

# P.E. Civil Exam Review: Hydrology

N.R. Bhaskar

Email: [bhaskar@louisville.edu](mailto:bhaskar@louisville.edu)

Phone: 502-852-4547



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# Hydrologic Cycle and Terms

- **Precipitation/Rainfall.**

Discharge of water from the atmosphere.

- **Rainfall excess.**

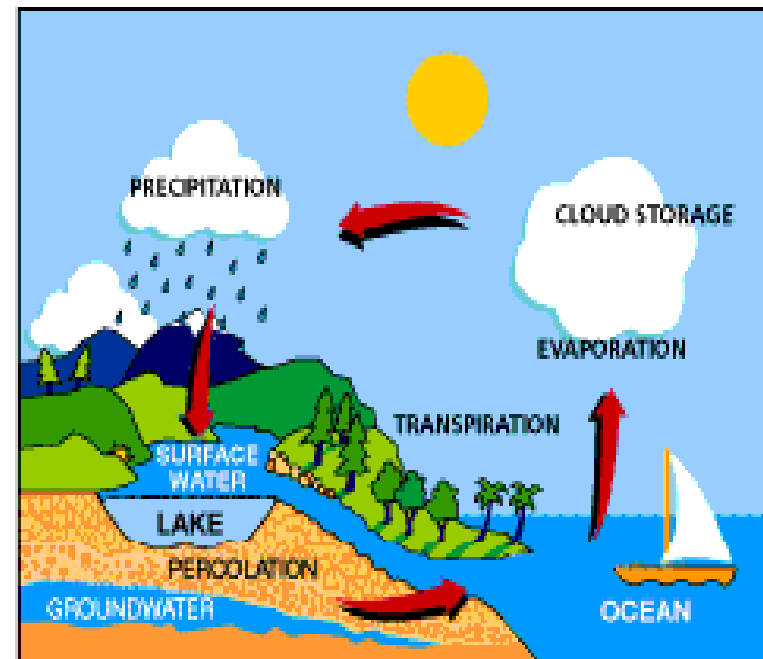
Rainfall minus *interception*  
*depression storage*  
*evaporation*  
*infiltration*

- **Runoff.**

Rainfall that appears in surface streams  
(includes subsurface quick return flow).

- **Surface runoff.**

Runoff which travels over the soil surface to the nearest stream.



# Commonly Used Units

- **Acre-foot:** Volume of water equal to 1 ft. depth of water covering 1.0 acre area = 43,560 ft<sup>3</sup>.
- **Cfs:** or cubic feet per second (ft<sup>3</sup>/s) - unit of discharge .
- **Cumec:** or cubic meters per second (m<sup>3</sup>/s) - unit of discharge .
- **Cfs-day (sfd):** Volume from a flow of 1 cfs for 1-day (24 hours) = 86,400 cubic feet or 1.98 acre-feet.

# Hydrologic Budget

- Continuity Equation (Law of Conservation of mass):

$$\Delta S/\Delta t = \sum \text{Inflows} - \sum \text{Outflows}$$

or

$$\Delta S = S_2 - S_1 = \{ \sum I - \sum Q \} \Delta t$$

Change in Storage

Time Interval

- Can be applied to:

1. Watershed System
2. Reservoir System

# Hydrologic Budget: Watershed System

## Example 1: Watershed Runoff Computation

### Given:

- 1) Watershed size above a gage site along a river = 50 sq. mi;
- 2) Watershed annual rainfall = 60 inches/yr;
- 3) Assume Runoff = 55% of annual rainfall (i.e. 45% losses)

### Compute:

- 1) Volume of Annual Runoff from the watershed in acre-feet and sfd
- 2) Rate of runoff in cfs

### Solution:

Watershed area in  
Note: 1 sq. mi = 640 acres

Runoff depth in feet

Annual Volume of runoff =  $\{50 \times 640\} \times \{0.55 \times 60 / 12\} = 88,000$   
ac-feet

(in 1-year)  $= 88,000 \times 43,560 = 3833.28 \times 10^6 \text{ ft}^3$   
 $= 44,366.67 \text{ sfd (or cfs-day)}$

Rate of runoff  $= (3833.28 \times 10^6 \text{ ft}^3) / (365 \times 86400 \text{ s})$   
 $= 121.55 \text{ cfs}$

# Hydrologic Budget: Reservoir System

## Example 2: Reservoir Storage Computation

**Given:** During a 30 day period:

- 1) Streamflow into the reservoir,  $Q = 5.0 \text{ m}^3/\text{s}$
- 2) Water supply withdrawal,  $W = 136 \text{ mgd}$
- 3) Evaporation from the reservoir surface =  $9.40 \text{ cm}$
- 4) Average reservoir water surface =  $3.75 \text{ km}^2$
- 5) Beginning reservoir storage,  $S_1 = 12,560 \text{ ac-ft}$

**Continuity Equation:  $S_2 = S_1 + Q - W - E$**   
**(all volume units)**

**Note: Time period  $\Delta t = 30 \text{ days} = (30 \times 86,400) \text{ sec}$**

# Hydrologic Budget: Reservoir System

## Example 2: Reservoir Storage Computation (Cont.)

**Compute:** The month-end reservoir storage,  $S_2$  in  $m^3$  and ac- ft.

1)  $S_1 = (12,560 \text{ ac- ft})$

**Note: Streamflow Q is converted to volume for the 30 day period**

2)  $Q = (5.0 \text{ m}^3/\text{s})(30 \text{ days})(86,400\text{s/day})$   
 $= 12,960,000\text{m}^3 (35.314 \text{ ft}^3/\text{m}^3)(2.296 \times 10^{-5} \text{ ac-ft}/\text{ft}^3) = 10,508 \text{ ac-ft}$

3)  $W = (136 \times 10^6 \text{ gal/day})(0.003785 \text{ m}^3/\text{gal})(30 \text{ days})$   
 $= 15,442,800 \text{ m}^3 (35.314 \text{ ft}^3/\text{m}^3)(2.296 \times 10^{-5} \text{ ac-ft}/\text{ft}^3) = 12,521 \text{ ac-ft}$

4)  $E = (3.75\text{km}^2)(1 \times 10^6 \text{ m}^2/\text{km}^2)(9.4 \text{ cm})(0.01 \text{ m}/\text{cm})$   
 $= 352,500 \text{ m}^3 (35.314 \text{ ft}^3/\text{m}^3)(2.296 \times 10^{-5} \text{ ac-ft}/\text{ft}^3) = 286 \text{ ac-ft}$

5)  $S_2 = S_1 + Q - W - E \quad (\text{for } \Delta t = 30 \text{ days}) \text{ ac-ft}$   
 $= (12,560 + 10,508) - (12,521 + 286)$

**$S_2 = 10,261 \text{ ac-ft (loss of storage)}$**  (Note:  $S_1 = 12,560 \text{ ac-ft}$ )

**Note:** Change in storage,  $\Delta S = S_2 - S_1 = (10,261 - 12,560) = - 2299 \text{ ac- ft (loss)}$



# Problem 1: Reservoir Storage Computation

## Given:

- Reservoir located at the outlet of a 150 sq. mile watershed
- Mean **annual** rainfall,  $P = 38$  inches (use as inflow into reservoir)
- Mean **annual** watershed runoff (flow into reservoir),  $Q = 13$  inches
- Mean **annual** reservoir evaporation,  $E = 49$  inches
- Mean **daily** withdrawal from reservoir (draft),  $D = 100$  MGD
- Mean reservoir surface area,  $A_S = 4000$  acres

(Note: 1.0 sq. mi = 640 acres; 1.0 ac-ft = 43,560 ft<sup>3</sup>; 1.0 MGD = 3.07 ac-ft/day; 1.0 ft<sup>3</sup> = 7.48 gal; 1.0 cfs = 1.9835 ac-ft/day)

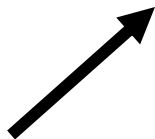
## Using a time frame, $\Delta t = 1$ year (365 days) determine:

1. Volume of water evaporated from lake in acre-ft/yr:

- a) 16,333                      b) 24,586                      c) 392,000                      d) 55,600

2. Watershed runoff or inflow into reservoir in acre-ft/yr:

- a) 104,000                      b) 1,248,000                      c) 99,667                      d) 266,580



Note: Watershed area not adjusted for reservoir area of 4000 acres

## Problem 1: Reservoir Storage Computation (cont.)

3. Watershed runoff or inflow into reservoir in cfs:

- a) 250.8                      b) 143.7                      c) 550.0                      d) 85.6

4. Volume of rainfall input, P, to reservoir in acre-ft/yr

- a) 152,000                      b) 304,000                      c) 12,667                      d) 85,000

5. Mean draft, D in ac-ft/yr:

- a) 100,000                      b) 112,055                      c) 185,250                      d) 265,500

6. Net loss/gain of reservoir storage,  $\Delta S$  in acre-ft/yr:

- a) -16,280                      b) 12,500                      c) -11,721                      d) 0

# Computation of Critical (Maximum) Reservoir Storage

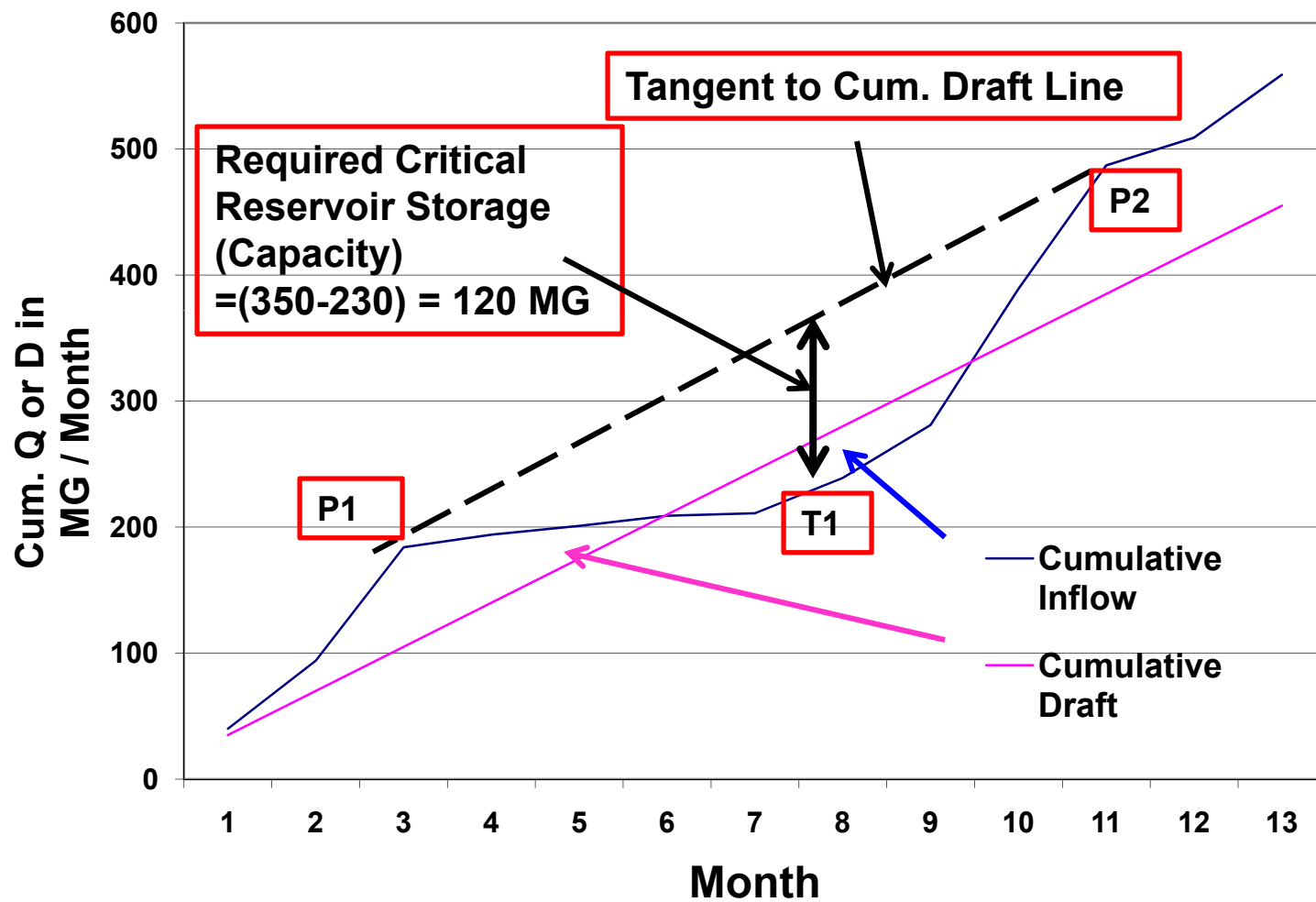
- **Graphical Method:**  
Mass Curve Analysis (Rippl Method)
- **Analytical Method:**  
Sequent Peak Method

## Example 3: Estimation of Critical or Maximum Reservoir Storage (Capacity)

**Problem:** The following mean monthly flows were measured in million gallons/month (MG/month) at a gage in a river during a 13 month critical drought period. Determine the maximum reservoir storage required in MG to provide a constant draft of 35 MG/month (Note: 1.0 MGD = 1.55 cfs or 1.0 MG/month = 46.5 cfs based on 30 days)

Month	Inflow, Q MG/month
1	40
2	54
3	90
4	10
5	7
6	8
7	2
8	28
9	42
10	108
11	98
12	22
13	50

### Example 3: Mass Curve Analysis for Estimating Critical Reservoir Capacity- Rippl Method (cont.)



## Example 3: Analytical Solution (Sequent Peak Method) (cont.)

Month	Inflow, Q MG/month	Draft, D MG/month	Cum. Q MG/month	Cum. D MG/month	Cum Q - Cum D MG/month
1	40	35	40	35	5
2	54	35	94	70	24
3	90	35	184	105	79 P1
4	10	35	194	140	54
5	7	35	201	175	26
6	8	35	209	210	-1
7	2	35	211	245	-34
8	28	35	239	280	-41 T1
9	42	35	281	315	-34
10	108	35	389	350	39
11	98	35	487	385	102 P2
12	22	35	509	420	89 T2
13	50	35	559	455	104 P3

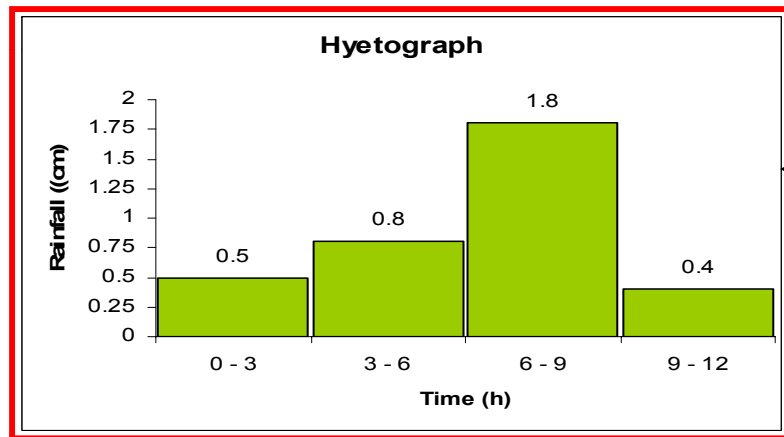
$$\begin{aligned}
 \text{Required Reservoir Storage} &= \text{Max}\{ (P1-T1) \text{ or } (P2 - T2) \} \\
 &= (79-(-41)) \text{ or } (102 - 89) \\
 &= 120 \text{ MG}
 \end{aligned}$$

# RAINFALL

## Two types of rainfall events:

### 1) HISTORICAL:

- Spatially and temporally averaged hyetographs rainfall events
- Based on measured rainfall depths using rain gages at a point



### A 12- hour historical rainfall event

#### Incremental depths:

Time (h):	0-3	3-6	6-9	9-12
Rainfall (cm):	0.50	0.80	1.80	0.40
Intensity (cm/hr):	0.17	0.27	0.60	0.13

### 2) DESIGN (or SYNTHETIC):

- Standardized temporal rainfall distributions (design storm hyetographs)
- Based on regionalized historical rainfall data for select duration and frequency

# Design Rainfall

## Intensity-Duration-Frequency (IDF)

Requires the following:

1) Frequency ( $F_R$ ) or average return period ( $T$ ) (see Appendix A)

Example: For  $T = 100$  years,  $F_R = 1/T = 1/100 = 0.01$  or 1%

2) Duration,  $t_d$

← Usually assumed equal to time of concentration,  $t_c$

3) Design rainfall depth = (intensity x duration)

Obtained from:

← Based on  $T$  and  $t_d$

a) NWS TP 40 Regionalized Rainfall

b) From Intensity Duration Frequency (IDF) Curves or Equations

4) Time (or temporal) distribution of rainfall depth

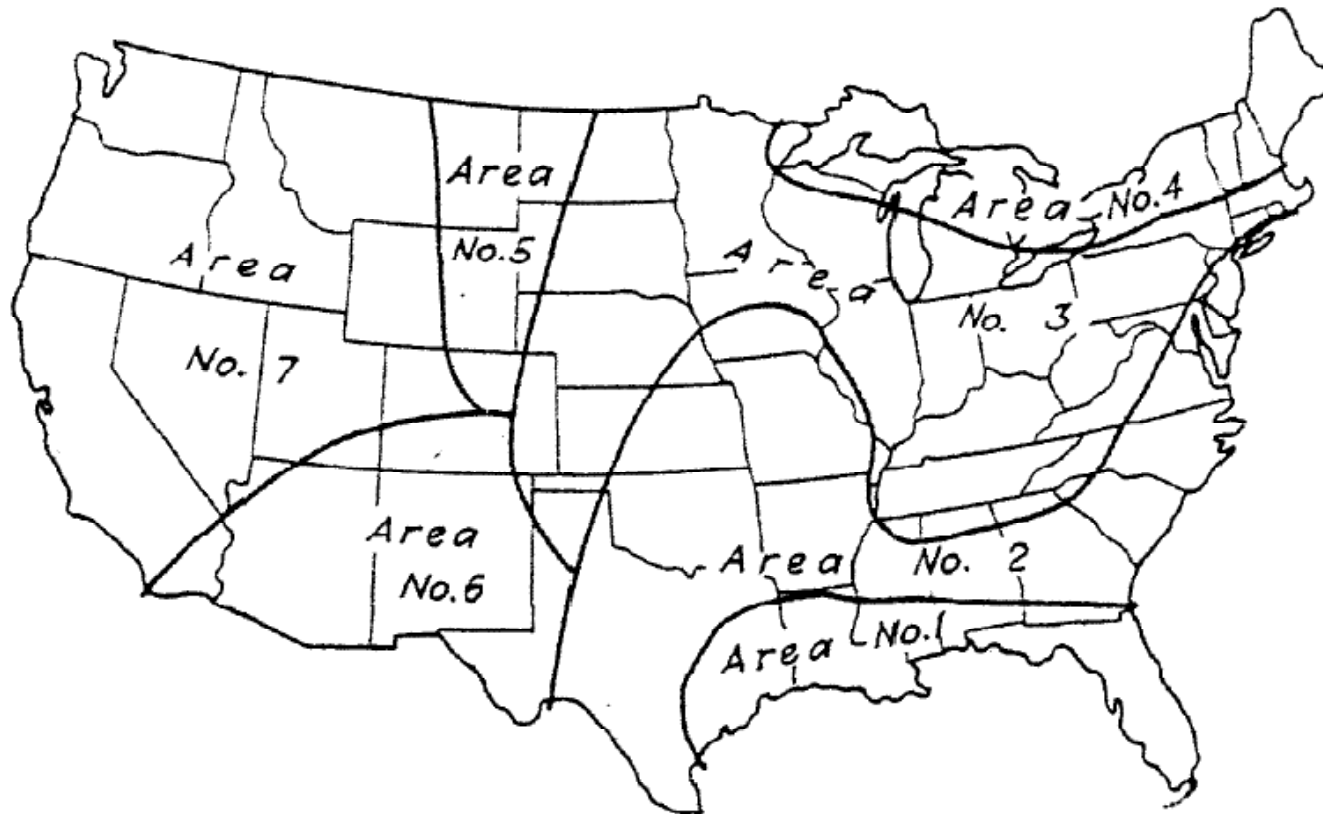
← Rainfall Hyetograph

5) Spatial variation rainfall depth over a catchment

← Handled using an average rainfall depth



## Figure: Regionalized IDF Map for IDF Equations



Map showing areas of approximately similar rainfall characteristics.

