P.E. Civil Exam Review: Strength of Materials

J.P. Mohsen

Email:jpm@louisville.edu

ASCE | KNOWLEDGE & LEARNING

Strength of Materials

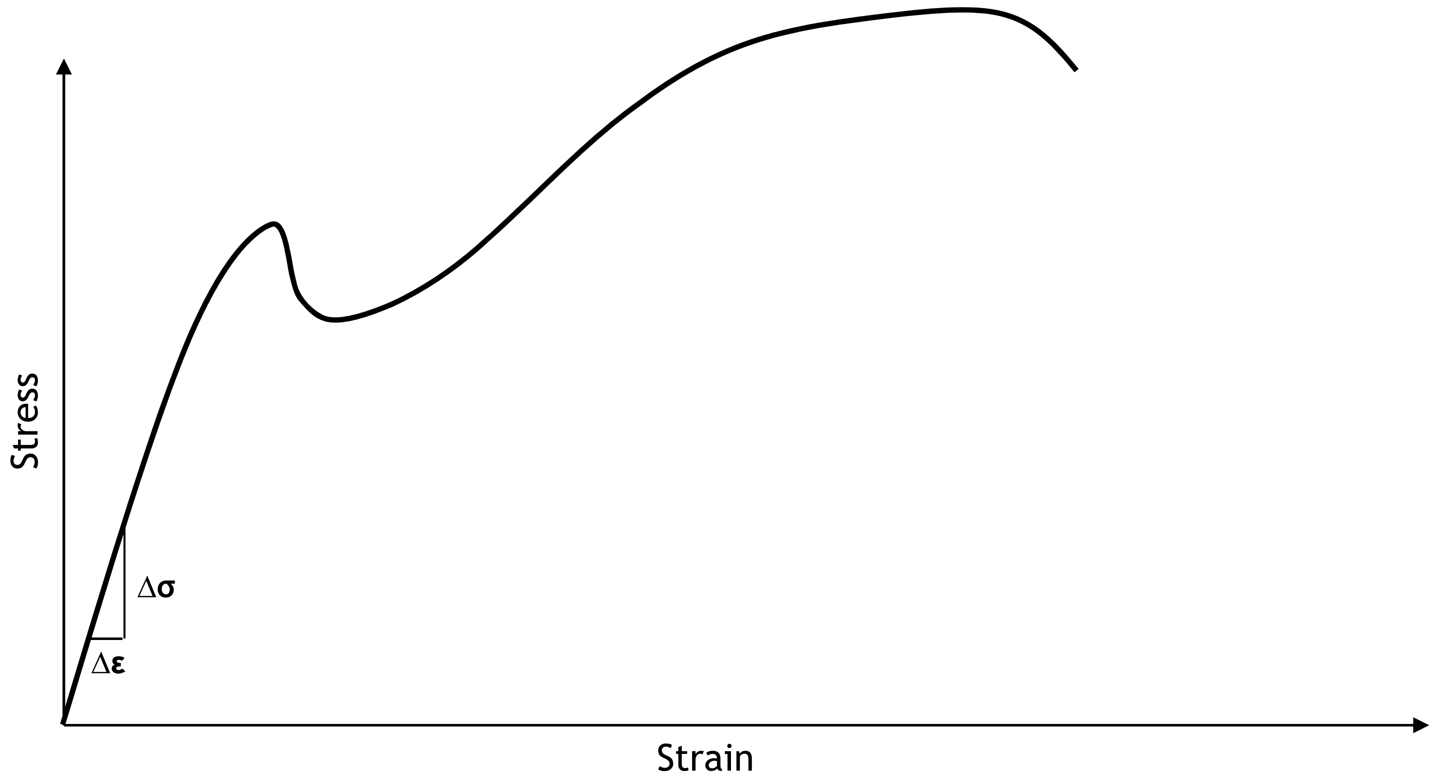
