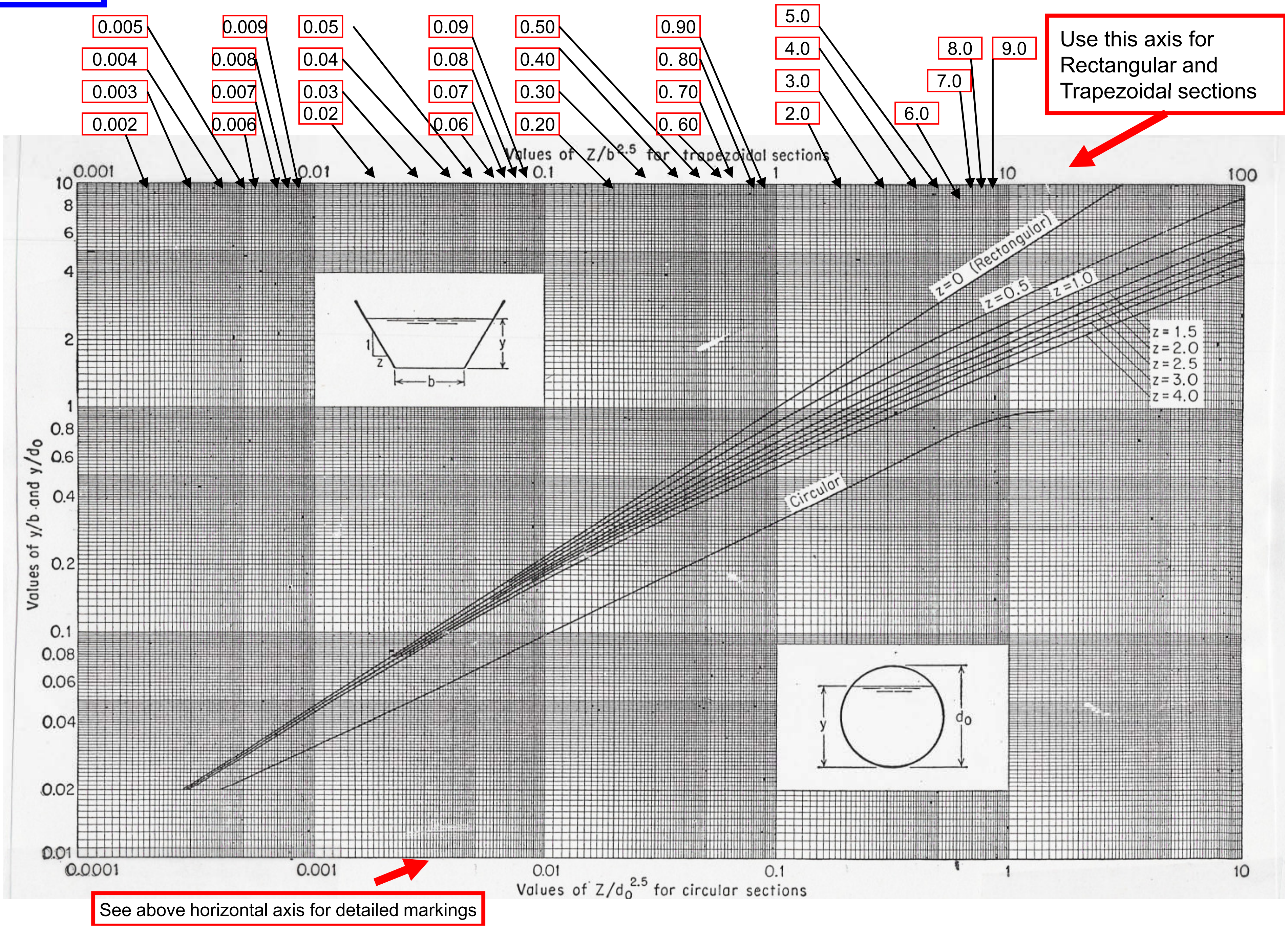
or

$$y_c = [\, q^2/(g/\alpha)\,]^{1/3} \qquad (28)$$

Given discharge/unit width, q solve for critical depth, y_c analytically from this equation.

where, q = discharge per unit width = Q/B.

Figure: Design Chart for Determining the Critical Depth, y_c (reproduced from Chow, 1959)



Example 10: Computation of Critical Depth: Design Chart Method

Compute the critical depth and velocity in a trapezoidal channel carrying a discharge, Q, of 400 cfs. The channel has a bottom width, B = 20 feet and side slope z = 2 (i.e. 2H:1V). Assume $\alpha = 1.10$.