# Table: Regionalized IDF Equations

Precipitation formulas for various parts of the United States  
( $i$ , mm/h;  $t$ , min)

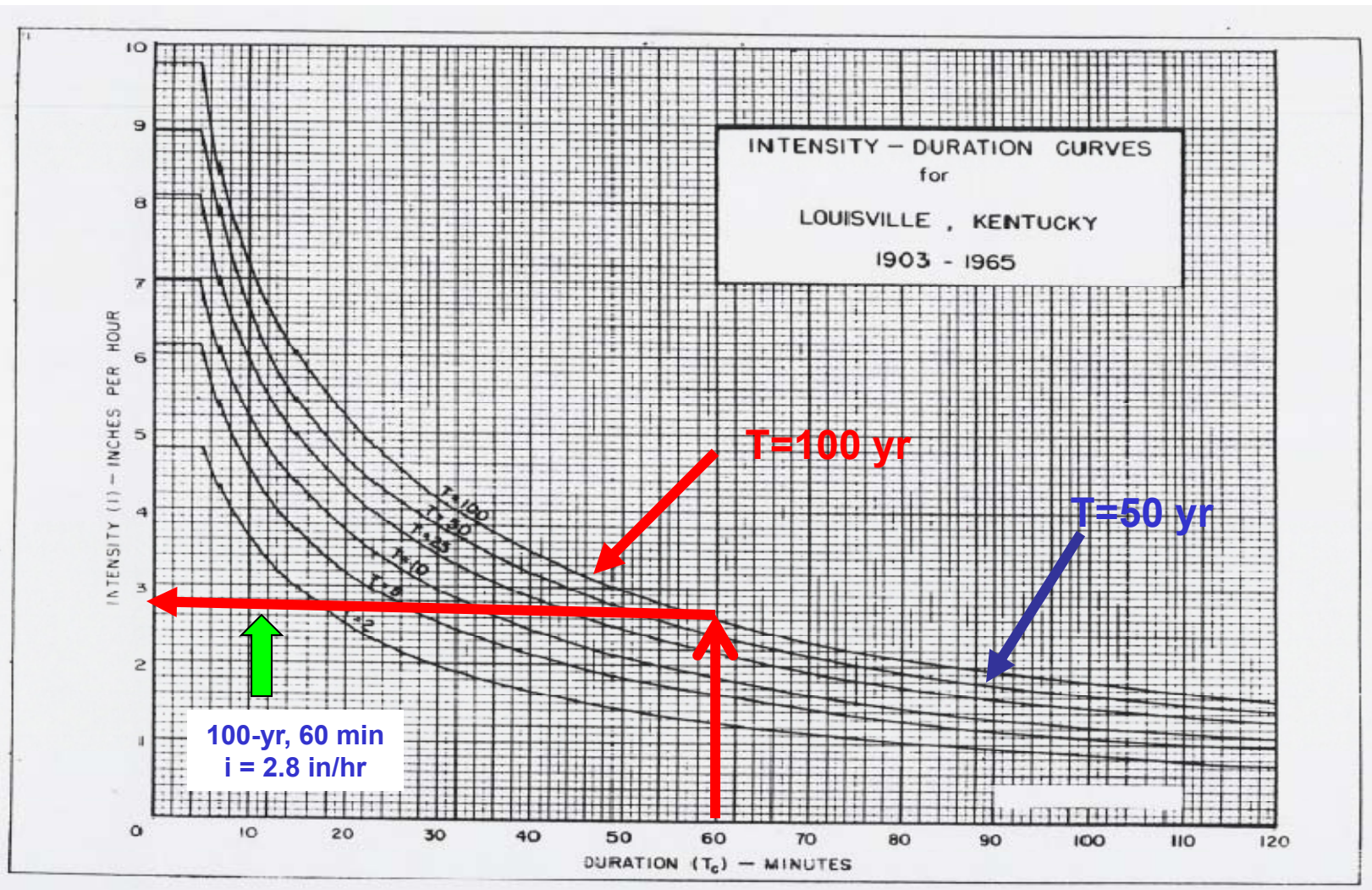
Fre- quency, years	Area 1	Area 2	Area 3	Area 4	Area 5	Area 6	Area 7
2	$i = \frac{5230}{t + 30}$	$i = \frac{3550}{t + 21}$	$i = \frac{2590}{t + 17}$	$i = \frac{1780}{t + 13}$	$i = \frac{1780}{t + 16}$	$i = \frac{1730}{t + 14}$	$i = \frac{810}{t + 11}$
5	$i = \frac{6270}{t + 29}$	$i = \frac{4830}{t + 25}$	$i = \frac{3330}{t + 19}$	$i = \frac{2460}{t + 16}$	$i = \frac{2060}{t + 13}$	$i = \frac{1900}{t + 12}$	$i = \frac{1220}{t + 12}$
10	$i = \frac{7620}{t + 36}$	$i = \frac{5840}{t + 29}$	$i = \frac{4320}{t + 23}$	$i = \frac{2820}{t + 16}$	$i = \frac{2820}{t + 17}$	$i = \frac{3100}{t + 23}$	$i = \frac{1520}{t + 13}$
25	$i = \frac{8300}{t + 33}$	$i = \frac{6600}{t + 32}$	$i = \frac{5840}{t + 30}$	$i = \frac{4320}{t + 27}$	$i = \frac{3300}{t + 17}$	$i = \frac{3940}{t + 26}$	$i = \frac{1700}{t + 10}$
50	$i = \frac{8000}{t + 28}$	$i = \frac{8890}{t + 38}$	$i = \frac{6350}{t + 27}$	$i = \frac{4750}{t + 24}$	$i = \frac{4750}{t + 25}$	$i = \frac{4060}{t + 21}$	$i = \frac{1650}{t + 8}$
100	$i = \frac{9320}{t + 33}$	$i = \frac{9520}{t + 36}$	$i = \frac{7370}{t + 31}$	$i = \frac{5590}{t + 28}$	$i = \frac{6100}{t + 29}$	$i = \frac{5330}{t + 26}$	$i = \frac{1960}{t + 10}$

**Example 4: Estimate the 100-yr, 1 hour design rainfall for Louisville, KY**

a) From the IDF Curves for Louisville, Kentucky:

Rainfall Intensity,  $i = 2.8$  in/hr

Design Rainfall depth =  $i \times t_d = 2.8 \times 1.0 = 2.8$  inches



**Example 4: Estimate the 100-yr, 1 hour design rainfall for Louisville, KY (cont.)**

**b) From the IDF Map and Table (Slides 17 and 18) Louisville is in Region 3.**

Regional IDF equation for  $T = 100$  yr :

$i$  (mm/hr) = { 7370/(t+31) } where  $t = t_d$  in minutes

$$i = 7370/(60+31)$$

$$= 80.989 \text{ mm/hr}$$

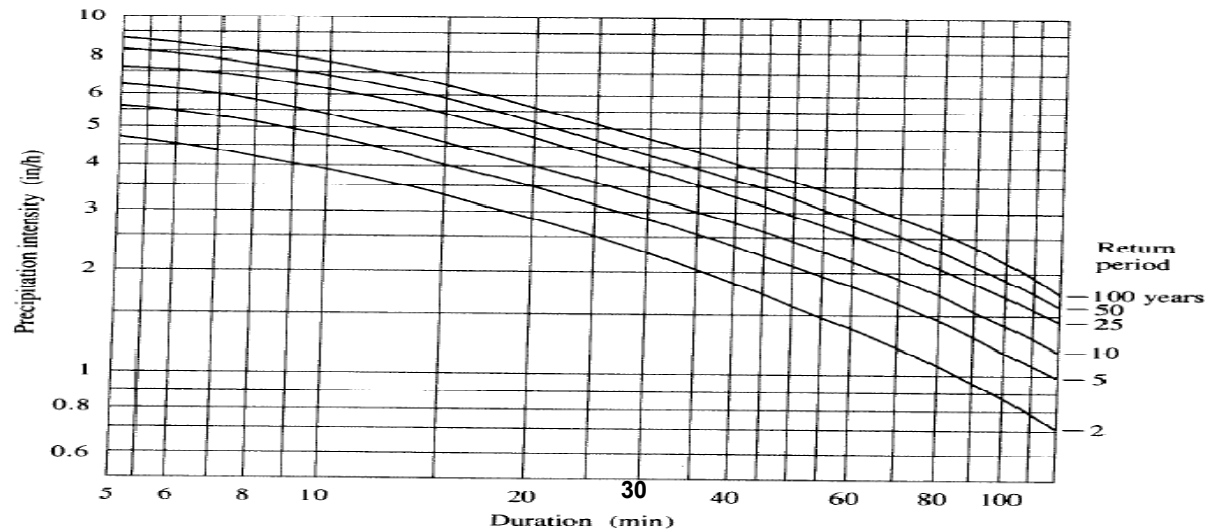
$$= 3.2 \text{ in/hr}$$

Rainfall Depth = 3.2 in

From Louisville IDF Curve  $P = 2.80$  in  
(see previous slide)

- Note the difference between using a locally developed IDF curve versus using a regional equation

## Problem 2: Using IDF Curves: Chicago, Illinois (Chow, 1988)



**FIGURE 14.2.1**  
Intensity-duration-frequency curves of maximum rainfall in Chicago, U. S. A.

### Determine:

1. The 100-year, 60 min rainfall intensity for Chicago in in/hr is:  
a) 2.5;                      b) 3.2;                      c) 3.0;                      d) 6.0
2. The 100-year, 30 min rainfall intensity (in in/hr) using the Chicago IDF Equation  $i \text{ (mm/hr)} = 7370/(t+31)$  (from region 3; see slides 17 and 18):  
a) 4.5;                      b) 3.5;                      c) 2.8;                      d) 4.8
3. The 100-year, 30 min rainfall depth in inches is:  
a) 3.0;                      b) 4.0;                      c) 2.4;                      d) 1.5

# Abstractions

Effective Rainfall (or rainfall excess),  $P_e$  equals

Rainfall (P) minus

Abstractions due to:

**Infiltration**

**Depression Storage**

**Interception**

**Rainfall Excess = Volume of Direct Runoff, Q**



## Abstractions (Cont.)

- **Main Abstraction Process is Infiltration**
- **Interception and Depression Storage occur in the early stages of event and can be considered as initial losses**
- **Some methods for infiltration:**
  - **Infiltration Indices:  $\Phi$  index**
  - **Runoff Coefficients: Rational Method – C**
  - **SCS Curve Number Method, CN**
  - **Infiltration Capacity Curves: Horton's**

## Example 5: Rainfall Excess and Volume of Direct Runoff

### Given:

- A direct runoff hydrograph from a 7.0 sq. mi watershed.
- A uniform loss rate,  $\Phi = 0.50$  in/hr (or 0.25 in/1/2hr).
- Calculate volume of direct runoff in inches.
- Show volume of direct runoff = volume of rainfall excess

Time Interval,  $\Delta t = 0.5$  hrs

Note: Rainfall intensity,  $i > \Phi$  in all time intervals.

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Time Period (1/2 hr):	1	2	3	4	5	6	7	8	9	10	11
Rainfall Intensity, $i$ (in/hr):	2.7	4.30	4.10								
Direct Runoff, $Q$ (cfs):	430	1920	5300	9130	10625	7830	3920	1845	1400	830	310

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## Example 5: Rainfall Excess and Volume of Direct Runoff (cont.)

### Solution:

#### Volume of Rainfall Excess, $P_e$

$$\begin{aligned} &= \{(2.70-0.50) \times 0.5\} + \{(4.30-0.50) \times 0.5\} + \{(4.1-0.50) \times 0.5\} \\ &= \mathbf{4.8 \text{ inches}} \end{aligned}$$

#### Volume of Direct Runoff, $V_d = \sum Q_n \times \Delta t$

$$\begin{aligned} &= (430 + 1920 + 5300 + 9130 + 10625 + 7830 + 3920 + \\ &\quad 1845 + 1400 + 830 + 310) \times 0.5 \\ &= (43,540 \times 0.5) = 21,770 \text{ cfs-hrs} \\ &= (21,770 \times 3600) = 78,372,000 \text{ ft}^3 \end{aligned}$$

#### Direct Runoff Depth, $r_d$ (inches)

$$\begin{aligned} &= (\text{direct runoff volume in ft}^3 / \text{drainage area in ft}^2) \\ &= 78,372,000 / (7 \times 5280^2) = 0.40 \text{ ft.} \\ &= \mathbf{4.8 \text{ inches}} \end{aligned}$$

Note: Rainfall Excess Depth,  $P_e =$  Direct Runoff Depth,  $r_d = 4.8 \text{ in.}$

# Runoff Methods


## 1. Peak Discharge, $Q_p$ , Methods:

- Rational Method
- SCS Curve Number Method  
(and TR 55 Graphical Peak Discharge Method)

## 2. Unit Hydrograph Method

# Runoff – Small Catchments

## Rational Method

- Ideal for small catchments (less than 100 acres)
- Used widely in Urban Storm water sewer design
- Requires the following design variables:
  - a) **Runoff Coefficient C (refer to slide 29)**
  - b) **Maximum Rainfall intensity,  $i$  (in in/hr or mm/hr)**  
(obtained from IDF Curves or IDF Equations for a specified duration usually equal to time of concentration,  $t_c$ )
  - c) **Time of Concentration,  $t_c$**   Refer to Slides 17 and 18
  - d) **Catchment Area,  $A$  (in acres or km<sup>2</sup> or Hectares (Ha))**

# Rational Method-Peak Flow Formulae

- **US units:**  $Q_p = C i A$  (in cfs)

$i$  in in/hr  
Area  $A$  in acres

- **SI units:**  $Q_p = 0.278 C i A$  (in  $m^3/s$ )

$i$  in mm/hr  
Area  $A$  in  $km^2$

$$Q_p = 2.78 C i A \text{ (SI units) (in liters/s),}$$

$i$  in mm/hr  
Area  $A$  in Ha

Note: 1.0 Ha = 2.471 acres = 10,000  $m^2$  ; 1.0  $km^2$  = 100 Ha

## Table: Runoff Coefficients, C For the Rational Method:

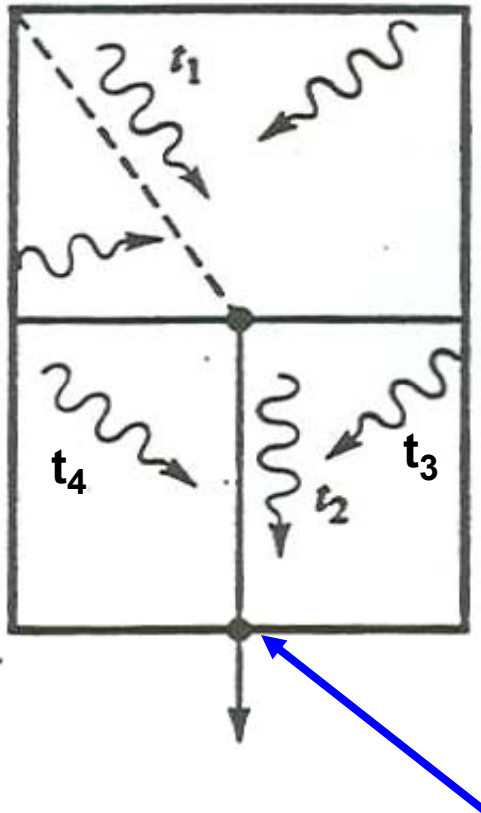
Character of surface	Return Period (years)						
	2	5	10	25	50	100	500
<b>Developed</b>							
Asphaltic	0.73	0.77	0.81	0.86	0.90	0.95	1.00
Concrete/roof	0.75	0.80	0.83	0.88	0.92	0.97	1.00
Grass areas (lawns, parks, etc.)							
<i>Poor condition (grass cover less than 50% of the area)</i>							
Flat, 0–2%	0.32	0.34	0.37	0.40	0.44	0.47	0.58
Average, 2–7%	0.37	0.40	0.43	0.46	0.49	0.53	0.61
Steep, over 7%	0.40	0.43	0.45	0.49	0.52	0.55	0.62
<i>Fair condition (grass cover on 50% to 75% of the area)</i>							
Flat, 0–2%	0.25	0.28	0.30	0.34	0.37	0.41	0.53
Average, 2–7%	0.33	0.36	0.38	0.42	0.45	0.49	0.58
Steep, over 7%	0.37	0.40	0.42	0.46	0.49	0.53	0.60
<i>Good condition (grass cover larger than 75% of the area)</i>							
Flat, 0–2%	0.21	0.23	0.25	0.29	0.32	0.36	0.49
Average, 2–7%	0.29	0.32	0.35	0.39	0.42	0.46	0.56
Steep, over 7%	0.34	0.37	0.40	0.44	0.47	0.51	0.58
<b>Undeveloped</b>							
Cultivated Land							
Flat, 0–2%	0.31	0.34	0.36	0.40	0.43	0.47	0.57
Average, 2–7%	0.35	0.38	0.41	0.44	0.48	0.51	0.60
Steep, over 7%	0.39	0.42	0.44	0.48	0.51	0.54	0.61
Pasture/Range							
Flat, 0–2%	0.25	0.28	0.30	0.34	0.37	0.41	0.53
Average, 2–7%	0.33	0.36	0.38	0.42	0.45	0.49	0.58
Steep, over 7%	0.37	0.40	0.42	0.46	0.49	0.53	0.60
Forest/Woodlands							
Flat, 0–2%	0.22	0.25	0.28	0.31	0.35	0.39	0.48
Average, 2–7%	0.31	0.34	0.36	0.40	0.43	0.47	0.56
Steep, over 7%	0.35	0.39	0.41	0.45	0.48	0.52	0.58

# Time of Concentration, $t_c$

## Definition:

$t_c = \sum$  travel times from the hydraulically remotest point in a catchment  
= (Overland flow time)  
+ (Channel or Pipe flow) time

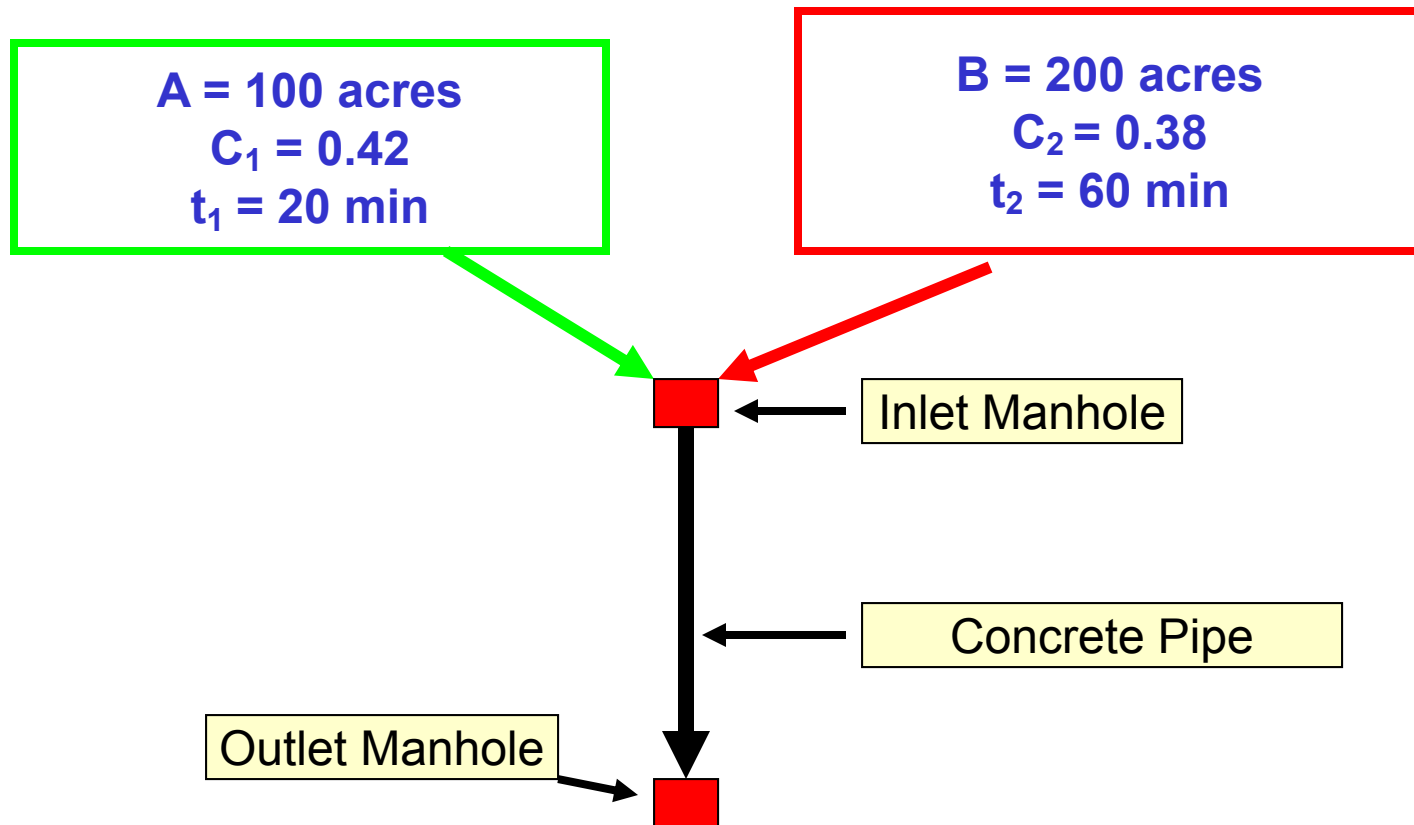
Refer to Appendix A for Methods for Computing Time of Concentration