Stress vs. Strain

σ → Stress = Force / Area

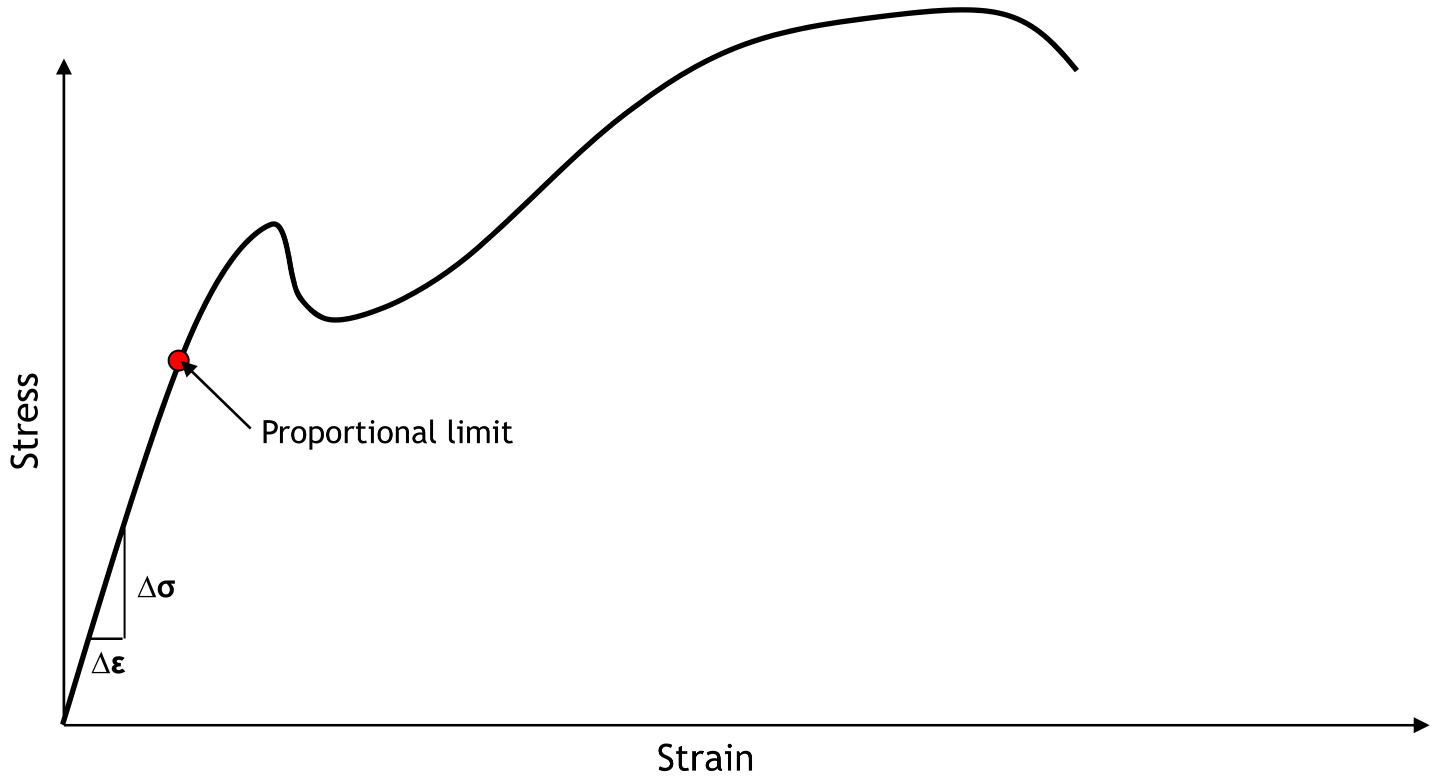
$$\sigma = \frac{P}{A}$$

ε → Strain = Change in Length / Original Length