Solution:

See Equation 26 Slide 61

1. **Section factor in critical flow**

$$Z_c = Q/(g/\alpha)^{0.5} = 400\ (32.2/1.1)^{0.5} = 73.93$$

2. **Compute the dimensionless value,**

$$Z_c\ /\ B^{2.5} = 73.93\ /\ (20)^{2.5} = 0.0413$$

3. **From the Design Chart for $Z_c / B^{2.5} = 0.0413$**

$$y/B = 0.11$$

or critical depth, $y_c = 0.11\ (20) = 2.22$ feet

Uniform Flow In Open Channels

- **The flow depth, water area, velocity and discharge in the channel are constant.**
- **The flow depth is called *uniform flow depth* or *normal depth, y_n*.**
- **Uniform flow exists when *gravitational forces are balanced by frictional forces*.**
- **Friction slope, S_f = channel bottom slope, S_0.**
- **Basis for channel design.**

Note: If a channel is allowed to flow without any physical obstructions or changes in channel cross section, the flow would occur under uniform flow conditions.

Friction Formula In Uniform Flow

Manning Formula:

- Most widely formula in design of open channels.

Note: friction slope, S_f = channel slope, S_0

$$V = (1.49/n)\, R^{2/3} S_0^{1/2} \quad \text{(US Units)} \qquad (29)$$

or

$$Q = (1.49/n)\, A R^{2/3} S_0^{1/2} \quad \text{(US Units)} \qquad (30)$$

- In SI units the constant 1.49 is replaced with 1.0.

Manning Roughness Coefficient, n

Table: Average Values of the Manning Roughness, n

Boundary Material	Manning, n
Planed Wood	0.012
Unplaned Wood	0.013
Finished Concrete	0.012
Unfinished Concrete	0.014
Cast iron	0.015
Brick	0.016
Riveted Steel	0.018
Corrugated Metal	0.022
Rubble	0.025
Earth	0.025
Earth with stones or weeds	0.035
Gravel	0.029

Computing Channel Normal Flow Depth, y_n
(or Uniform Flow Depth, y_n)

Method 1: Analytical method (Trial and Error):

- Method based on section factor $AR^{2/3} = nQ/(1.49S_0^{1/2})$

(Note: In SI units 1.49 is replaced with 1.0)

- Solve for y_n expressing A and R in terms of flow depth.

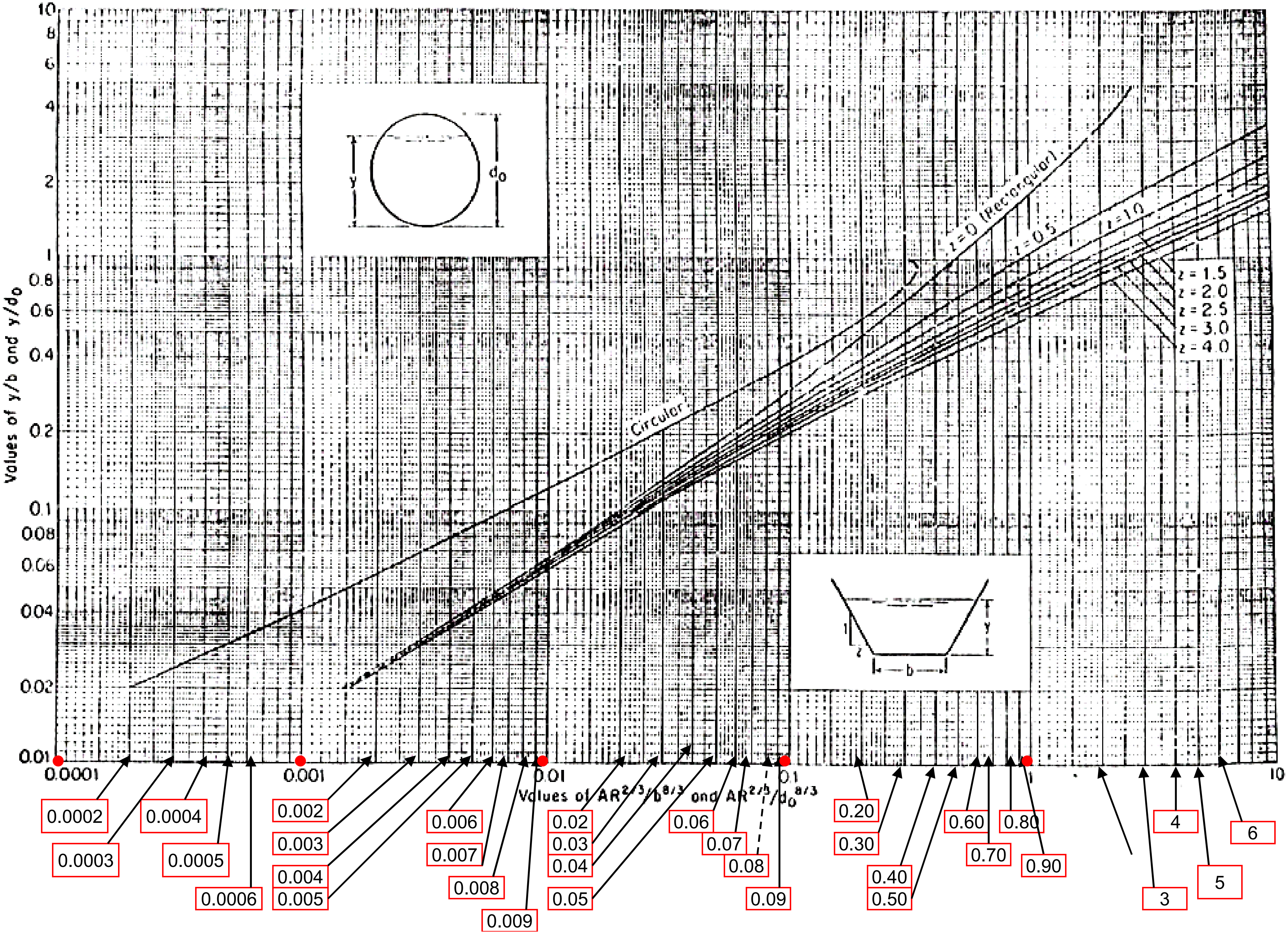
(See Appendix for details)