Example:  $t_c$  at this outlet  
=  $\max(t_1 + t_2)$  or  $t_3$  or  $t_4$

# Example 6: Rational Method for Catchment Runoff Peak

- Two watersheds A and B (located in Tallahassee, Florida) drain at a common outlet. Determine the peak outflow,  $Q_p$ , in cfs for a design return period  $T = 25$  years. Also calculate the size of a circular concrete pipe (Manning's  $n = 0.025$ , slope = 0.005) which can handle the peak flow. IDF Equation for Tallahassee, Florida,  $i$  (in mm/hr) =  $\{ 8300/(t+33) \}$  with  $t$  in min



## Example 6: Rational Method for Catchment Runoff Peak (cont.)

- Composite  $C_c = (C_1A_1 + C_2A_2)/(A_1+A_2)$
- $= (0.42 \times 100 + 0.38 \times 200)/(100+200) = 0.39$

Note for multiple areas:  
 $Q_p = C_c i A_T = (\sum C_i i / \sum A_i) A_T = i \sum C_i A_i$   
where total area,  $A_T = \sum A_i$

- Time of Concentration,  $t_c = \max(t_1, t_2) = \max(20, 60) = 60 \text{ min}$

Tallahassee, FL, IDF Eq. Area 1 (slide 18)  
 $i \text{ (mm/hr)} = \{8300/(t+33)\}$  with  $t$  in min

- 25-year Rainfall intensity  $I$  for  $t_c = 60 \text{ min} = 3.5 \text{ in/hr}$

Rational Method Formula (US units)

- Peak Flow  $Q_p = C_c i A = 0.39 \times 3.5 \times 300 = 409.5 \text{ cfs}$

From Manning's (US Units)  
 $D = (2.16 \times Q_p n / \sqrt{S_o})^{3/8}$

- Pipe diameter,  $D = \{ 2.16 \times 409.5 \times 0.020 / (0.005)^{0.5} \}^{3/8} = 7.93 \text{ feet}$   
 $= 95.2 \text{ inches (Use 96 inch pipe)}$



# Problem 3: Application of Rational Method in Storm Sewer Design

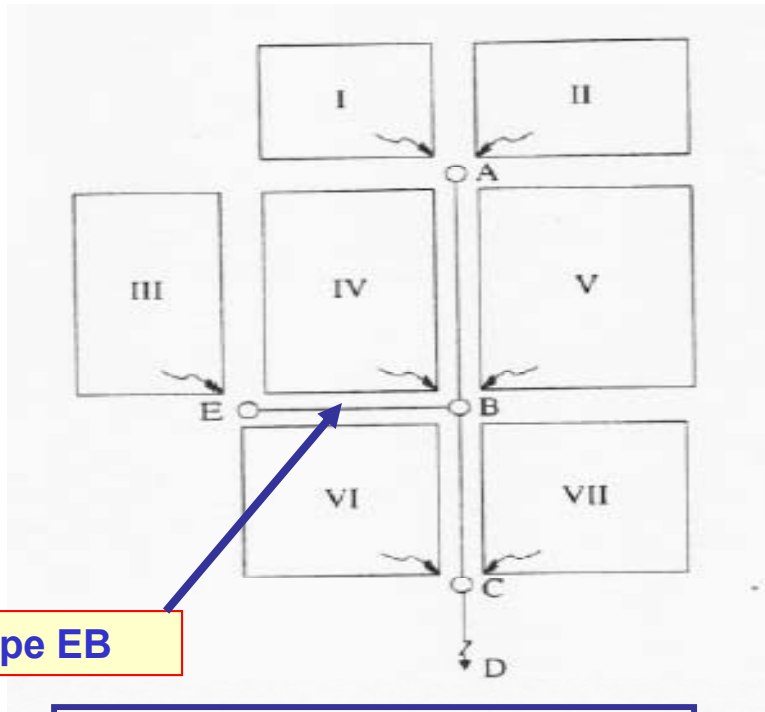
**Given:**

**IDF Equation for the area is:**

$$i \text{ (in/hr)} = \frac{120T^{0.175}}{(t_d + 27)}$$

**where T = Return Period in years**

**$t_d$  = Storm duration = time of concentration,  $t_c$  (in minutes)**



**Pipe EB**

Elv. E = 498.43 ft; Elv. B = 495.55  
Length of pipe EB = 450 ft

Catchment	Area A (acres)	Runoff coefficient C	Inlet time $t_i$ (min)
I	2	0.7	5
II	3	0.7	7
III	4	0.6	10
IV	4	0.6	10
V	5	0.5	15
VI	4.5	0.5	15
VII	4.5	0.5	15

**Determine Peak Flow from Sub-area Area III and Pipe size EB Using Rational Method**

### Problem 3: Application of Rational Method in Storm Sewer Design (cont.)

Answer the following questions for a rainfall event with a return period  $T = 5\text{-yrs}$  :

1) The maximum rainfall intensity  $i$  in in/hr is:

- a) 2.5;                      b) 5.2;                      c) 4.3;                      d) 3.5

2) The peak flow  $Q_p$  (in cfs) using rational method formula) from Area III into inlet E is:

- a) 5.5;                      b) 10.3;                      c) 15.2;                      d) 75.5

$$Q_p = CiA$$

3) The slope  $S_0$  of pipe EB is:

- a) 0.005;                      b) 0.00034;                      c) 0.0002;                      d) 0.0064

4) The required diameter of sewer pipe EB (in inches) to handle peak flow  $Q_p$  is (assume  $n = 0.015$ ):

- a) 15.2;                      b) 32.6;                      c) 25.8;                      d) 20.5

From Manning's (US Units)

$$D = (2.16xQ_p n / \sqrt{S_0})^{3/8}$$

### Problem 3: Application of Rational Method in Urban Storm Sewer Design- Complete Solution (cont.)

1 Sewer pipe	2 Length $L$ (ft)	3 Slope $S_o$ (ft/ft)	4 Total area drained (acres)	5 $\Sigma CA$	6 $t_c$ (min)	7 Rainfall intensity $i$ (in/hr)	8 Design discharge $Q$ (cfs)	9 Computed sewer diameter (ft)	10 Pipe size used (ft)	11 Flow velocity $Q/A$ (ft/s)	12 Flow time $L/V$ (min)
EB	450	0.0064	4	2.4	10.0						
AB	550	0.0081	5	3.5	7.0	4.68	16.4	1.94	2.00	5.21	1.76
BC	400	0.0064	18	10.8	15.0	3.79	40.9	2.87	3.00	5.78	1.15
CD	450	0.0064	27	15.3	16.2	3.68	56.3	3.22	3.50	5.85	1.28

Solution in the previous slide

Note for multiple areas:

$$\text{Peak Flow, } Q = C_c i A_T = (\Sigma C_i i / \Sigma A_i) A_T = i \Sigma C_i A_i$$

where total area,  $A_T = \Sigma A_i$

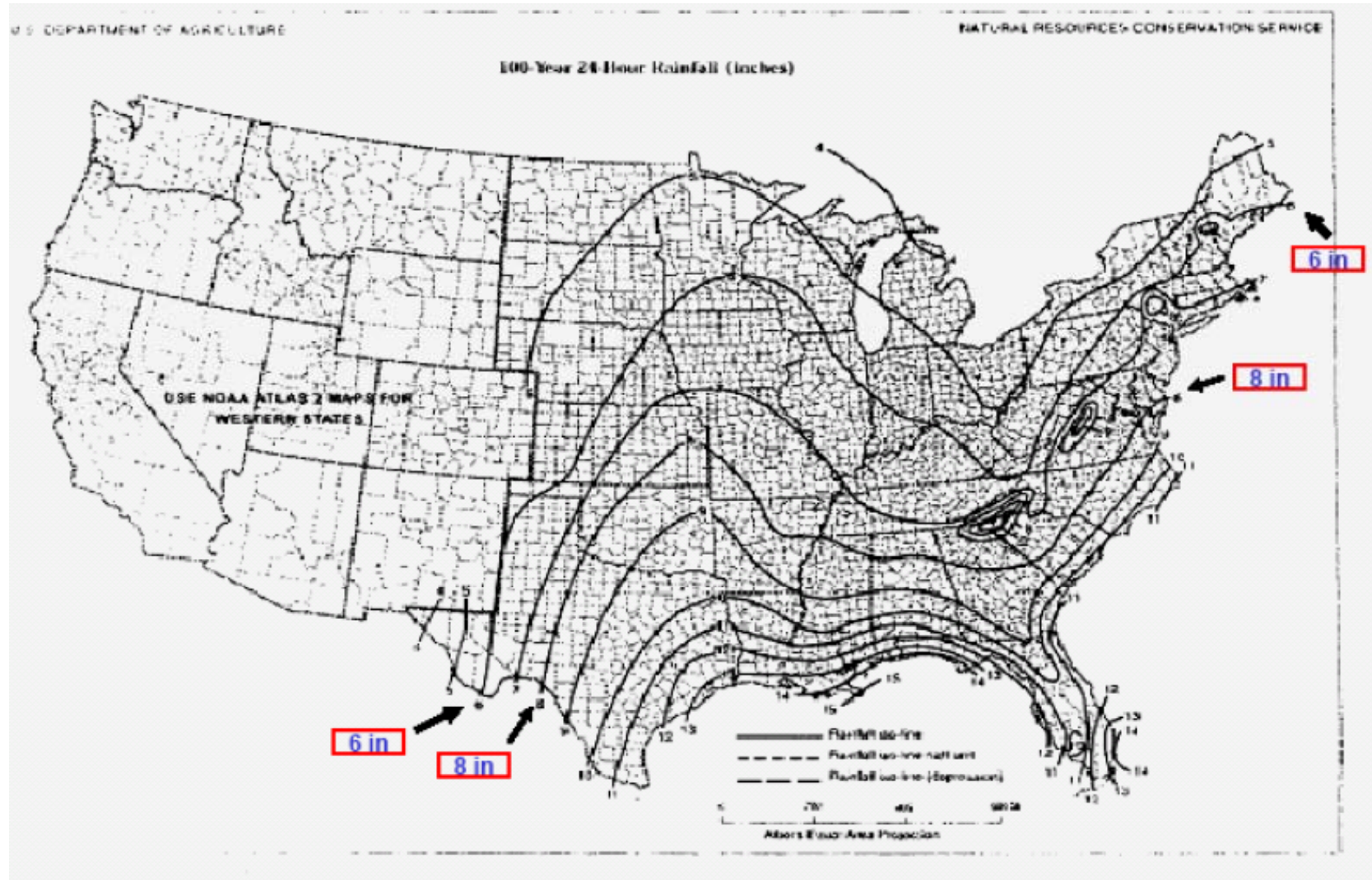
# **Runoff - Midsize Catchments:**

## **SCS TR55 Method**

# Runoff - Midsized Catchments: SCS TR55 Method

## Requires:

- Cumulative 24-hour Design Rainfall depth,  $P$  (in inches) for a selected return period,  $T$  (frequency)





## Runoff - Midsize Catchments: SCS TR55 Method (cont.)

**Requires:** SCS Curve Number, CN, based on Land use, Soil, and Antecedent Moisture Condition (AMC)

**Runoff curve numbers for selected agricultural, suburban, and urban land uses (antecedent moisture condition II,  $I_a = 0.25$ )**

Land Use Description	Hydrologic Soil Group			
	A	B	C	D
Cultivated land <sup>1</sup> : without conservation treatment	72	81	88	91
with conservation treatment	62	71	78	81
Pasture or range land: poor condition	68	79	86	89
good condition	39	61	74	80
Meadow: good condition	30	58	71	78
Wood or forest land: thin stand, poor cover, no mulch	45	66	77	83
good cover <sup>2</sup>	25	55	70	77
Open Spaces, lawns, parks, golf courses, cemeteries, etc. good condition: grass cover on 75% or more of the area	39	61	74	80
fair condition: grass cover on 50% to 75% of the area	49	69	79	84
Commercial and business areas (85% impervious)	89	92	94	95
Industrial districts (72% impervious)	81	88	91	93
Residential <sup>3</sup> :				
Average lot size	Average % impervious <sup>4</sup>			
1/8 acre or less	65	77	85	90
1/4 acre	38	61	75	83
1/3 acre	30	57	72	81
1/2 acre	25	54	70	80
1 acre	20	51	68	79
Paved parking lots, roofs, driveways, etc. <sup>5</sup>	98	98	98	98
Streets and roads:				
paved with curbs and storm sewers <sup>5</sup>	98	98	98	98
gravel	76	85	89	91
dirt	72	82	87	89

<sup>1</sup>For a more detailed description of agricultural land use curve numbers, refer to Soil Conservation Service, 1972, Chap. 9

<sup>2</sup>Good cover is protected from grazing and litter and brush cover soil.

<sup>3</sup>Curve numbers are computed assuming the runoff from the house and driveway is directed towards the street with a minimum of roof water directed to lawns where additional infiltration could occur.

<sup>4</sup>The remaining pervious areas (lawn) are considered to be in good pasture condition for these curve numbers.

<sup>5</sup>In some warmer climates of the country a curve number of 95 may be used.

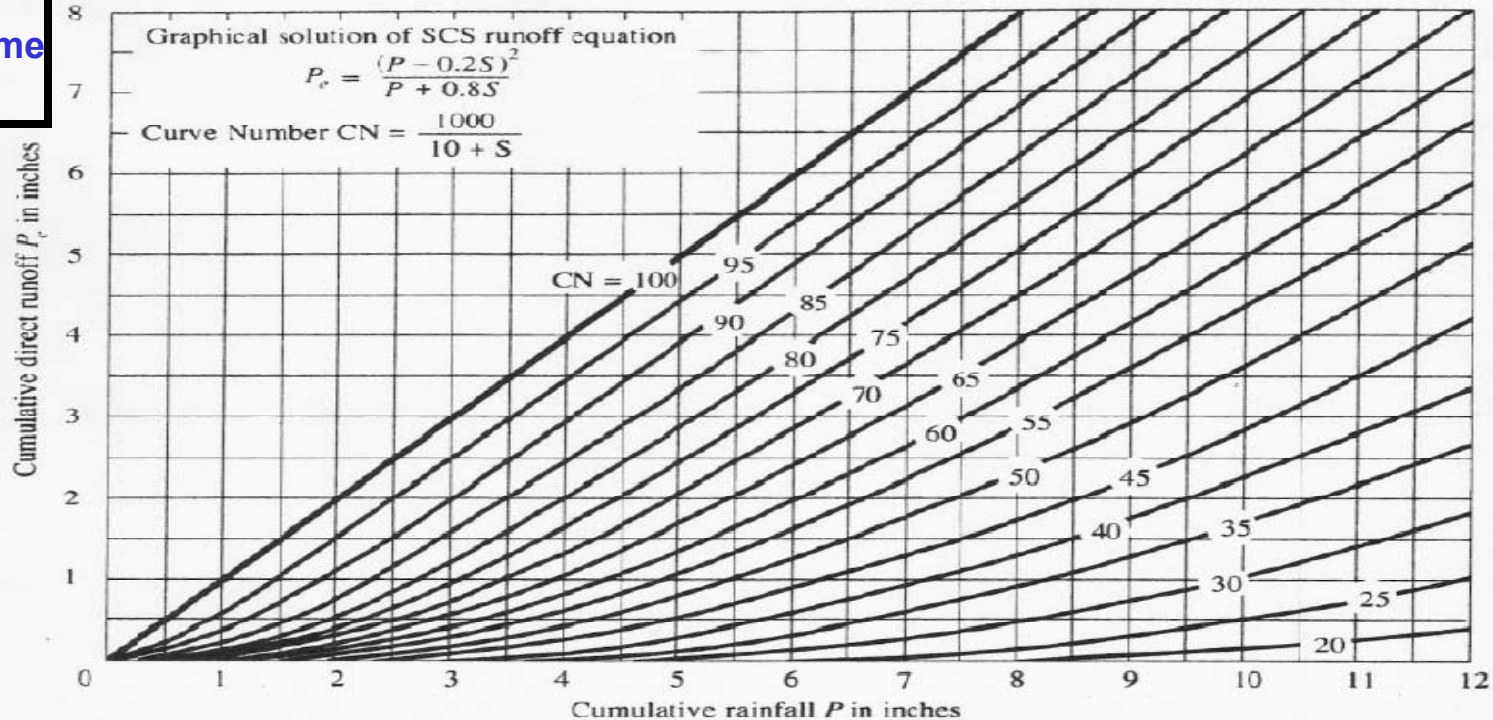
# Runoff - Midsize Catchments: SCS TR55 Method (cont.)

## Cumulative Direct Runoff Volume (inches), Q

Based on Cumulative Design Rainfall P, total Direct Runoff Volume is computed using an equation or figure shown below:

$$Q = (P - I_a)^2 / (P - I_a + S); \quad \text{Sorptivity (inches): } S = (1000 / CN) - 10$$
$$\text{Initial Abstraction (inches): } I_a = 0.2 S$$

Note:  
 $P_e$  is same  
as Q



Solution of the SCS runoff equations. (Source: Soil Conservation Service, 1972, Fig. 10.1, p. 10.21)

## Example 7: SCS CN and Direct Runoff Calculations (Chow et al., 1988)

- **Given:**

Note: 1.0 sq. mile = 259 ha = 640 acres  
or 1.0 ha = 2.471 acres

Rain falls on a 0.05 square miles (32 acres or 12.95 ha) urban catchment with an intensity of 0.85 in/hr (2.16 cm/hr) for a duration of 3 hours. The **soil** is classified as SCS hydrologic soil group (HSG) C in the entire catchment.

**Land use** within the catchment is as follows:

- 1) 20% area is business district (85% impervious; HSG =C)
- 2) 80% area is residential district (1/3 acre lots with 30% impervious)

- **Determine total runoff volume in inches and ft<sup>3</sup>.**




## Example 7: SCS CN and Direct Runoff Calculations (cont.) (Chow et al., 1988)

### Solution:

#### 1. Determine SCS Composite CN:

Note: From CN Table for HSG C (Slide 38):  
Imperious Area CN = 98  
Open space in good condition CN = 74



- Business District

$$\text{CN} = ( 0.85 \times 98 + 0.15 \times 74 ) = 94$$

- Residential District (1/3 acre lots)

$$\text{CN} = ( 0.30 \times 98 + 0.70 \times 74 ) = 81$$

- Composite CN for Catchment

$$\text{CN} = ( 0.20 \times 94 + 0.80 \times 81 ) = 83.6$$

Note: In computing Composite CN in urban areas any area not urbanized is assumed to be open space in good hydrologic condition (see SCS Curve Number, CN Table foot-note 4 – Slide 38).