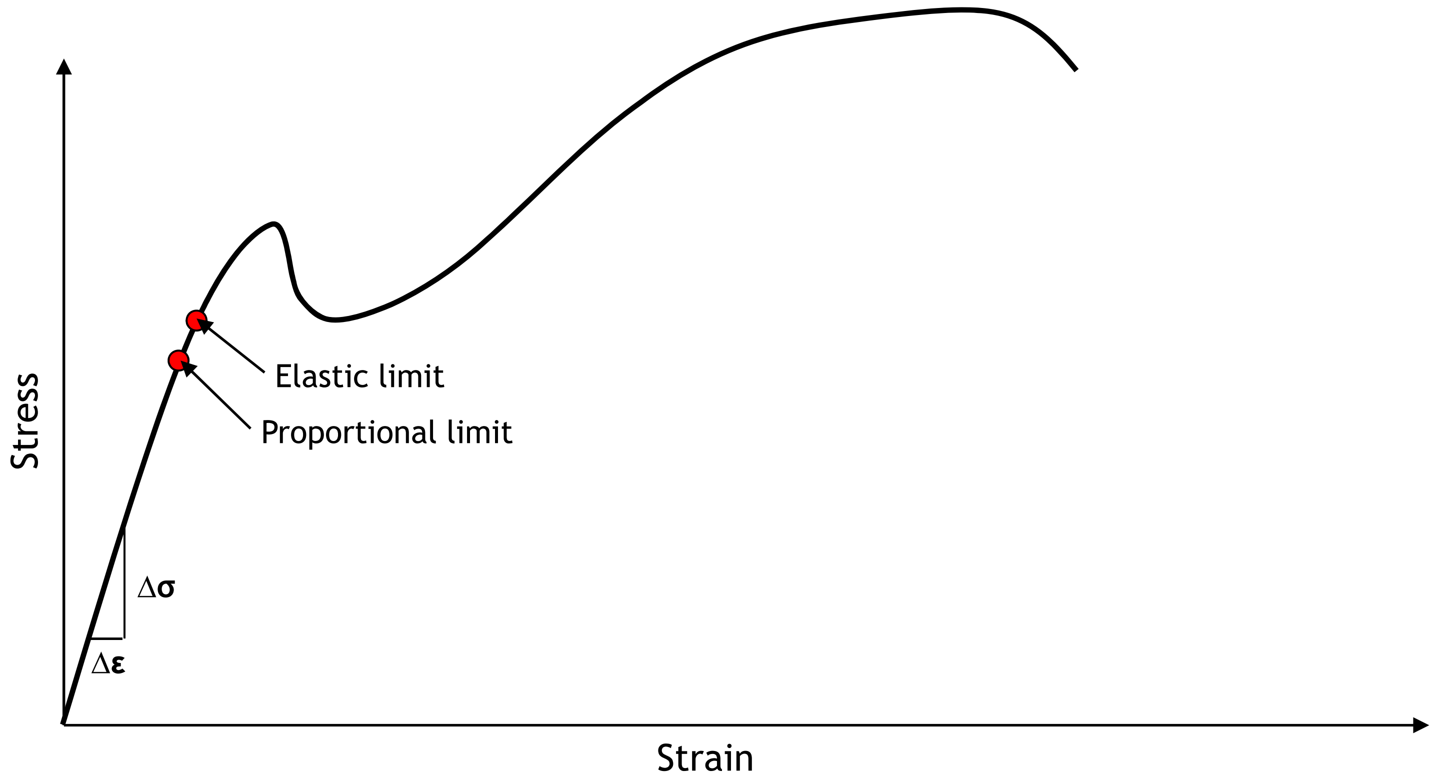
$$\varepsilon = \frac{\Delta L}{L}$$



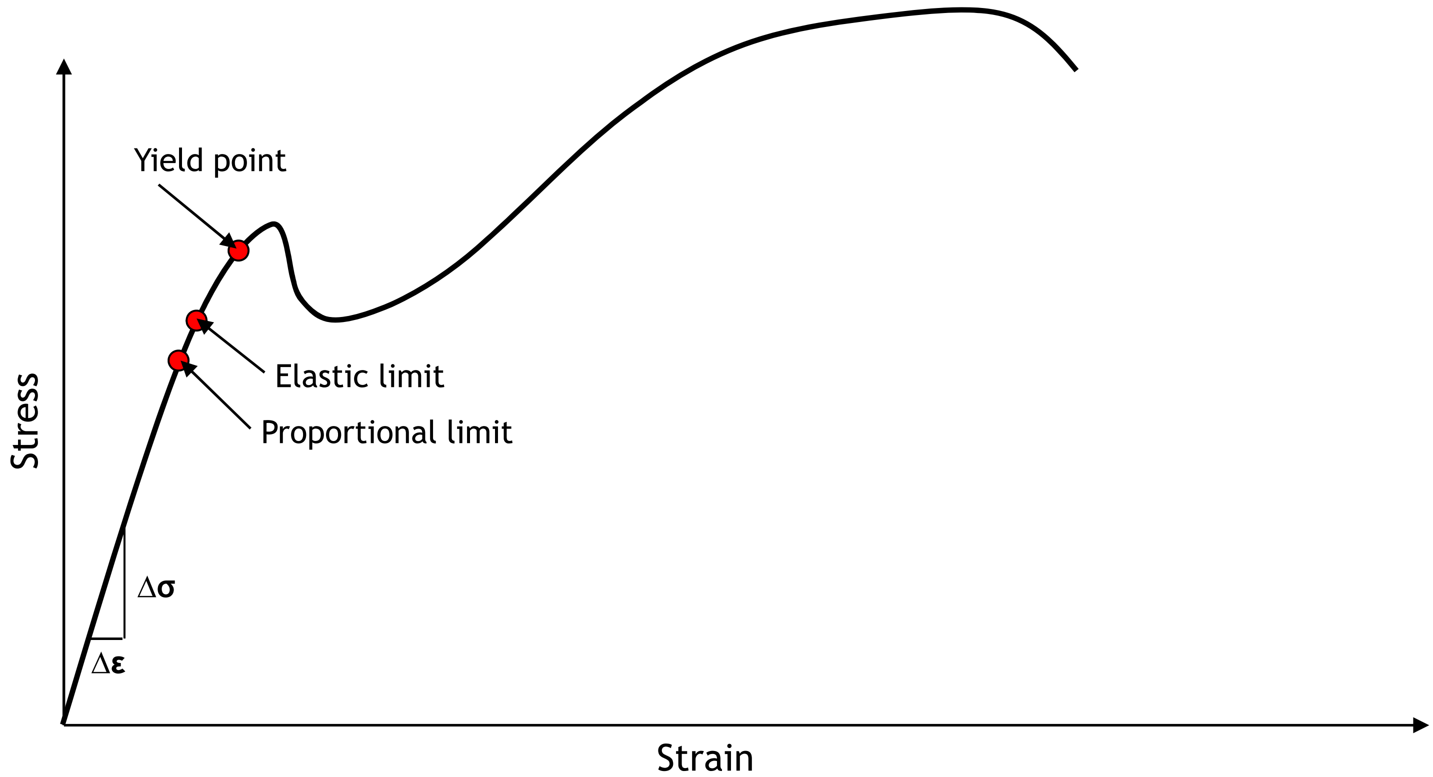
Stress – Strain Diagram   Typical for Ductile Materials