Method 2: Method of Design Chart (Slide 68)

- Design Chart is used for computing normal depth, y_n
- Procedure requires a dimensionless ratio $AR^{2/3}/B^{8/3}$

$AR^{2/3}$ is Section Factor
B is channel bottom width

Figure 6.1: Design Chart for Determining Normal or Uniform Flow Depth, y_n (Chow, 1959)



Example 11: Computation of Normal Depth, y_n

A trapezoidal channel (bottom width B = 20 feet, side slope z = 2, slope S_0 = 0.0016, and n = 0.025) carries a discharge, Q = 400 cfs.

Find the normal depth, y_n, and the average velocity, V.

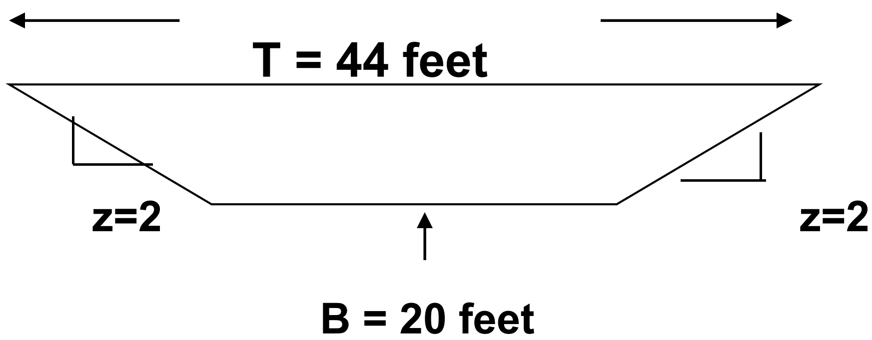


Figure: Channel Cross-Section

Example 11 : Computation of Normal Depth, y_n (cont..)

Design Chart Method:

- **Step 1:** **Compute the Section Factor in Uniform Flow**

$$AR^{2/3} = nQ/(1.49S_0^{1/2}) = (0.025 \times 400)/(1.49 \times 0.0016^{1/2}) = 167.7$$

- **Step 2:** **Compute the dimensionless ratio:**

$$AR^{2/3}/B^{8/3} = 167.7/20^{8/3} = 0.0569$$

Step 3: **Using trapezoidal channel section with a side slope z = 2 curve (Slide 68) and for $AR^{2/3}/B^{8/3} = 0.0569$,**

$$y/B = 0.168$$

or **Normal Depth $y_n = 0.168 \times 20 = 3.36$ feet**

Problem 7: Computation of Section Factor, Uniform and Critical Flow Depths

Given: Uniform flow conditions.
Concrete lined rectangular channel (manning's n = 0.015)
Longitudinal slope, $S_0 = 0.004$ m/m
Bottom width, B = 8.25m;
Design flow Q = 150 m^3/s

Note: $AR^{2/3} = nQ/S_o^{1/2}$
in SI units

1) the section factor in uniform flow, $AR^{2/3}$ is:

a) 50.2 b) 35.6 c) 45.6 d) 64.2

2) the uniform or normal depth, y_n in meters:

a) 2.5 b) 3.5 c) 3.0 d) 4.2

3) the critical depth, y_c, in meters:

a) 2.8 b) 3.2 c) 4.0 d) 3.8

4) state of flow:

a) subcritical b) supercritical c) critical d) none of these

Thank you for listening to the presentation.