## Example 7: SCS CN and Direct Runoff Calculations (cont.) (Chow et al., 1988)

### 2. Compute total rainfall:

Note: The design rainfall is not SCS 24 hr Type-II

$$P = 0.85 \text{ in/hr} \times 3 \text{ hours} = 2.55 \text{ inches}$$

### 3. Compute Runoff Volume Q (in inches):

$$\text{Sorptivity, } S = (1000/\text{CN}) - 10 = (1000/83.6) - 10 = 1.96 \text{ inches}$$

$$\text{Initial Abstraction, } I_a = 0.2S = 0.2 \times 1.96 = 0.392 \text{ inches}$$

$$\begin{aligned} \text{Direct runoff depth, } Q &= (P - I_a)^2 / (P - I_a + S) \\ &= (2.55 - 0.392)^2 / (2.55 - 0.392 + 1.96) \\ &= 1.13 \text{ inches} \end{aligned}$$

$$\begin{aligned} \text{Direct runoff volume } V_d &= (1.13/12) \times (0.05 \times 640 \times 43,560) \\ &= 131,260.8 \text{ ft}^3 \\ &= (0.02832 \times 131,260.8) = 3,717.3 \text{ m}^3 \end{aligned}$$

Note:  $1.0 \text{ ft}^3 = 0.02832 \text{ m}^3$

## Problem 4: Calculating SCS CN and Direct Runoff, Q

An undeveloped 1000 acre catchment currently is covered by pasture in good condition and is composed of hydrologic soil group C. This gives a **pre-development** composite SCS CN equal to 74

A proposed urban development (**post development**) will change the land use to:

- 1) 55% 1/3 acre lots (30% impervious), CN = 81;
- 2) 20 % in open space in good condition, CN = 74;
- 3) 25% in roads, sewers and parking lots, CN = 98.

**Note: Refer to SCS CN Table, Slide 38 for curve numbers.**



## Problem 4: Calculating SCS CN and Direct Runoff, Q (Cont.)

Assuming AMC II condition answer the following questions:

1) The **post-development** composite SCS CN is:

- a) 90.2                      b) 78.6                      c) 83.9                      d) 89.5

2) The **pre-development** direct runoff Q associated with a 10-year, 24-hour rainfall of  $P = 3.2$  inches is (**Note: CN = 74**) :

(Use Figure – Slide 39 or Equation:  $Q = P_e = (P - I_a)^2 / (P - I_a + S)$ ) :

- a) 2.5                      b) 1.5                      c) 1.0                      d) 2.1

3) The **post-development** direct runoff Q associated with a 10-year, 24-hour rainfall of 3.2 inches is:

(Use Figure – Slide 39 or Equation:  $Q = P_e = (P - I_a)^2 / (P - I_a + S)$ ) :

- a) 3.2                      b) 2.8                      c) 1.3                      d) 1.7

Note: For Q. 2 & 3:  $S = (1000/CN) - 10$ ;  $I_a = 0.2S$

## Runoff - Midsize Catchments: SCS TR55 Method

### Peak Discharge, $Q_p$ computation:

- **Peak Flow (cfs):**  $Q_p = q_u A Q F$

where:

$q_u$  = unit peak discharge (cfs/sq. mi/in)

$A$  = watershed size in sq. miles

$Q$  = Volume of direct runoff in inches

$F$  = Pond Factor (depends on % natural storage in ponds and lakes. Assume 1.0 if storage negligible)

•Requires:

- Unit peak discharge,  $q_u$ , based on Graphical Method

- Time of Concentration,  $t_c = (\sum \text{Overland} + \text{Channel Flow})$

# **Runoff - Midsize Catchments: SCS TR55 Method**

## **Computing Catchment's Time of Concentration**

**SCS TR55 uses the following flow paths for computing catchment's time of concentration:**

- 1. Overland Sheet Flow (<300 feet)**
- 2. Overland Shallow Concentrated Flow**
- 3. Channel or Pipe Flow**

## SCS TR55 Method (cont.)

### 1) Equations for Computing Overland Sheet Flow Time:

**US Units:** 
$$t = \frac{0.007(nL)^{0.8}}{P_2^{0.5}S^{0.4}}$$

**SI Units:** 
$$t = \frac{0.02887(nL)^{0.8}}{P_2^{0.5}S^{0.4}}$$

where:

**t** = travel time in **hours**;

**S** = average land slope in **feet/foot** (or meters/meter in SI)

**n** = Manning's overland roughness coefficient (**see Slide 48**)

**L** = overland flow distance in **feet** (or meters for SI units)

**P<sub>2</sub>** = 2-Year, 24-hour rainfall depth in **inches** (or cms for SI units)

## Table: TR55 Manning n Values for Overland Sheet Flow

Surface Description	Manning <i>n</i>
Smooth surfaces (concrete, asphalt, gravel, or bare soil)	0.011
Fallow (no residue)	0.05
Cultivated ground (residue cover less than or equal to 20%)	0.06
(residue cover greater than 20%)	<b>0.17</b>
Grass	
Range, short prairie	0.15
Dense	0.24
Bermuda	0.43
Range	0.13
Woods	
Light underbrush	0.40
Dense underbrush	0.80

*Note:* Dense grass includes weeping lovegrass, bluegrass, buffalo grass, blue gamma grass, native grass mixture, alfalfa, and the like.



## SCS TR55 Method (cont.)

### 2) Computing Shallow Concentrated Overland Flow Time

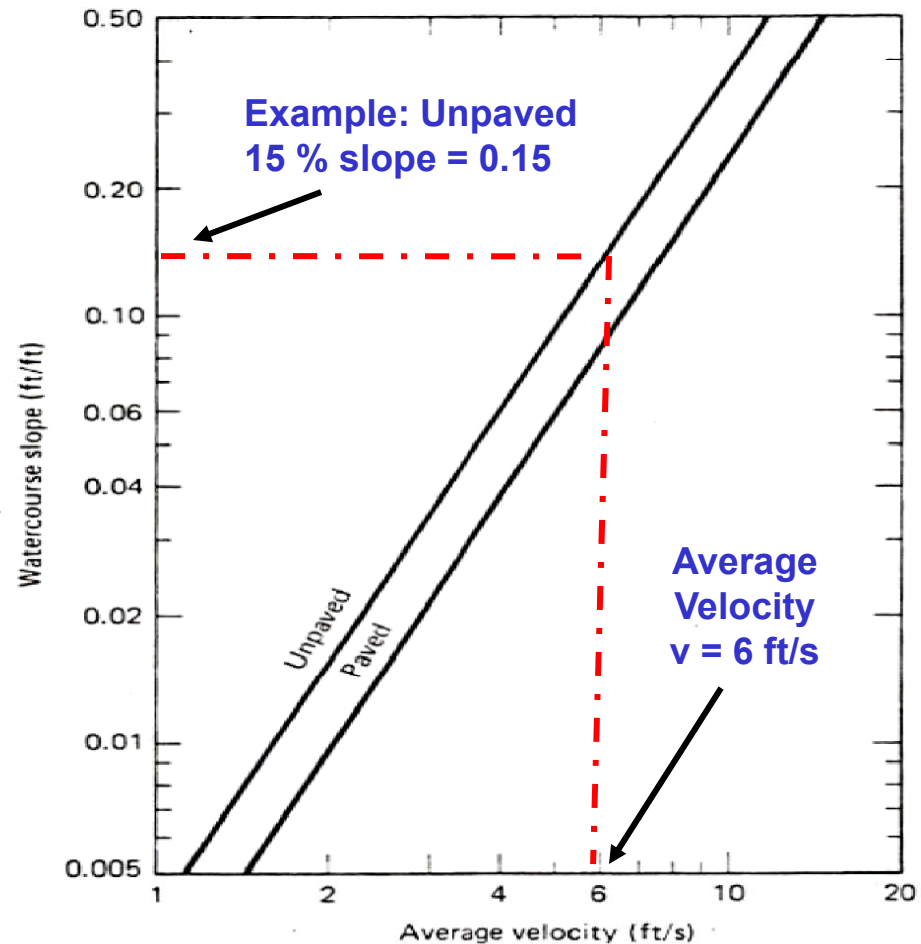
Figure: SCS TR55 Average Velocity for Shallow Concentrated Overland Flow

#### Shallow Concentrated

Overland Flow time is calculated as:

1. Given **overland flow path slope** use Figure to get overland flow velocity,  $v$  given the overland flow surface as **paved** or **unpaved**.
2. Given the length  $L$  of the overland flow path Travel time =

$$t_2 = L / V$$



## SCS TR55 Method (cont.)

### 3) Computing Average Velocity for Channel or Pipe Flow Time

#### Procedure:

Step 1: Use Manning's Formula to compute velocity in pipe or channel:

$$V = (1.49/n) R^{2/3} S_0^{1/2} \quad (\text{US Units})$$

$$V = (1.0/n) R^{2/3} S_0^{1/2} \quad (\text{SI Units})$$

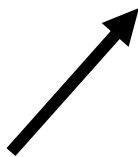
(Note: For full pipe flow condition  $R = d_0/4$  where  $d_0$  pipe diameter)

Step 2: Compute travel time = Pipe or channel length / velocity =  $L/V$

## Example 8: TR-55 Time of Concentration Computation

A 300 acre watershed drains along the path ED→DC→CB→BA shown in the table below. Determine the time of concentration,  $t_c$  using SCS TR55 method

<u>Hydraulic Path</u>	<u>Type of Flow</u>	<u>Slope (%)</u>	<u>Length (ft)</u>
ED	Overland Sheet Flow	5.0	100
DC	Overland Gutter Flow (unpaved)	1.5	300
CB	Pipe Flow ( $d_0 = 24$ in; $n = 0.015$ )	1.0	3000
BA	Open Channel Flow ( $y = 2$ ft; $n = 0.02$ ) (For a wide rectangular: Hydraulic Radius $R =$ flow depth, $y$ )	0.5	5000



**Note:** The pipe is 24 inches in diameter with a Manning's  $n = 0.015$   
 The open channel is wide rectangular with main bank flow depth = 2.0 ft.  
 and Manning's  $n = 0.020$ .

## Example 8: TR-55 Time of Concentration Computation (cont.)

### 1. Path ED (Overland Sheet flow):

Bermuda Grass – roughness coefficient from slide 48,  $n = 0.43$

2-yr, 24 hour cumulative rainfall depth,  $P_2 = 3.2$  inches

Overland surface slope,  $S = 0.05$

Overland flow length,  $L = 100$  feet

From Equation – Slide 47

$$\begin{aligned} \text{Travel time, } t_{ED} &= \frac{0.007(nL)^{0.8}}{P_2^{0.5}S^{0.4}} = \frac{0.007(0.43 \times 100)^{0.8}}{(3.2)^{0.5}(0.05)^{0.4}} \\ &= 0.263 \text{ hrs} = 15.8 \text{ min} = 946.3 \text{ s} \end{aligned}$$

### 2. Path DC (Overland Concentrated Flow – Unpaved Surface):

Overland flow slope,  $S = 0.015$

Overland flow velocity,  $V = 1.8$  ft/s

From Figure – Slide 49

$$\text{Travel time, } t_{DC} = L/V = 300/1.8 = 166.67 \text{ s} = 2.78 \text{ min} = 0.046 \text{ hrs.}$$

## Example 8: TR-55 Time of Concentration Computation (cont.)

### 3. Path CB (Pipe Flow):

Mannings,  $n = 0.015$ ;

Pipe slope,  $S = 0.01$ ;

Pipe length,  $L = 3000$  ft

Pipe diameter = 24 inches = 2.0 feet

Pipe cross sectional area,  $A = 3.14 \times 2^2 / 4 = 3.14$  ft<sup>2</sup>

Pipe wetted perimeter,  $P = 3.14 \times 2 = 6.28$  ft

Pipe Hydraulic Radius,  $R = A/P = 3.14/6.28 = 0.50$

Velocity,  $V = (1.49/0.015) \times 0.5^{2/3} \times 0.01^{1/2} = 6.26$  ft/s

From Manning's Formula  
(US units) – Slide 40

Travel time in pipe,  $t_{CB} = L/V = 3000/6.26 = 479.2$  s  
 $= 7.987$  min = 0.113 hrs

## Example 8: TR-55 Time of Concentration Computation (cont.)

### 4. Path BA (Open Channel Flow):

From Manning's Formula  
(US units) - Slide 40

Mannings,  $n = 0.02$ ; Channel slope,  $S = 0.005$ ;

Channel length,  $L = 5000$  ft

Flow depth,  $y = 2.0$  feet

Hydraulic Radius,  $R = y = 2$  feet (wide rect. channel)

Velocity,  $V = (1.49/0.02) \times 2.0^{2/3} \times 0.005^{1/2} = 8.36$  ft/s

Travel time in pipe,  $t_{CB} = L/V = 5000/8.36$

$= 598.1$  s  $= 9.968$  min  $= 0.166$  hrs

**Watershed Time of Concentration,  $t_c =$  total travel time**

$$= \sum t = t_{ED} + t_{DC} + t_{CB} + t_{BA}$$

$$= 0.263 + 0.046 + 0.113 + 0.166 = 0.588 \text{ hours} = 35.28 \text{ min}$$

## Problem 5: SCS TR-55 - Time of Concentration Computation

Using SCS TR55 Method, calculate the time of concentration of a watershed given the following flow path:

- 1) **overland sheet flow** on dense grass, length  $L = 100$  ft; slope  $S = 0.01$ , 2-yr 24-hr rainfall  $P_2 = 3.6$  inches;
- 2) **shallow concentrated flow** on unpaved surface, length  $L = 1400$  ft, slope  $S = 0.01$ ;
- 3) **streamflow**, Manning's  $n = 0.05$ ; flow area  $A = 27$  ft<sup>2</sup>, wetted perimeter  $P = 28.2$  ft, slope  $S = 0.005$  and length  $L = 7300$  ft.

## Problem 5: TR-55 Time of Concentration Computation (cont.)

### 1) Overland Flow Time, $t_1$ (refer to slides 47 and 48):

$n = 0.24$  (dense grass);  $L = 100$  ft;  $S = 0.01$  and  $P_2 = 3.6$  in

$$t_1 = [0.007(nL)^{0.8}] / (P_2^{0.5} S^{0.4}) = \underline{\hspace{2cm}} \text{ min} =$$

### 2) Shallow Concentrated Flow, $t_2$ (use Figure – Slide 49) :

Unpaved;  $L = 1400$  ft;  $S = 0.01$

Average Velocity =        ft/s

Travel time,  $t_2 = L/V = \underline{\hspace{2cm}}$  sec =        min

### 3) Stream/ Channel Flow (use Manning's equation – Slide 50):

Manning's  $n = 0.05$ ;  $A = 27$  ft<sup>2</sup> ;  $P = 28.2$  ft;  $S = 0.005$ ;  $L = 7300$  ft

$$V = (1.49/n)R^{2/3}S^{1/2} = \underline{\hspace{2cm}} \text{ ft/s}$$

Travel time  $t_3 = L/V = \underline{\hspace{2cm}}$  sec =        min

**Total Travel Time (or time of concentration)**

$$= t_1 + t_2 + t_3 = t_c = \underline{\hspace{2cm}} \text{ min } \underline{\hspace{2cm}} \text{ hrs.}$$



# SCS TR55 Method – Peak Discharge Computation

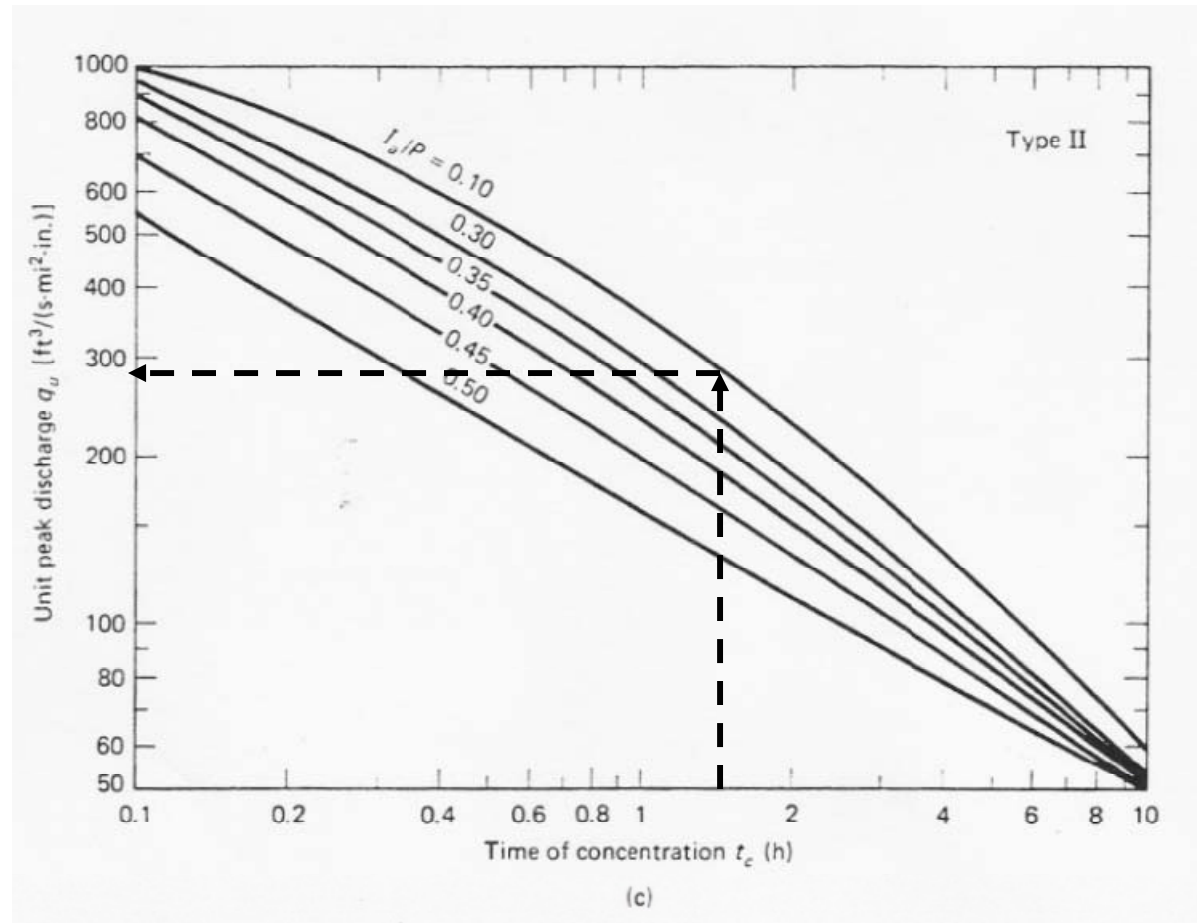
## Steps:

1. Compute watershed composite curve number CN;
2. Compute sorptivity,  $S$  (inches), initial abstraction,  $I_a$  (inches) and  $I_a/P$  ratio;
3. Compute direct runoff volume  $Q$  (inches);
4. Compute unit peak discharge,  $q_u$  (cfs/sq. mi/inch) given time of concentration,  $t_c$  and  $I_a/P$  ratio from Figure in Slide 58 (or similar curve for Type I and III )
5. Determine the pond factor,  $F$ .
6. Compute peak discharge  $q_p = q_u Q A F$

# SCS TR55 Method – Peak Discharge Computation

## Figure: Unit Peak Discharge Curves-Type II

Refer to USDA TR55 Manual for Type I-A, Type I-B and Type III Curves



## **Example 9: TR-55 Computation of Peak Flow Using SCSTR55 Graphical Peak Discharge Method**

### **Given:**

- **250 acre (0.39 sq. miles) watershed;**
- **25-year, 24hour Type II design rainfall  $P = 6$  inches;**
- **watershed time of concentration,  $t_c = 1.50$  hours;**
- **composite SCS Curve number  $CN = 75$**
- **Neglect storage in lakes and ponds.**