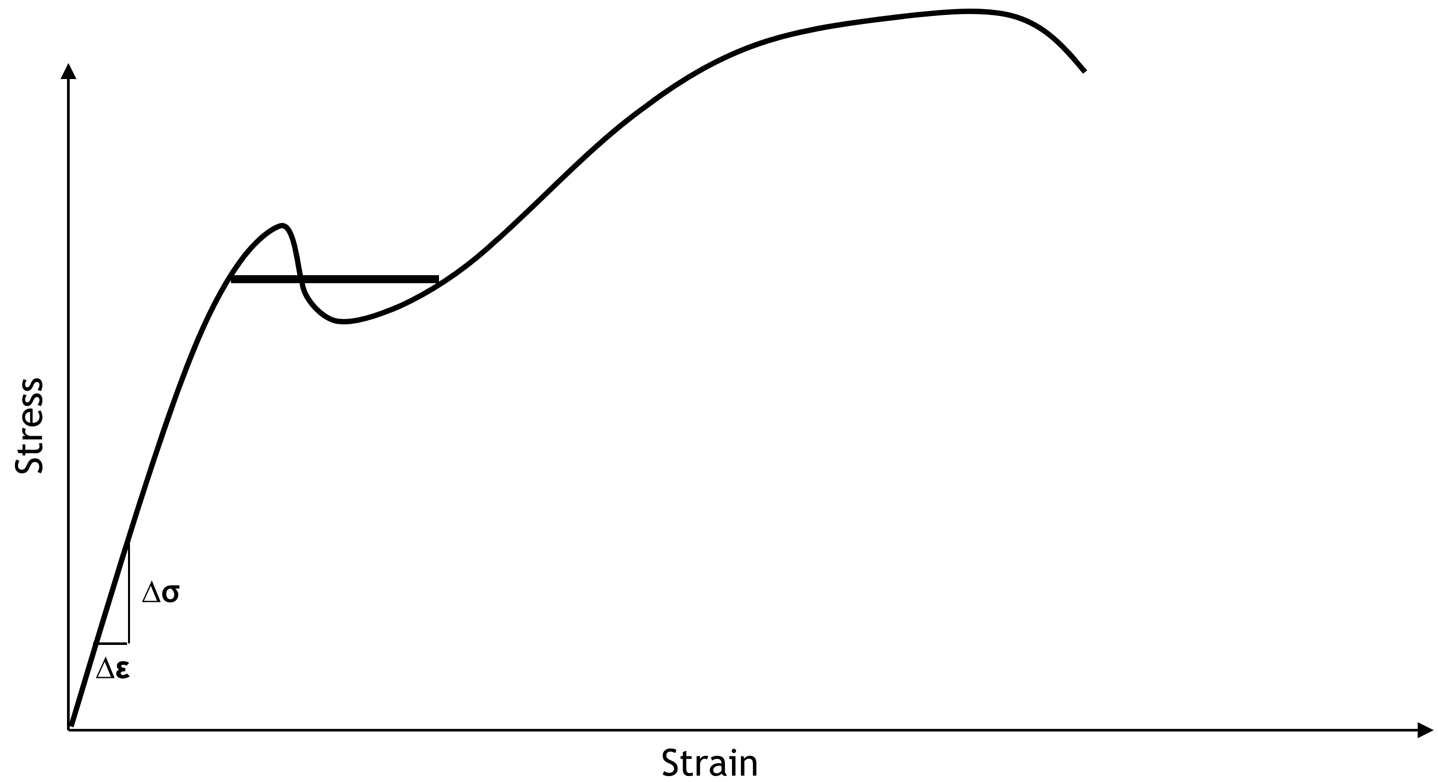
Stress – Strain Diagram