Good luck on the P.E. Exam

QED

References:

1. Gupta, R., 1989. Hydrology and Hydraulic Systems. Prentice Hall, Inc., Englewood, NJ, pp. 739.
2. Streeter, V. L., and Wylie, B. E., 1979. Fluid Mechanics (seventh edition). McGraw Hill, Inc, New York, pp. 562.
3. Viessman, W., and Hammer, M. J., 2005. Water Supply and Pollution Control (seventh edition), Pearson Prentice Hall, Inc., Upper Saddle River, NJ, pp.865.
4. Chow, V. T., 1959. Open-Channel Hydraulics. McGraw Hill, Inc, New York, pp. 680.

Answers to Problems

• Problem 1(a) (Slide 9):	1) c;	2) b;	3) d;	4) a
• Problem 1(b) (Slide 10):	1) b;	2) c:	3) d.	
• Problem 2 (Slide 18):	1) d;	2) b.		
• Problem 3 (Slide 27):	1) c;	2)d;	3) b;	4) d.
• Problem 4 (Slide 28):	1) c;	2) d;	3) b.	
• Problem 5 (Slide 38):	1) c;	2) d;	3) a;	4) c.
• Problem 6 (Slide 45):	1) b;	2) c;	3) d.	
• Problem 7 (Slide 71):	1) b;	2) c;	3) b;	4) b.

APPENDIX
Open Channel Flow

Analytical Methods for Computing Critical and Normal Depths

Involves a trial and error procedure

Example: Computation of Critical Depth
Analytical Method

Compute the critical depth and velocity in a trapezoidal channel carrying a discharge, Q, of 400 cfs. The channel has a bottom width, B = 20 feet and side slope z = 2 (i.e. 2H:1V). Assume α = 1.10.

Solution:

Step 1: Compute section factor Z_c:

$$Z_c = Q/(g/\alpha)^{0.5} = 400/(32.2/1.10)^{0.5} = 73.93$$

Step 2: Since $Z_c = A\sqrt{D}$; or $Z_c^2 = A^2 D$, substituting for A and D= A/T in terms of depth for a trapezoidal channel gives:

$$\{y\,(20 + 2y)\}^2 \{y\,(20 + 2y) / (20 + 4y)\} = Z_c^2 = (73.93)^2 = 5465.84$$

Step 3: Solving Eq. above by trial and error gives
the critical depth, y_c = 2.22 feet.

Methods for Computing Uniform Flow Depth (or Normal Flow Depth, y_n) in a Channel

Analytical method (Trial and Error)

- Method based on section factor $AR^{2/3}$
- Solve for y_n expressing A and R in terms of flow depth.

Example: Computation of Normal Depth, y_n

Analytical Method (Trial and Error):

Step 1: Compute the Section Factor in Uniform Flow:

$AR^{2/3} = nQ/(1.49S_0^{1/2}) =(0.025 \times 400)/(1.49 \times 0.0016^{1/2}) = 167.7$

Step 2: Substitute equations for water flow area A and Hydraulic radius, R:

$$(20 + 2y)y \;\; x \{ (20 + 2y)y / (20+2y(5)^{0.5}) \}^{2/3} = 167.7$$

Step 3. Solving for flow depth y by trial and error gives:

Normal depth, y_n = 3.36 feet

Step 4: Compute average velocity, V and other hydraulic variables:

Flow area $A = (20 + 2x3.36) \times 3.36 = 89.80\ ft^2$

Av. Velocity, $V = Q/A = (400/89.80) = 4.45$ ft/s

Top width, $T = (B + 2\ y(1+z^2)^{0.5}) = (20+2x(3.36)x(5)^{0.5}) =35.026$ feet

Hydraulic depth, $D = A/T = 89.80/35.026 = 2.563$ feet

Froude Number, $Fr = V/(gD)^{0.5}$

$= 4.45/((32.2x2.563)^{0.5}) = 0.49$ (subcritical flow)