**Compute the peak discharge  $q_p$ .**

## Example 9: TR-55 Computation of Peak Flow Using SCSTR55 Graphical Peak Discharge Method (Cont..)

### Solution:

- $S = (1000/CN) - 10 = (1000/75) - 10 = 3.33$  inches;
- $I_a = 0.2 S = 0.2 \times 3.33 = 0.667$  inches;
- Ratio  $I_a / P = (0.667/6.0) = 0.11$ ;
- Runoff volume  $Q = (P - I_a)^2 / (P - I_a + S)$   
 $= (6.0 - 0.667)^2 / (6.0 - 0.667 + 3.33) = 3.28$  inches;
- Unit peak discharge,  $q_u = 285$  cfs/sq.mi/inch (Slide 58);
- Pond factor  $F = 1$ ;

for  $t_c = 1.5$  hours

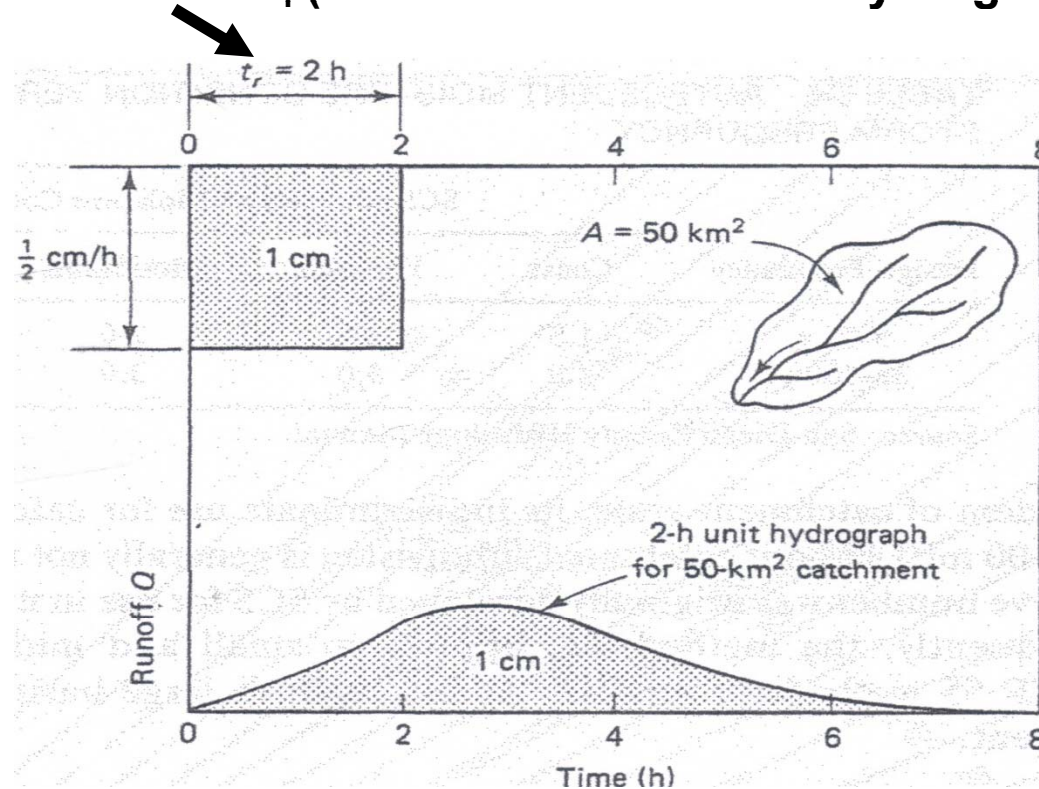
$$\begin{aligned} \text{Peak Discharge, } q_p &= q_u Q A F \\ &= 285 \times 3.28 \times 0.39 \times 1 = 364.6 \text{ cfs.} \end{aligned}$$

# Runoff Midsize Catchments

## Unit Hydrograph Method

- Unit Hydrograph Definition:**

The unit hydrograph of a watershed is defined a direct runoff hydrograph (DRH) resulting from 1 inch (or 1 cm in SI units) of excess rainfall generated uniformly over the drainage area at a constant rate for an effective duration  $t_r$  (referred to as the Unit Hydrograph duration)

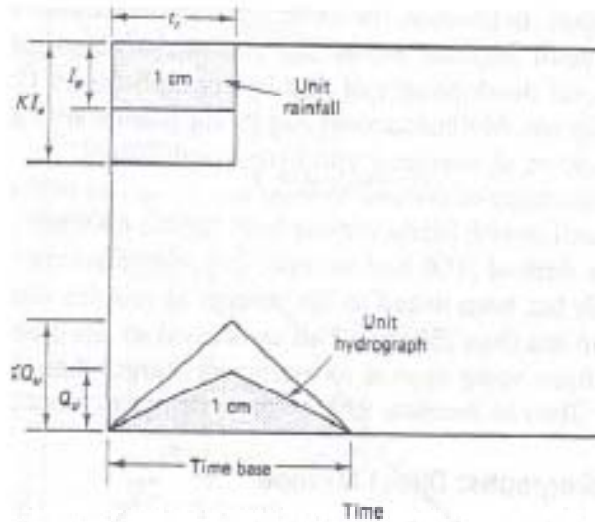


(Adapted from Ponce 1989)

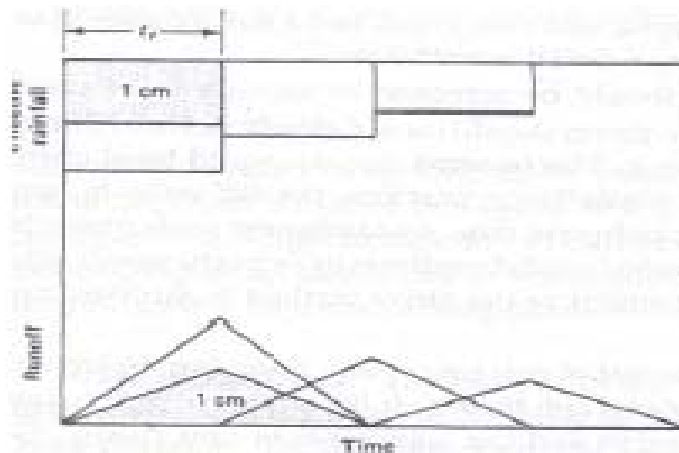
# Unit Hydrograph Properties

(Adapted from Ponce 1989)

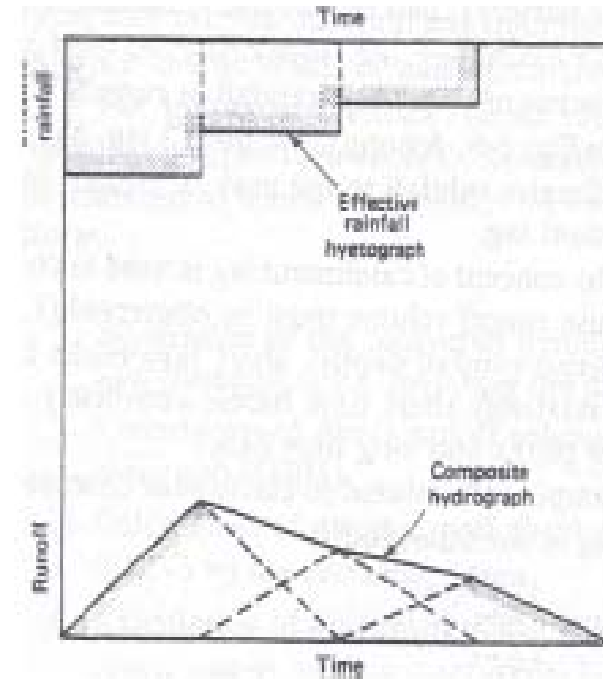
Linearity



Lagging



Superposition



## Direct Runoff Convolution (Adapted from Chow et al (1988))

The set of equations for discrete time convolution  $Q_n = \sum_{m=1}^{n \leq M} P_m U_{n-m+1};$   
 $n = 1, 2, \dots, N$

---


$$\begin{aligned}
 Q_1 &= P_1 U_1 \\
 Q_2 &= P_2 U_1 + P_1 U_2 \\
 Q_3 &= P_3 U_1 + P_2 U_2 + P_1 U_3 \\
 &\dots \\
 Q_M &= P_M U_1 + P_{M-1} U_2 + \dots + P_1 U_M \\
 Q_{M+1} &= 0 + P_M U_2 + \dots + P_2 U_M + P_1 U_{M+1} \\
 &\dots \\
 Q_{N-1} &= 0 + 0 + \dots + 0 + 0 + \dots + P_M U_{N-M} + P_{M-1} U_{N-M+1} \\
 Q_N &= 0 + 0 + \dots + 0 + 0 + \dots + 0 + P_M U_{N-M+1}
 \end{aligned}$$


---

In the equations above:

$P_m$  = rainfall excess pulses (inches or cms)

$U_j$  = unit hydrograph ordinates cfs/in or m<sup>3</sup>/s/cms)

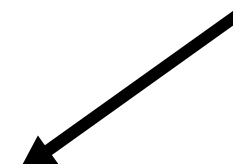
$Q_n$  = Direct runoff ordinates (cfs)

$N$  = number of non-zero direct runoff ordinates,  $Q_n$

$M$  = number of rainfall excess pulses in the hyetograph,  $P_m$

$J$  = number of Unit Hydrograph ordinates,  $U_j$

$N = M + J - 1$



## **Example 10: Derivation of Direct Runoff and Streamflow Hydrographs Using a Given Unit Hydrograph (Chow et al 1988)**

### **Given:**

- **1/2 hour unit hydrograph**
- **Storm of 6 inches total rainfall excess depth with:**
  - 2 inches first half hour ( $P_1$ ),**
  - 3 inches in the second half hour ( $P_2$ )**
  - 1 inch in the third half hour ( $P_3$ )**
- **Base Flow = 500 cfs**
- **Determine:**
  - Direct Runoff and the Streamflow Hydrograph.**



# Example 10: Derivation of Direct Runoff and Stream Flow Hydrographs Using a Given Unit Hydrograph (cont.)

Solution by Convolution where  $N = M + J - 1 = 3 + 9 - 1 = 11$  ordinates

Time Interval  $\Delta t = \frac{1}{2}$  hr    **J = 9**     $\frac{1}{2}$  Hour Unit Hydrograph    Includes base flow = 500 cfs

Time (h)	Excess Precipitation (in)	Unit hydrograph ordinates (cfs/in)									Direct runoff (cfs)	Streamflow* (cfs)
		1	2	3	4	5	6	7	8	9		
		404	1079	2343	2506	1460	453	381	274	173		
n = 1	2.00	808									808	1308
2	3.00	1212	2158								3370	3870
3	1.00	404	3237	4686							8327	8827
4			1079	7029	5012						13,120	13,620
5				2343	7518	2920					12,781	13,281
6					2506	4380	906				7792	8292
7						1460	1359	762			3581	4081
8							453	1143	548		2144	2644
9								381	822	346	1549	2049
10									274	519	793	1293
11										173	173	673
											Total	54,438

Baseflow = 500 cfs.

# Problem 6: Derivation of Direct Runoff and Streamflow Hydrographs Using a Given Unit Hydrograph

Given: Rainfall hietograph, one-hour unit hydrograph and abstractions a constant rate of  $\Phi = 0.5$  in/hr.

Time (hrs):	1	2	3	4
Rainfall intensity, i (in/hr):	1.0	1.5	0.5	
1-hr Unit Hydrograph (cfs/in):	50	200	150	50

Answer the following questions by completing the spaces shown in green:

- a) Time (hrs): 1 2 3  
Rainfall Excess (inches):
- b) Time (hrs): 1 2 3 4 5  
Direct Runoff, Q (cfs)       
(see slide 65)
- c) Volume of Direct Runoff in cfs-hours =
- d) Watershed size in sq. mile =

**Thank you for listening to the presentation.**

**Remember to Review the following  
Appendix for:**

- 1. Hydrologic Routing**
- 2. Groundwater Hydrology – Well Hydraulics**
- 3. Additional examples on Unit Hydrograph**

**Good luck on the P.E. Exam**

**QED**

# REFERENCES

1. **Chow, Maidment, Mays, 1988. Applied Hydrology**
2. **Chow, ed. 1964. Handbook of Applied Hydrology**
3. **Domenico, Schwartz, 1998. Physical and Chemical Hydrogeology.**
4. **Freeze, Cherry, 1979. Groundwater**
5. **Maidment, ed. 1993. Handbook of Hydrology**
6. **Ponce, 1989. Engineering Hydrology**

# Answers

- **Problem 1 (Slide 9):** 1) a; 2) a .
- **Problem 1 (Slide 10):** 3) b; 4) c; 5) b; 6) c .
- **Problem 2 (Slide 21):** 1) b; 2) d; 3) c.
- **Problem 3 (Slide 34):** 1) c; 2) b; 3) d; 4) d.
- **Problem 4 (Slide 44):** 1) c; 2) c; 3) d.
- **Problem 5 (Slide 55):**
  - 1) Overland sheet flow: 0.296 hrs;
  - 2) Overland Shallow Concentrated flow: 0.229 hrs
  - 3) Stream Flow: 0.991 hrs
  - 4) Total Time of Concentration = 1.515 hours.
- **Problem 6 (Slide 66):**
  - a) Time (hrs): 1 2 3
  - Rainfall excess (inches): 0.5 1.0 0 (M=2)
  - b) Time (hrs): 1 2 3 4 5
  - Direct runoff (cfs): 25 150 275 175 50 (N=5)
  - c) Volume of direct runoff,  $V_d = 675 \text{ cfs-hours} = 2430,000 \text{ ft}^3$
  - d) Drainage Area = 0.697 sq. miles
- (Hint: use volume under unit hydrograph = 1 inch or Volume under direct runoff hydrograph)

# **APPENDIX A**

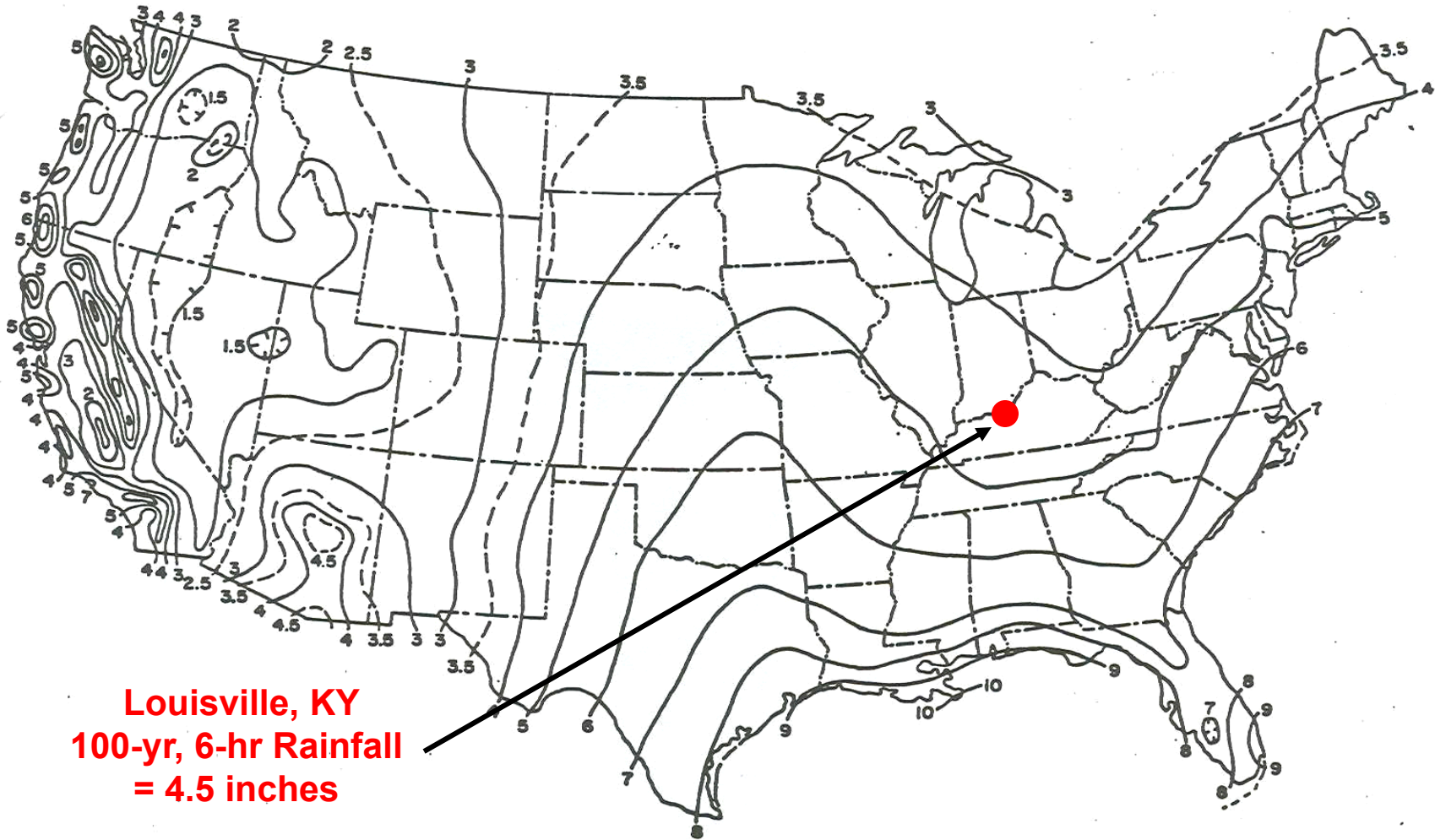
## **Hydrologic Design Components for SCS TR55**

# Table: Design Frequency or Return Period for Various Hydraulic Structures

## Generalized design criteria for water-control structures

Type of structure	Return period (years)
Highway culverts	
Low traffic	5-10
Intermediate traffic	10-25
High traffic	50-100
Highway bridges	
Secondary system	10-50
Primary system	50-100
Farm drainage	
Culverts	5-50
Ditches	5-50
Urban drainage	
Storm sewers in small cities	2-25
Storm sewers in large cities	25-50
Airfields	
Low traffic	5-10
Intermediate traffic	10-25
High traffic	50-100
Levees	
On farms	2-50
Around cities	50-200
Dams with no likelihood of loss of life (low hazard)	
Small dams	50-100
Intermediate dams	100+
Large dams	—
Dams with probable loss of life (significant hazard)	
Small dams	100+
Intermediate dams	—
Large dams	—
Dams with high likelihood of considerable loss of life (high hazard)	
Small dams	—
Intermediate dams	—
Large dams	—

## Figure: Rainfall Depth & Intensity



**Louisville, KY**  
**100-yr, 6-hr Rainfall**  
**= 4.5 inches**

100-year 6-hr rainfall (inches). (U.S. Weather Bureau [38].)  
adapted from Chow, Handbook of Applied Hydrology, McGraw - Hill 1964



# Methods for Computing Time of Concentration

- **Formulas**

- **Example: Kirpich, SCS Average Velocity Charts**

- **Approximate Velocities**

# Table: Formulas for Time of Concentration, $t_c$ (source: Chow et al, 1988)

## Summary of time of concentration formulas

Method and Date	Formula for $t_c$ (min)	Remarks
Kirpich (1940)	$t_c = 0.0078L^{0.77}S^{-0.385}$ $L$ = length of channel/ditch from headwater to outlet, ft $S$ = average watershed slope, ft/ft	Developed from SCS data for seven rural basins in Tennessee with well-defined channel and steep slopes (3% to 10%); for overland flow on concrete or asphalt surfaces multiply $t_c$ by 0.4; for concrete channels multiply by 0.2; no adjustments for overland flow on bare soil or flow in roadside ditches.
California Culverts Practice (1942)	$t_c = 60(11.9L^3/H)^{0.385}$ $L$ = length of longest <i>Source:</i> Kibler, 1982, Copyright by the American Geophysical Union. ----- between divide and outlet, ft	Essentially the Kirpich formula; developed from small mountainous basins in California (U. S. Bureau of Reclamation,
Izzard (1946)	$t_c = \frac{41.025(0.0007i + c)L^{0.33}}{S^{0.333}i^{0.667}}$ $i$ = rainfall intensity, in/h $c$ = retardance coefficient $L$ = length of flow path, ft $S$ = slope of flow path, ft/ft	Developed in laboratory experiments by Bureau of Public Roads for overland flow on roadway and turf surfaces; values of the retardance coefficient range from 0.0070 for very smooth pavement to 0.012 for concrete pavement to 0.06 for dense turf; solution requires iteration; product $i$ times $L$ should be $\leq 500$ .
Federal Aviation Administration (1970)	$t_c = 1.8(1.1 - C)L^{0.50}/S^{0.333}$ $C$ = rational method runoff coefficient $L$ = length of overland flow, ft $S$ = surface slope, %	Developed from air field drainage data assembled by the Corps of Engineers; method is intended for use on airfield drainage problems, but has been used frequently for overland flow in urban basins.

*Source:* Kibler, 1982, Copyright by the American Geophysical Union.

## Table: Formulas for Time of Concentration, $t_c$ (Cont.) (source: Chow et al, 1988)

### Summary of time of concentration formulas

Method and Date	Formula for $t_c$ (min)	Remarks
Kinematic wave formulas Morgali and Linsley (1965) Aron and Erborge (1973)	$t_c = \frac{0.94L^{0.6}n^{0.6}}{(i^{0.4}S^{0.3})}$ $L$ = length of overland flow, ft $n$ = Manning roughness coefficient $i$ = rainfall intensity in/h $S$ = average overland slope ft/ft	Overland flow equation developed from kinematic wave analysis of surface runoff from developed surfaces; method requires iteration since both $i$ (rainfall intensity) and $t_c$ are unknown; superposition of intensity-duration-frequency curve gives direct graphical solution for $t_c$
SCS lag equation (1973)	$t_c = \frac{100 L^{0.8}[(1000/CN) - 9]^{0.7}}{1900 S^{0.5}}$ $L$ = hydraulic length of watershed (longest flow path), ft CN = SCS runoff curve number $S$ = average watershed slope, %	Equation developed by SCS from agricultural watershed data; it has been adapted to small urban basins under 2000 acres; found generally good where area is completely paved; for mixed areas it tends to overestimate; adjustment factors are applied to correct for channel improvement and impervious area; the equation assumes that $t_c = 1.67 \times$ basin lag.
SCS average velocity charts (1975, 1986)	$t_c = \frac{1}{60} \sum \frac{L}{V}$ $L$ = length of flow path, ft $V$ = average velocity in feet per second from Fig. 3-1 of TR 55 for various surfaces	Overland flow charts in Fig. 3-1 of TR 55 show average velocity as function of watercourse slope and surface cover. (See also Table 5.7.1)

Source: Kibler, 1982. Copyright by the American Geophysical Union.

Used in SCS TR55 Method

**Table: Average Velocities for Different Flow Paths**  
(source: Chow et al. 1988)

**Approximate average velocities in ft/s of runoff flow for calculating time of concentration**

Description of water course	Slope in percent			
	0-3	4-7	8-11	12-
<b>Unconcentrated*</b>				
Woodlands	0-1.5	1.5- 2.5	2.5- 3.25	3.25-
Pastures	0-2.5	2.5- 3.5	3.5- 4.25	4.25-
Cultivated	0-3.0	3.0- 4.5	4.5- 5.5	5.5-
Pavements	0-8.5	8.5-13.5	13.5-17	17-
<b>Concentrated**</b>				
Outlet channel—determine velocity by Manning's formula				
Natural channel not well defined	0-2	2-4	4-7	7-

\*This condition usually occurs in the upper extremities of a watershed prior to the overland flows accumulating in a channel.

\*\*These values vary with the channel size and other conditions. Where possible, more accurate determination should be made for particular conditions by the Manning channel formula for velocity.

(Source: Drainage Manual, Texas Highway Department, Table VII, p. II-28, 1970.)

# **APPENDIX B**

## **Hydrologic Routing**

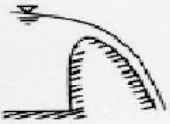
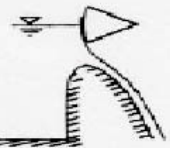
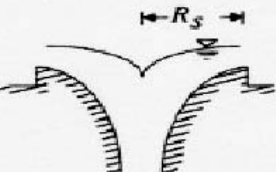
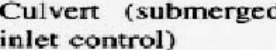
# Reservoir or Detention Basin Routing

- **Storage – Indication Method:** Used for routing flood hydrographs through detention basins or reservoirs.
- **Information Required:**
  1. **Storage-Elevation Data** for the reservoir (obtained from site topographic map)
  2. **Storage-Discharge Relationship** (depends on the hydraulics of outflow control structures such as a spillway) (See Figure – Slide 67)
  3. **Steps 1 and 2 are combined to develop a Storage-Indication curve:**  
 **$2S/\Delta t + Q$  vs  $Q$  curve (See Figure – Slide 68)**
- **Uses discrete form of equation of continuity as:**

$$\left( \frac{2S_{j+1}}{\Delta t} - Q_{j+1} \right) = \left( \frac{2S_j}{\Delta t} + Q_j \right) - 2Q_j$$

## Reservoir Routing (cont.)

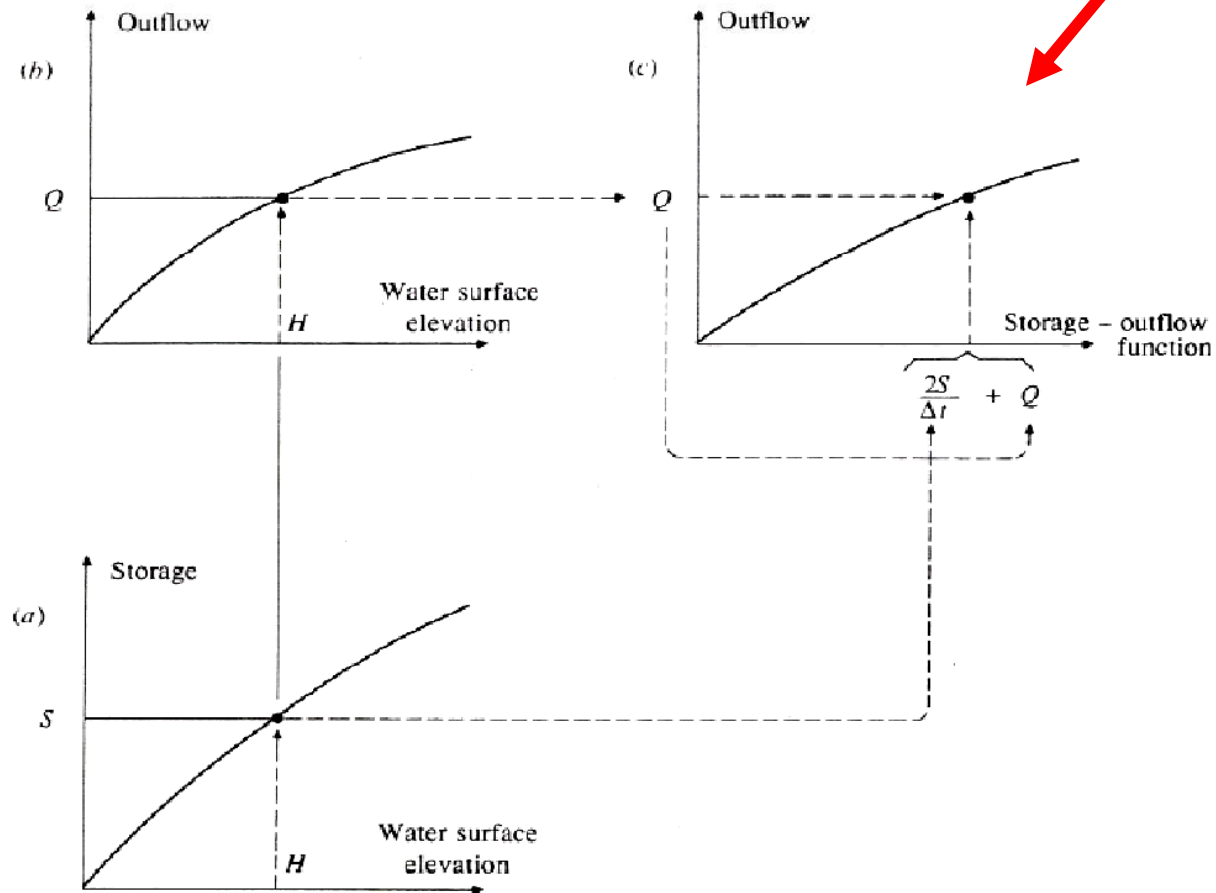
**Figure: Discharge-Elevation Relationships for Various Types of Spillway Structures**  
(Adapted from: Chow et al, 1988)

Spillway type	Equation	Notation
Uncontrolled over-flow ogee crest 	$Q = CLH^{3/2}$	$Q$ = discharge, cfs $C$ = variable coefficient of discharge $L$ = effective length of crest $H$ = total head on the crest including velocity of approach head.
Gate controlled ogee crest 	$Q = \frac{2}{3} \sqrt{2g} CL (H_1^{3/2} - H_2^{3/2})$	$H_1$ = total head to bottom of the opening $H_2$ = total head to top of the opening $C$ = coefficient which differs with gate and crest arrangement
Morning glory spillway 	$Q = C_o(2\pi R_s)H^{3/2}$	$C_o$ = coefficient related to $H$ and $R_s$ $R_s$ = radius of the overflow crest $H$ = total head
Culvert (submerged inlet control) 	$Q = C_d WD \sqrt{2gH}$	$W$ = entrance width $D$ = height of opening $C_d$ = discharge coefficient

Source: *Design of Small Dams*, Bureau of Reclamation, U. S. Department of the Interior, 1973.

## Reservoir Routing (cont.)

Figure: Development of Storage Indication Curve  
(Chow et al, 1988)





## Reservoir Routing (cont.)

### Example 8: Developing a Storage Indication Curve.

#### Problem Statement:

Storage vs outflow characteristics for a proposed reservoir are given below. Calculate the storage-outflow function  $2S/\Delta t + Q$  vs  $Q$  for each of the tabulated values if  $\Delta t = 2$  hours. Plot a graph of the storage-outflow function.

---

Storage, $S$ ( $10^6$ m <sup>3</sup> ):	75	81	87.5	100	110.2
Outflow, $Q$ (m <sup>3</sup> /s):	57	227	519	1330	2270

---

#### Solution:

Storage ( $10^6$ m <sup>3</sup> )	Outflow (m <sup>3</sup> /s)
75	57
81	227
87.5	519
100	1330
110.2	2270

$2S/\Delta t + Q$ (m <sup>3</sup> /s)
20890
22727
24825
29108
32881

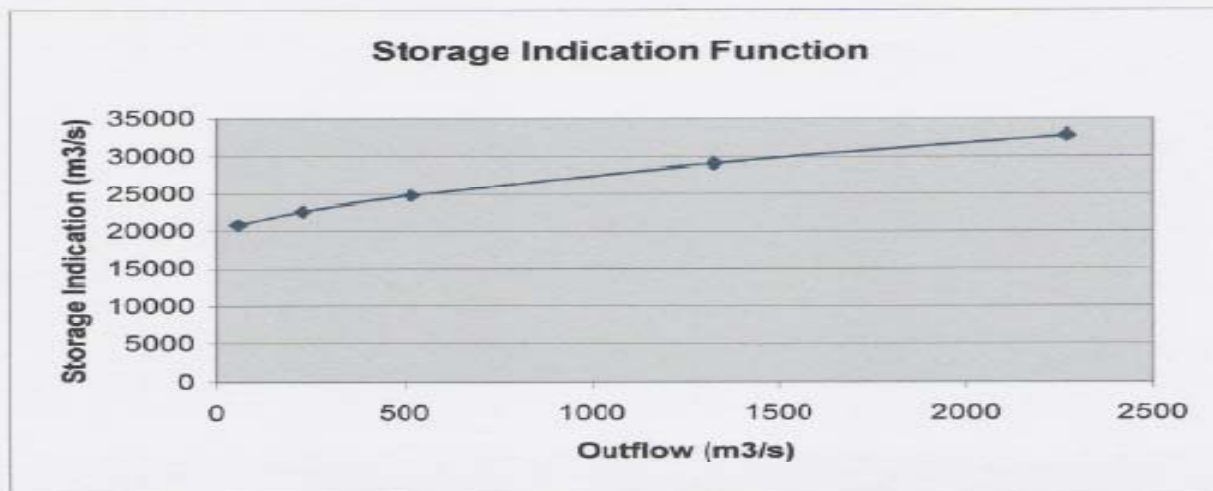
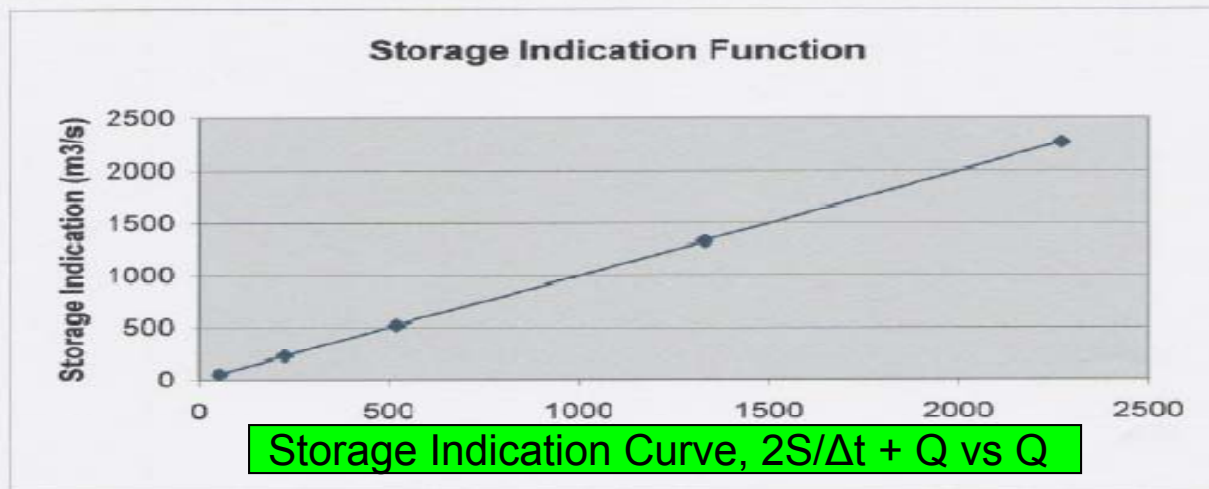
$$\begin{aligned}\Delta t &= 2 \text{ hr} \\ \Delta t &= 7200 \text{ s}\end{aligned}$$

# Reservoir Routing

## Example 8: Developing a Storage Indication Curve (cont.)

(Adapted from: Chow et al, 1988)

Storage-Outflow, S vs Q



## Example 9: Reservoir Routing (Cont.) (Chow et al, 1988)

Using the following storage indication data route the given inflow hydrograph using the storage indication method

Column: 1	2	3
Time index $j$	Time (min)	Inflow (cfs)
1	0	0
2	10	60
3	20	120
4	30	180
5	40	240
6	50	300
7	60	360
8	70	320
9	80	280
10	90	240
11	100	200
12	110	160
13	120	120
14	130	80
15	140	40
16	150	0
17	160	
18	170	
19	180	
20	190	
21	200	
22	210	

Column:	1	2	3	4
	Elevation $H$ (ft)	Discharge $Q$ (cfs)	Storage $S$ (ft <sup>3</sup> )	$(2S/\Delta t)^* + Q$ (cfs)
	0.0	0	0	0
	0.5	3	21,780	76
	1.0	8	43,560	153
	1.5	17	65,340	235
	2.0	30	87,120	320
	2.5	43	108,900	406
	3.0	60	130,680	496
	3.5	78	152,460	586
	4.0	97	174,240	678
	4.5	117	196,020	770
	5.0	137	217,800	863
	5.5	156	239,580	955
	6.0	173	261,360	1044
	6.5	190	283,140	1134
	7.0	205	304,920	1221
	7.5	218	326,700	1307
	8.0	231	348,480	1393
	8.5	242	370,260	1476
	9.0	253	392,040	1560
	9.5	264	413,820	1643
	10.0	275	435,600	1727

\*Time interval  $\Delta t = 10$  min.

## Example 9 Reservoir Routing (Chow et al, 1988) (cont.):

Solution:

From Storage Indication Equation

$$(2S_{j+1}/\Delta t + Q_{j+1}) = (I_{j+1} + I_j) + (2S_j/\Delta t - Q_j)$$

Column: 1	2	3	4	5	6	7
Time index $j$	Time (min)	Inflow (cfs)	$I_j + I_{j+1}$ (cfs)	$\frac{2S_j}{\Delta t} - Q_j$ (cfs)	$\frac{2S_{j+1}}{\Delta t} + Q_{j+1}$ (cfs)	Outflow (cfs)
1	0	0		0.0		0.0
2	10	60	60	55.2	60.0	2.4
3	20	120	180	201.1	235.2	17.1
4	30	180	300	378.9	501.1	61.1
5	40	240	420	552.6	798.9	123.2
6	50	300	540	728.2	1092.6	182.2
7	60	360	660	927.5	1388.2	230.3
8	70	320	680	1089.0	1607.5	259.3
9	80	280	600	1149.0	1689.0	270.0
10	90	240	520	1134.3	1669.0	267.4
11	100	200	440	1064.4	1574.3	254.9
12	110	160	360	954.1	1424.4	235.2
13	120	120	280	820.2	1234.1	206.9
14	130	80	200	683.3	1020.2	168.5
15	140	40	120	555.1	803.3	124.1
16	150	0	40	435.4	595.1	79.8
17	160		0	338.2	435.4	48.6
18	170			272.8	338.2	32.7
19	180			227.3	272.8	22.8
20	190			194.9	227.3	16.2
21	200			169.7	194.9	12.6
22	210				169.7	9.8

From Storage Indication Curve Table Slide 73

inflow hydrograph

outflow hydrograph

Solve this equation for next time step  $j = 3$

$$\left(\frac{2S_{j+1}}{\Delta t} - Q_{j+1}\right) = \left(\frac{2S_j}{\Delta t} + Q_j\right) - 2Q_j$$

Column 5

Column 6

# Reservoir Routing

## Example : Reservoir Routing Using Storage-Indication Method.

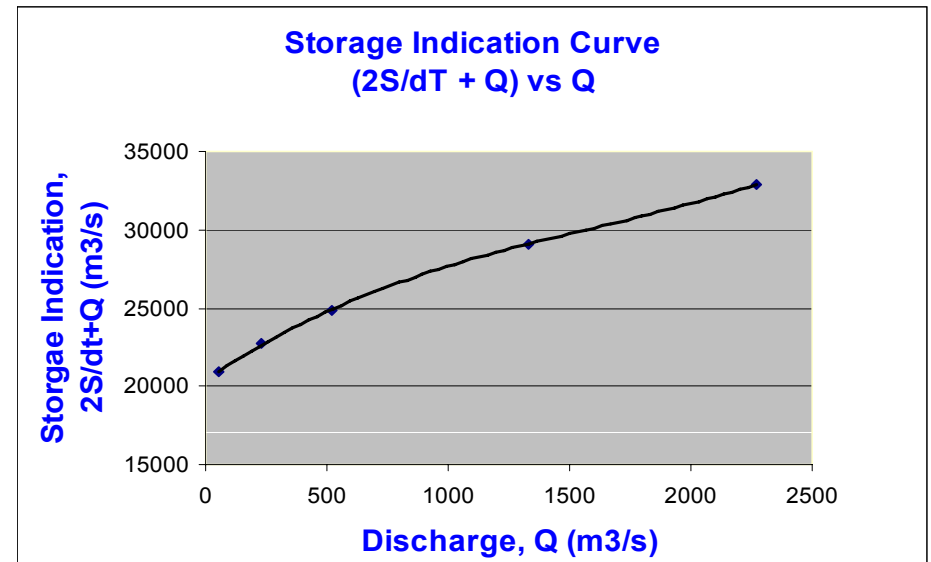
(Adapted from: Chow et al, 1988)

### Problem Statement:

Use the level pool routing method to route the hydrograph given below through the reservoir whose storage-outflow characteristics are given in Prob. 8.2.1. What is the maximum reservoir discharge and storage? Assume that the reservoir initially contains  $75 \times 10^6 \text{ m}^3$  of storage.

Time (h)	0	2	4	6	8	10	12	14	16	18
Inflow ( $\text{m}^3/\text{sec}$ )	60	100	231	300	520	1,310	1,930	1,460	930	650

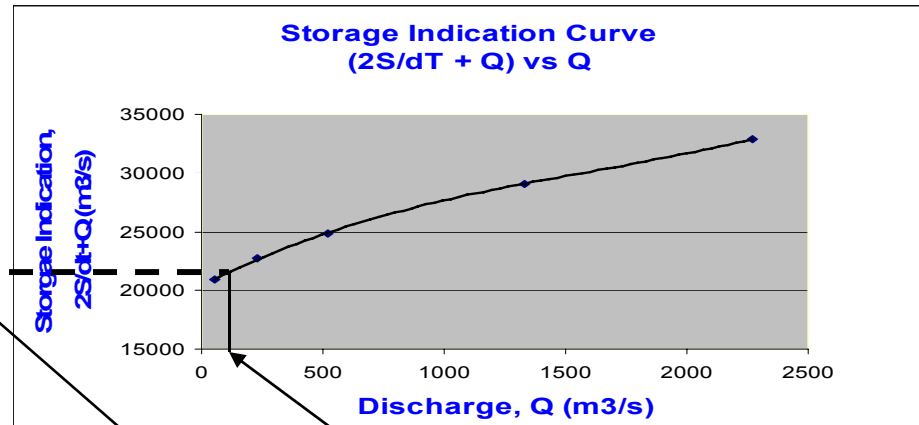
Storage ( $10^6 \text{ m}^3$ )	Outflow ( $\text{m}^3/\text{s}$ )	$2S/\Delta t + O$ ( $\text{m}^3/\text{s}$ )
75	57	20890
81	227	22727
87.5	519	24825
100	1330	29108
110.2	2270	32881



# Reservoir Routing (Cont.)

## Example: Reservoir Routing Using Storage-Indication Method (cont.)

(Adapted from: Chow et al, 1988)



Solution:

Column:

1	2	3	4	5	6
Time (min)	Inflow (cfs)	$I_j + I_{j+1}$ (cfs)	$2S_j/\Delta t - Q_j$ (cfs)	$2S_{j+1}/\Delta t + Q_{j+1}$ (cfs)	Outflow, $Q_j$ (cfs)
.00	60.00				
120.00	100.00	160.00	20776.11	20936.33	67.26
240.00	232.00	332.00	20813.82	21145.82	85.65
360.00	300.00	532.00	20984.52	21514.52	114.96
480.00	520.00	820.00	21286.60	22106.60	169.58
600.00	1310.00	1830.00	21767.45	23597.45	348.18
720.00	1930.00	3240.00	22801.10	26161.10	768.28
840.00	1460.00	3390.00	24604.54	27994.54	1119.22
960.00	930.00	2390.00	25756.11	28146.11	1147.91
1080.00	650.00	1580.00	25850.28	27430.28	1012.38
1200.00	60.00	710.00	25405.53	26115.53	763.44
1320.00	60.00	120.00	24588.45	24708.65	502.87
1440.00	60.00	120.00	23702.92	23822.92	379.56
1560.00	60.00	120.00	23083.80	23183.80	290.59
1680.00	60.00	120.00	22602.62	22722.62	226.59
1800.00	60.00	120.00	22269.43	22389.43	195.75
1920.00	60.00	120.00	21997.92	22117.92	170.62

$$= \left( \frac{2S_{j+1}}{\Delta t} + Q_{j+1} \right) - 2Q_{j+1}$$

# **APPENDIX C**

## **Groundwater: Well Hydraulics**

# GROUND WATER HYDROLOGY

## Terminology

- **Aquifer** – water bearing strata capable of transmitting water (fluid) at a rate as to be suitable for water supply
- **Confined Aquifer** – an aquifer located between two layers of low permeability or impermeable strata (aquitard)
- **Unconfined Aquifer** – water table aquifer, phreatic aquifer, water level exposed to atmospheric pressure
- **Vadose zone** – unsaturated region above the water table
- **Potentiometric or Piezometric surface** – hydraulic head,  $h$ , level at a point or across a region of aquifer



# Fundamental Principles

## Darcy's Law: $q = K i$

$q \equiv$  specific discharge  $\{L/T\}$

$K \equiv$  hydraulic conductivity  $\{L/T\}$ ,  
(ft/s, m/s)

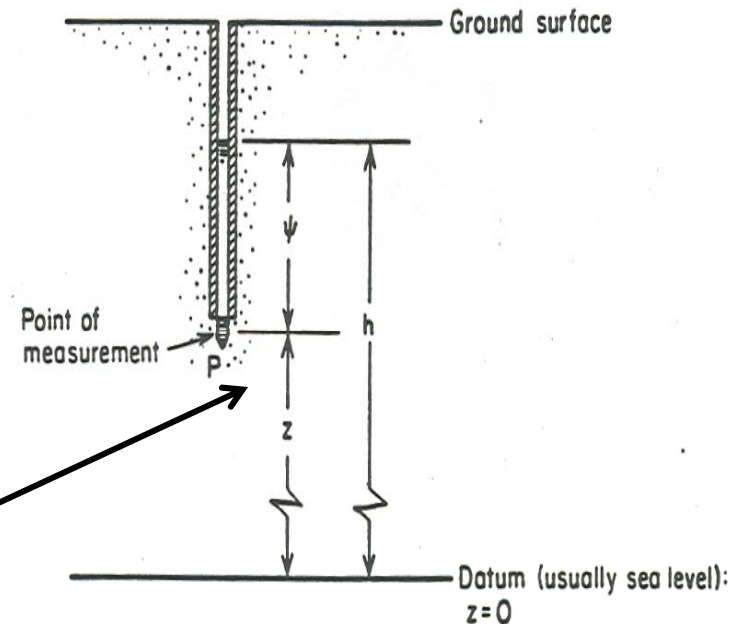
$i \equiv$  hydraulic gradient  $= dh/dL$   
 $\{\text{dimensionless, } L/L\}$

$h =$  hydraulic head,  $h$

$$= \psi + z$$

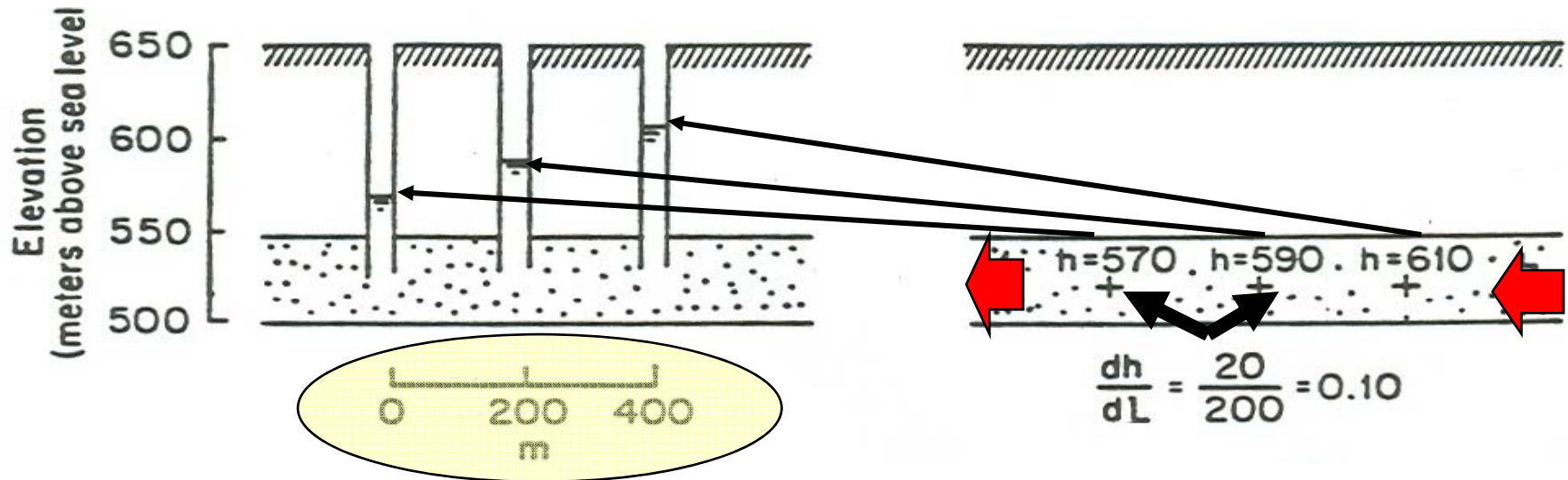
$\Psi \equiv$  gage pressure head or water  
pressure  $= p_{\text{gage}}/\gamma \{L\}$

$z \equiv$  elevation head  $\{L\}$



Note:  $\Psi = 0$  in saturated zone

# Fundamental Principles (cont.)



**Darcy's Law:  $v = q = K i$**

$v = q =$  flux rate or velocity (ft/s or m/s)

$i =$  hydraulic gradient,  $\frac{dh}{dL} = \frac{(h_2 - h_1)}{(L_2 - L_1)}$

$K =$  hydraulic conductivity (ft/s or m/s)

**Volume flow rate:  $Q = q A$**

$A =$  flow area

# Well Hydraulics

## Thiem's Steady State Solution

- **Unconfined Aquifer:** →

$$Q = \frac{\pi K_f (h_2^2 - h_1^2)}{\ln(r_2/r_1)}$$

- **Confined Aquifer:** →

$$Q = 2\pi K_f m \frac{h_2 - h_1}{\ln(r_2/r_1)}$$

where,

Q = Discharge from pumping well in cfs (m<sup>3</sup>/s or gpm);

h = hydraulic or piezometric head = p/γ in ft (or m);

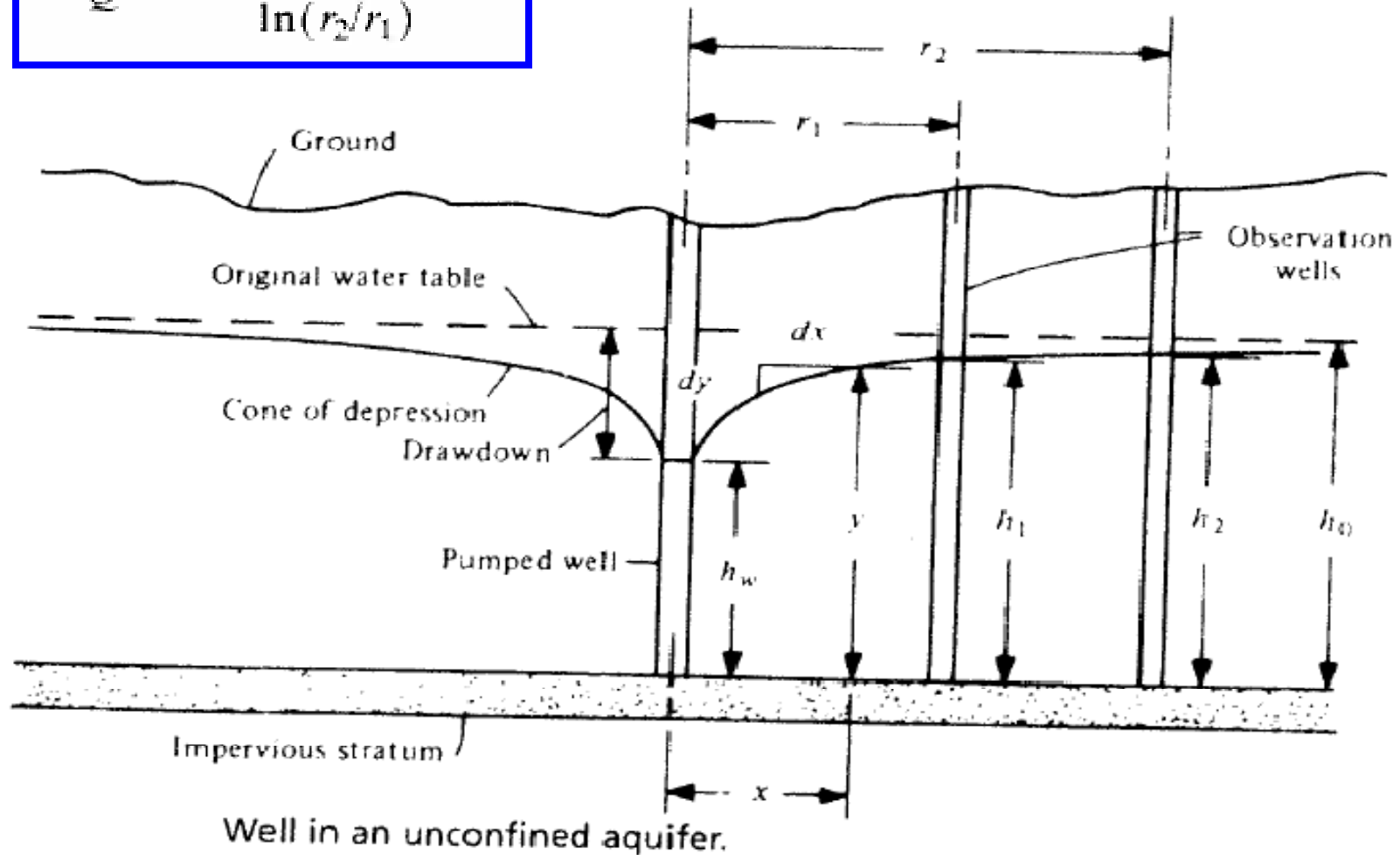
K<sub>f</sub> = hydraulic conductivity in gpd/ft<sup>2</sup> (or ft/s);

r = distance to observation well from center of the pumping well in ft (or m)

m = thickness of the confined aquifer in ft (or m)

## Example: Thiem's Steady State Solution - Unconfined Aquifer

$$Q = \frac{\pi K_f (h_2^2 - h_1^2)}{\ln(r_2/r_1)}$$



## Example: Thiem's Steady State Solution - Unconfined Aquifer

### Given:

- A 20 inch diameter well fully penetrates a 100 ft deep **unconfined** aquifer;
- Drawdowns at two observation wells located at 90 ft and 240 ft from the pumping well are 23 ft and 21.5 ft respectively;
- Hydraulic Conductivity of the aquifer is  $K_f = 1400 \text{ gpd/ft}^2 \times ((1.55 \times 10^{-6} \text{ cfs/gpd}) = 2.17 \times 10^{-3} \text{ ft/s}$
- Determine: The discharge  $Q$  from the pumping well in gpm

### Solution:

Hydraulic heads:  $h_1 = 100 - 23 = 77 \text{ ft}$  (at  $r_1 = 90 \text{ ft}$ );

$h_2 = 100 - 21.5 = 78.5 \text{ ft}$ . (at  $r_2 = 240 \text{ ft}$ );

$K_f = 1400 \text{ gpd/ft}^2 \times ((1.55 \times 10^{-6} \text{ cfs/gpd}) = 2.17 \times 10^{-3} \text{ ft/s}$

$$Q = \frac{\pi K_f (h_2^2 - h_1^2)}{\ln(r_2/r_1)}$$

$$\begin{aligned} Q &= 3.14 K_f (h_2^2 - h_1^2) / \log_e(r_2/r_1) - \text{all in consistent units} \\ &= 3.14 \times 2.17 \times 10^{-3} (78.5^2 - 77^2) / \log_e(240/90) \\ &= 1.62 \text{ cfs} = 725.9 \text{ gpm} \end{aligned}$$

## Example : Thiem's Steady State Solution – Confined Aquifer

$$Q = 2\pi K_f m \frac{h_2 - h_1}{\ln(r_2/r_1)}$$

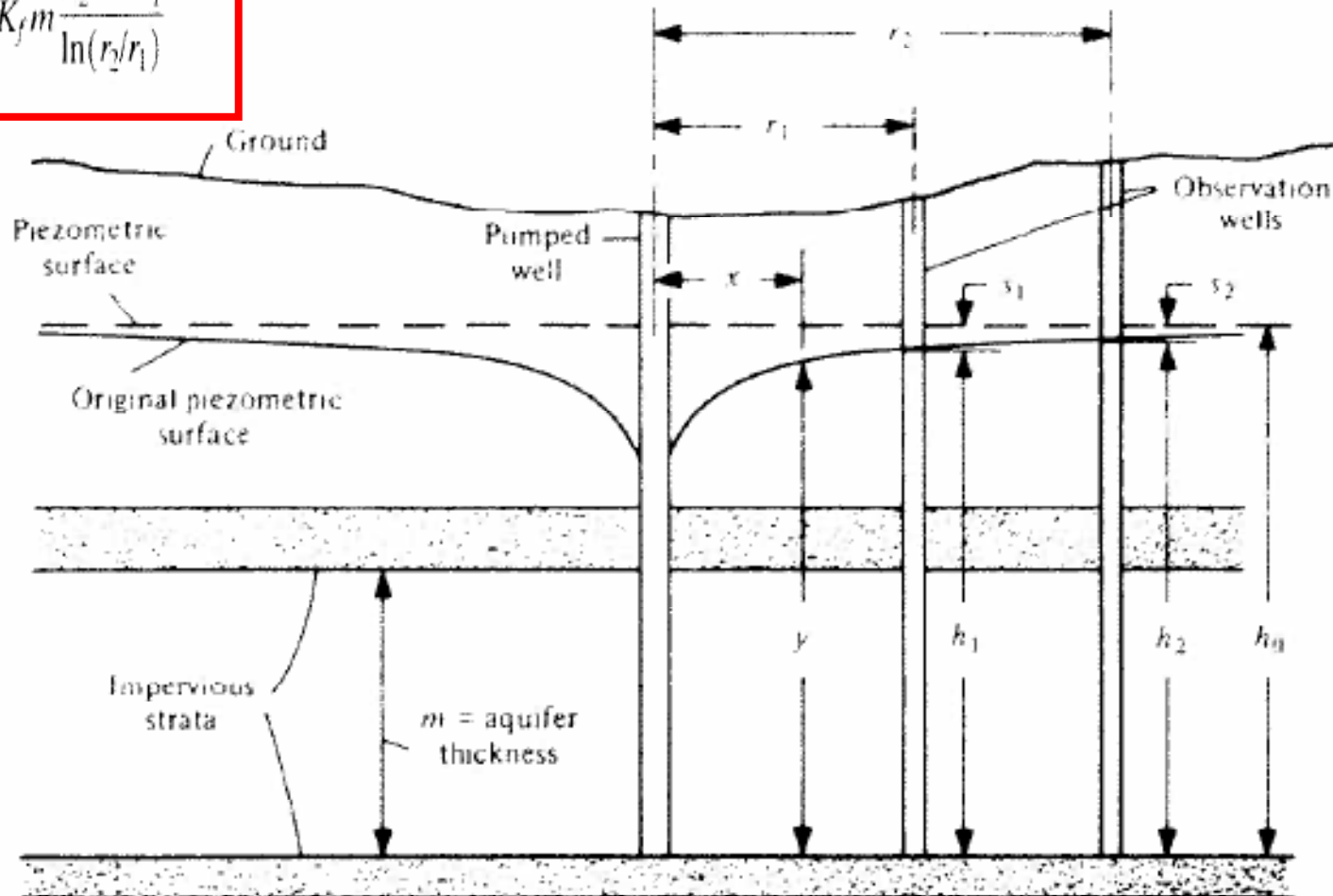


FIGURE 3.9 Radial flow to a well in a confined aquifer.

## Example : Thiem's Steady State Solution – Confined Aquifer

**Problem:** Determine the hydraulic conductivity,  $K_f$ , of an artesian aquifer (confined aquifer) pumped by a fully penetrating well.

**Given:**

- 1) Aquifer thickness,  $m = 100$  feet
- 2) Steady state pumping rate  $Q = 1000$  gpm = 2.232 cfs
- 3) Drawdowns,  $s$ , at observation wells:  
Well 1:  $r_1 = 50$  ft;  $s_1 = 10$  ( $h_1 = 100 - 10 = 90$  ft)  
Well 2:  $r_2 = 500$  ft;  $s_2 = 1$  ft ( $h_2 = 100 - 1 = 99$  ft)

**Solution:**

Solve for Hydraulic Conductivity,  $K_f$  from Thiem's steady state equation for confined aquifer:

$$Q = 2\pi K_f m \frac{h_2 - h_1}{\ln(r_2/r_1)}$$

$$\begin{aligned} K_f &= \{Q (\log_e(r_2/r_1))\} / \{(2 \pi m (h_2 - h_1))\} - \text{all in consistent units} \\ &= \{2.232 \times \log_e(500/50)\} / \{2 \times 3.14 \times 100 \times (99 - 90)\} \\ &= 9.093 \times 10^{-4} \text{ ft/s or ft}^3/\text{s.ft}^2 \\ &= (9.093 \times 10^{-4} \text{ ft}^3/\text{s.ft}^2) \times 646,323 \text{ gpd/ft}^2 \\ &= 587.7 \text{ gpd/ft}^2 \end{aligned}$$

# Theis Unsteady State Solution

## ASSUMPTIONS

1. The aquifer is homogeneous, isotropic, and of infinite extent (this is a built-in assumption of the groundwater flow equation).
2. The transmissivity of the aquifer is practically constant.
3. The water derived is entirely from storage and is released instantaneously with decline of head.
4. The well penetrates the entire thickness of the aquifer, and its diameter is very small compared to pumping rates, so that storage in well is negligible.

## INITIAL AND BOUNDARY CONDITIONS

1. At time = 0, drawdown = 0, at any distance.
2. At time  $> 0$ , drawdown = 0, at infinite distance.



# Theis Unsteady State Solution - Equations

$$h_0 - h = s = \frac{Q}{4\pi T} W(u)$$

$$u = \frac{r^2 S}{4 T t}$$

## Variables:

Drawdown:  $s = h_0 - h$

hydraulic conductivity:  $K$

aquifer transmissivity:  $T = K b$

aquifer thickness (confined) or saturated thickness (unconfined) :  $b$

Storativity:  $S$

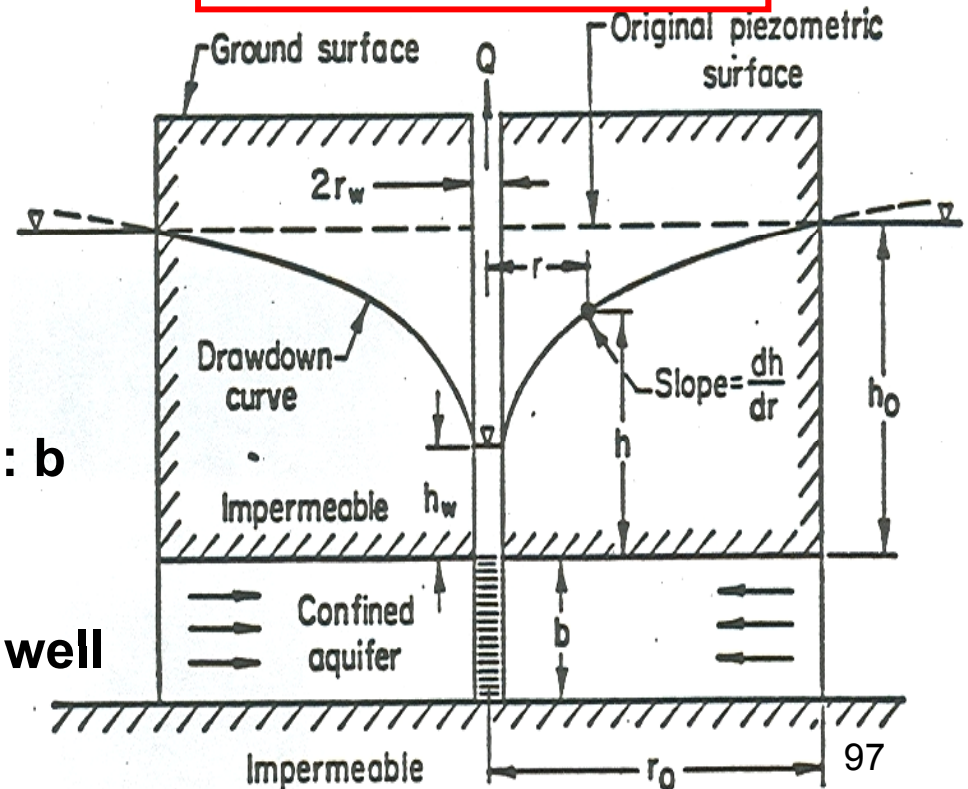
well function:  $W(u)$

$r$  = radial distance from center of well

In US practice – Equations are used with  $Q$  in gpm,  $T$  in gpd/ft and time  $t$  = days

$$s = \frac{114.6Q}{T} \int_{u}^{\infty} \frac{e^{-u}}{u} du$$

$$u = \frac{1.87r^2 S_c}{Tt}$$



## Theis Unsteady State Solution – Well Function $W(u)$

Well function,  $W(u)$  a function of the term  $u$

Tabulated to provide a convenient method to estimate drawdown for given aquifer conditions and steady-state pumping rate,  $Q$

Values of  $W(u)$  for Various Values of  $u$

$u$	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
$\times 1$	0.219	0.049	0.013	0.0038	0.0011	0.00036	0.000038	0.000012	0.000012
$\times 10^{-1}$	1.82	1.22	0.91	0.70	0.56	0.45	0.37	0.31	0.26
$\times 10^{-2}$	4.04	3.35	2.96	2.68	2.47	2.30	2.15	2.03	1.92
$\times 10^{-3}$	6.33	5.64	5.23	4.95	4.73	4.54	4.39	4.26	4.14
$\times 10^{-4}$	8.63	7.94	7.53	7.25	7.02	6.84	6.69	6.55	6.44
$\times 10^{-5}$	10.94	10.24	9.84	9.55	9.33	9.14	8.99	8.86	8.74
$\times 10^{-6}$	13.24	12.55	12.14	11.85	11.63	11.45	11.29	11.16	11.04
$\times 10^{-7}$	15.54	14.85	14.44	14.15	13.93	13.75	13.60	13.46	13.34
$\times 10^{-8}$	17.84	17.15	16.74	16.46	16.23	16.05	15.90	15.76	15.65
$\times 10^{-9}$	20.15	19.45	19.05	18.76	18.54	18.35	18.20	18.07	17.95
$\times 10^{-10}$	22.45	21.76	21.35	21.06	20.84	20.66	20.50	20.37	20.25
$\times 10^{-11}$	24.75	24.06	23.65	23.36	23.14	22.96	22.81	22.67	22.55
$\times 10^{-12}$	27.05	26.36	25.96	25.67	25.44	25.26	25.11	24.97	24.86
$\times 10^{-13}$	29.36	28.66	28.26	27.97	27.75	27.56	27.41	27.28	27.16
$\times 10^{-14}$	31.66	30.97	30.56	30.27	30.05	29.87	29.71	29.58	29.46
$\times 10^{-15}$	33.96	33.27	32.86	32.58	32.35	32.17	32.02	31.88	31.76

Source: After L. K. Wenzel, "Methods for Determining Permeability of Water Bearing Materials with Special Reference to Discharging Well Methods." U.S. Geological Survey, Water-Supply Paper 887, Washington, DC, 1942.

# Example: Theis Method

## Given:

A well is pumped at  $Q = 5400 \text{ m}^3/\text{day}$

Aquifer properties:

$S = 0.0003$ ;

$T = 2200 \text{ m}^2/\text{day}$  ( $0.0025 \text{ m}^2/\text{s}$ )

## Compute:

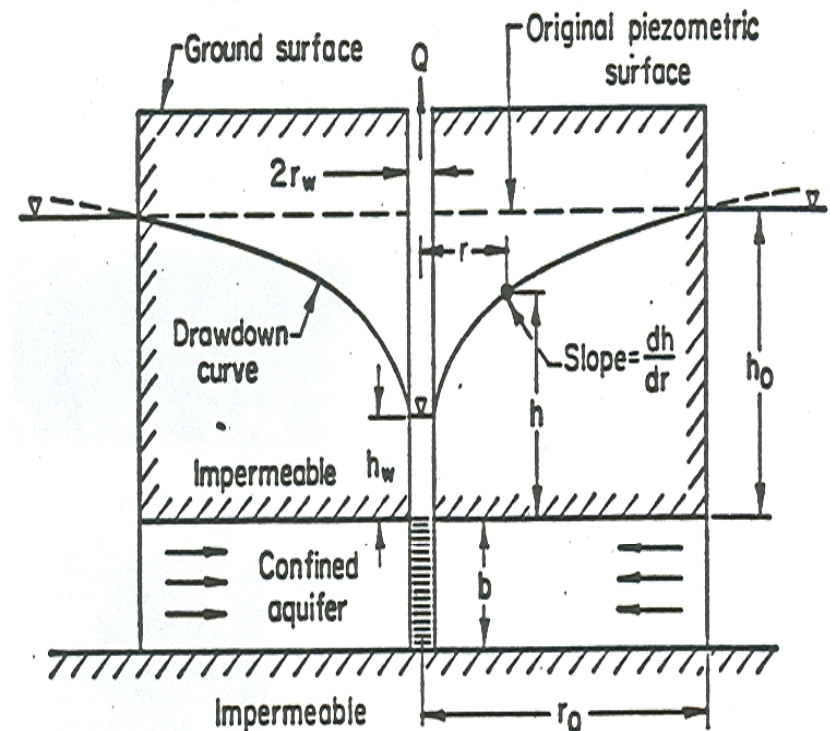
Drawdown,  $s = h_0 - h$ ,  
at  $t = 10$  days,  $r = 20$  m

## Solution:

Compute  $u = 1.36 \cdot 10^{-6}$

Well Function:  $W(u) = 12.99$

$$h_0 - h = s = \frac{Q}{4\pi T} W(u)$$



Drawdown,  $s = \{5400 / (4 \times 3.14 \times 2200)\} \times 12.99 = 2.53 \text{ m}$

## **Problem 7: Drawdown by Theis method**

### **Given:**

•The following information for a confined aquifer:

- a) Transmissivity  $T = 1650 \text{ ft}^2/\text{day}$
- b) Storage coefficient  $S = 0.0005$ .
- c) Aquifer thickness = 200 feet
- d) Well delivers a discharge of  $Q = 500 \text{ gpm}$

•Determine the drawdown at an observation well located 150 feet away after  $t=12$  hours using Theis method. Use consistent equations.

(Note: 1 cfs = 448 gpm).

# **APPENDIX D**

## **Unit Hydrograph**

### **Additional Examples**

# Example: Unit Hydrograph Derivation

**Problem Statement:** Determine 1/2hr Unit Hydrograph using the excess rainfall hyetograph and Direct Runoff Hydrograph shown in the Table below. (Adapted from Chow et al (1988))

Excess rainfall hyetograph and direct runoff hydrograph

Time ( $\frac{1}{2}$ h)	Excess rainfall (in)	Direct runoff (cfs)
1	1.06	428
2	1.93	1923
3	1.81	5297
4		9131
5		10625
6		7834
7		3921
8		1846
9		1402
10		830
11		313

# Example: Unit Hydrograph Derivation (Cont.)

Solution by Deconvolution where  $N = M + J - 1$  or  $J = N - M + 1 = 11 - 3 + 1 = 9$  ordinates (Adapted from Chow et al (1988))

$$U_1 = \frac{Q_1}{P_1} = \frac{428}{1.06} = 404 \text{ cfs/in}$$

$$U_2 = \frac{Q_2 - P_2 U_1}{P_1} = \frac{1923 - 1.93 \times 404}{1.06} = 1079 \text{ cfs/in}$$

$$U_3 = \frac{Q_3 - P_3 U_1 - P_2 U_2}{P_1} = \frac{5297 - 1.81 \times 404 - 1.93 \times 1079}{1.06} = 2343 \text{ cfs/in}$$

and similarly for the remaining ordinates

$$U_4 = \frac{9131 - 1.81 \times 1079 - 1.93 \times 2343}{1.06} = 2506 \text{ cfs/in}$$

$$U_5 = \frac{10625 - 1.81 \times 2343 - 1.93 \times 2506}{1.06} = 1460 \text{ cfs/in}$$

$$U_6 = \frac{7834 - 1.81 \times 2506 - 1.93 \times 1460}{1.06} = 453 \text{ cfs/in}$$

$$U_7 = \frac{3921 - 1.81 \times 1460 - 1.93 \times 453}{1.06} = 381 \text{ cfs/in}$$

$$U_8 = \frac{1846 - 1.81 \times 453 - 1.93 \times 381}{1.06} = 274 \text{ cfs/in}$$

$$U_9 = \frac{1402 - 1.81 \times 381 - 1.93 \times 274}{1.06} = 173 \text{ cfs/in}$$

**1/2 hr UHG**

$n$	1	2	3	4	5	6	7	8	9
$U_n$ (cfs/in)	404	1079	2343	2506	1460	453	381	274	173

## Problem : Derivation of Direct Runoff and Streamflow Hydrographs Using a Given Unit Hydrograph

### Problem Statement:

The six-hour unit hydrograph of a watershed having a drainage area equal to 393 km<sup>2</sup> is as follows:

Time (h)	0	6	12	18	24	30	36	42
Unit hydrograph (m <sup>3</sup> /s·cm)	0	1.8	30.9	85.6	41.8	14.6	5.5	1.8

For a storm over the watershed having excess rainfall of 5 cm for the first six hours and 15 cm for the second six hours, compute the streamflow hydrograph, assuming constant baseflow of 100 m<sup>3</sup>/s.



# Problem : Derivation of Direct Runoff and Streamflow Hydrographs Using a Given Unit Hydrograph (cont.)

## Solution:

**Q3**

Area = 393 km<sup>2</sup>  
Baseflow = 100 m<sup>3</sup>/s

Time (hr)	UH (m <sup>3</sup> /s-cm)	Rainfall		Direct	
		Excess (cm)	Runoff (cfs)	Runoff (cfs)	Streamflow (cfs)
0	0		0.0		100.0
6	1.8	5	9.0		109.0
12	30.9	15	181.5		281.5
18	85.6	0	891.5		991.5
24	41.8	0	1493.0		1593.0
30	14.6	0	700.0		800.0
36	5.5	0	246.5		346.5
42	1.8	0	91.5		191.5
48	0	0	27.0		127.0
54	0	0	0.0		100.0

$$Q_1 = P_1 U_1 = (5)(1.8) = 9$$

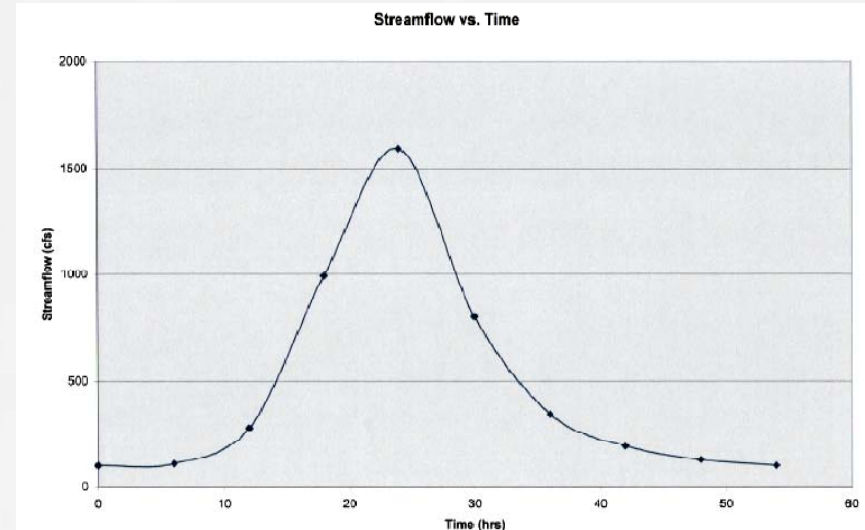
$$Q_2 = P_2 U_1 + P_1 U_2 = (15)(1.8) + (5)(30.9) = 181.5$$

$$Q_3 = P_3 U_1 + P_2 U_2 + P_1 U_3 = 0 + (15)(30.9) + (5)(85.6) = 891.5$$

$$Q_4 = P_2 U_3 + P_1 U_4 = (15)(85.6) + (5)(41.8) = 1493$$

### 2nd Method

Time (hr)	Rainfall		U <sub>1</sub>	U <sub>2</sub>	U <sub>3</sub>	U <sub>4</sub>	U <sub>5</sub>	U <sub>6</sub>	U <sub>7</sub>	Direct	
	Excess (cm)	Runoff (cfs)								Runoff (cfs)	Streamflow (cfs)
0										0.0	100.0
6	5	9								9.0	109.0
12	15	27		154.5						181.5	281.5
18	0			463.5	428					891.5	991.5
24	0				1284	209				1493.0	1593.0
30	0					627	73			700.0	800.0
36	0						219	27.5		246.5	346.5
42	0							82.5	9	91.5	191.5
48	0								27	27.0	127.0
54	0									0.0	100.0



# Example : Derivation of Direct Runoff and Streamflow Hydrographs Using a Given Unit Hydrograph

## Problem Statement:

- The six-hour unit hydrograph of a watershed having a drainage area equal to 393 km<sup>2</sup> is as follows:

---

<b>Time (hr):</b>	<b>0</b>	<b>6</b>	<b>12</b>	<b>18</b>	<b>24</b>	<b>30</b>	<b>36</b>	<b>42</b>
<b>Unit hydrograph (m<sup>3</sup>/s/cm):</b>	<b>0</b>	<b>1.8</b>	<b>30.9</b>	<b>15.6</b>	<b>41.8</b>	<b>14.6</b>	<b>5.5</b>	<b>1.8</b>

---

- For a storm over the watershed having the following excess rainfall depths, compute the streamflow hydrograph assuming a constant base flow of 100 m<sup>3</sup>/s.

---

<b>Time (hr):</b>	<b>0</b>	<b>6</b>	<b>12</b>
<b>Rainfall Excess (cm):</b>	<b>0</b>	<b>5</b>	<b>15</b>

---

# Example: Derivation of Direct Runoff and Streamflow Hydrographs Using a Given Unit Hydrograph (cont.)

## Solution:

**Q3**

Area = 393 km<sup>2</sup>  
 Baseflow = 100 m<sup>3</sup>/s

Time (hr)	UH (m <sup>3</sup> /s-cm)	Rainfall	Direct	Streamflow (cfs)
		Excess (cm)	Runoff (cfs)	
0	0		0.0	100.0
6	1.8	5	9.0	109.0
12	30.9	15	181.5	281.5
18	85.6	0	891.5	991.5
24	41.8	0	1493.0	1593.0
30	14.6	0	700.0	800.0
36	5.5	0	246.5	346.5
42	1.8	0	91.5	191.5
48	0	0	27.0	127.0
54	0	0	0.0	100.0

$$Q_1 = P_1 U_1 = (5)(1.8) = 9$$

$$Q_2 = P_2 U_1 + P_1 U_2 = (15)(1.8) + (5)(30.9) = 181.5$$

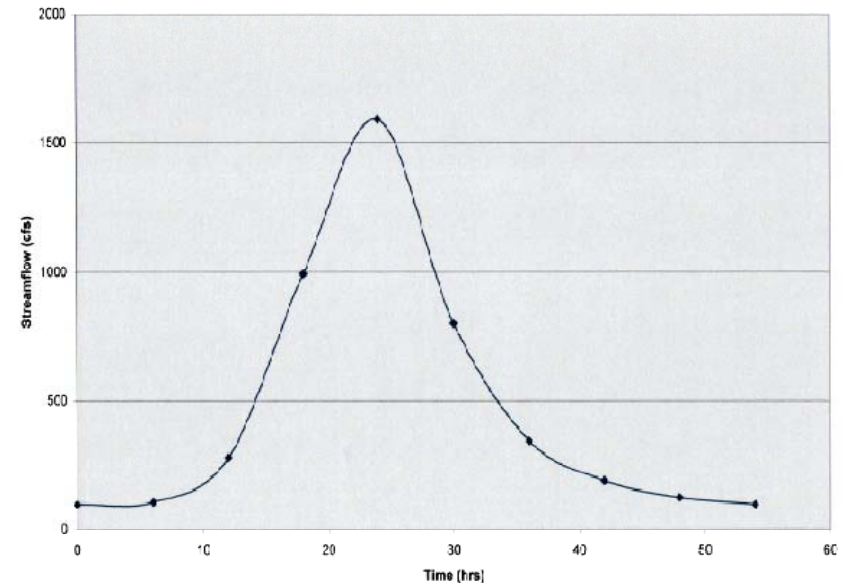
$$Q_3 = P_3 U_1 + P_2 U_2 + P_1 U_3 = 0 + (15)(30.9) + (5)(85.6) = 891.5$$

$$Q_4 = P_2 U_3 + P_1 U_4 = (15)(85.6) + (5)(41.8) = 1493$$

### 2nd Method

Time (hr)	Rainfall	U <sub>1</sub>	U <sub>2</sub>	U <sub>3</sub>	U <sub>4</sub>	U <sub>5</sub>	U <sub>6</sub>	U <sub>7</sub>	Direct	Streamflow (cfs)
	Excess (cm)								Runoff (cfs)	
0									0.0	100.0
6	5	9							9.0	109.0
12	15	27	154.5						181.5	281.5
18	0		463.5	428					891.5	991.5
24	0			1284	209				1493.0	1593.0
30	0				627	73			700.0	800.0
36	0					219	27.5		246.5	346.5
42	0						82.5	9	91.5	191.5
48	0							27	27.0	127.0
54	0								0.0	100.0

Streamflow vs. Time



# Example: Unit Hydrograph Derivation

**Problem Statement:** Determine 1/2hr Unit Hydrograph using the excess rainfall hyetograph and Direct Runoff Hydrograph shown in the Table below. (Adapted from Chow et al (1988))

Excess rainfall hyetograph and direct runoff hydrograph

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# Example: Unit Hydrograph Derivation (Cont.)

Solution by Deconvolution where  $N = M + J - 1$  or  $J = N - M + 1 = 11 - 3 + 1 = 9$  ordinates

(Adapted from Chow et al (1988))

$$U_1 = \frac{Q_1}{P_1} = \frac{428}{1.06} = 404 \text{ cfs/in}$$

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and similarly for the remaining ordinates

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$$U_8 = \frac{1846 - 1.81 \times 453 - 1.93 \times 381}{1.06} = 274 \text{ cfs/in}$$

$$U_9 = \frac{1402 - 1.81 \times 381 - 1.93 \times 274}{1.06} = 173 \text{ cfs/in}$$

**1/2 hr UHG**

$n$	1	2	3	4	5	6	7	8	9
$U_n$ (cfs/in)	404	1079	2343	2506	1460	453	381	274	173