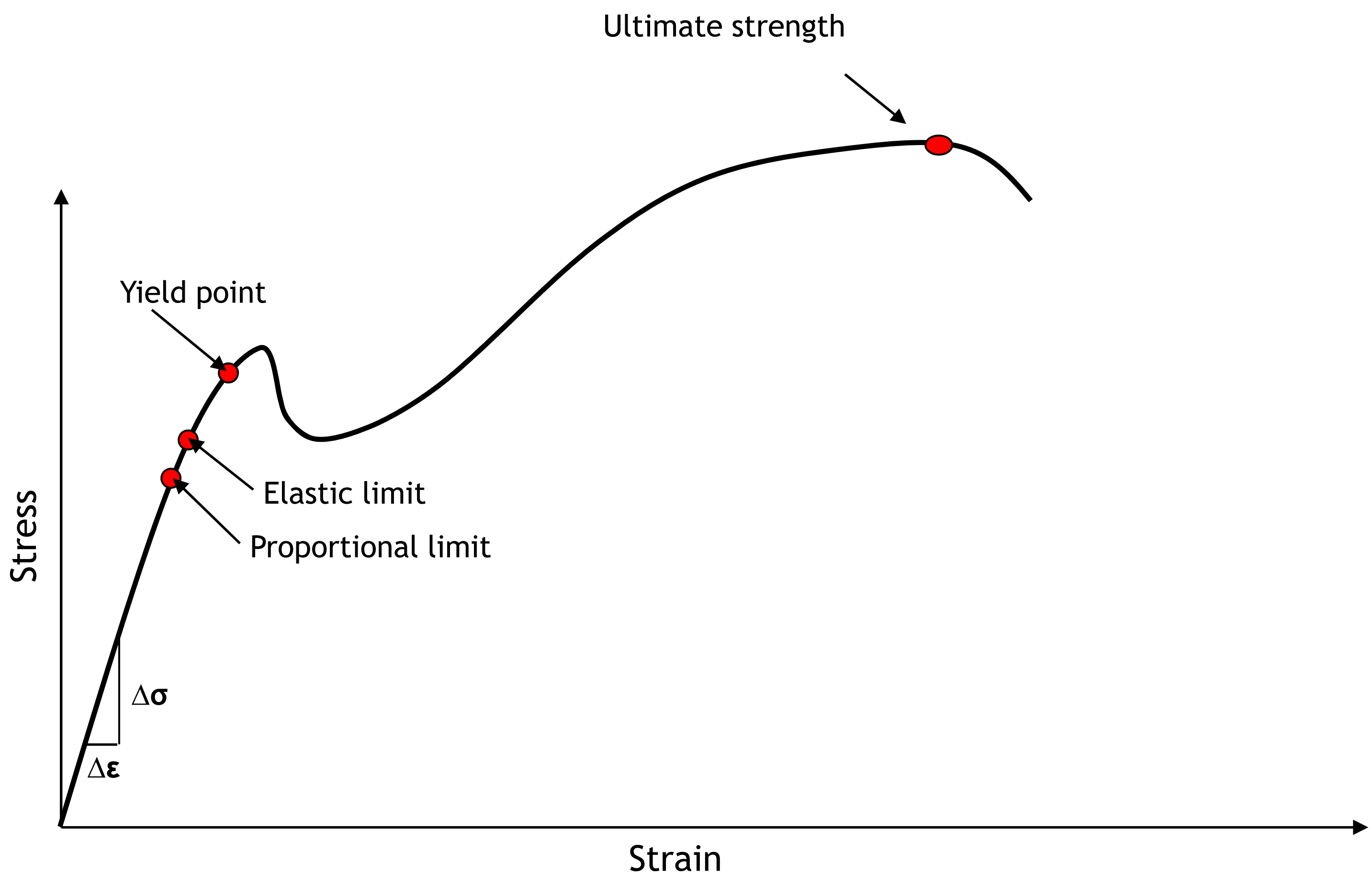
Stress – Strain Diagram



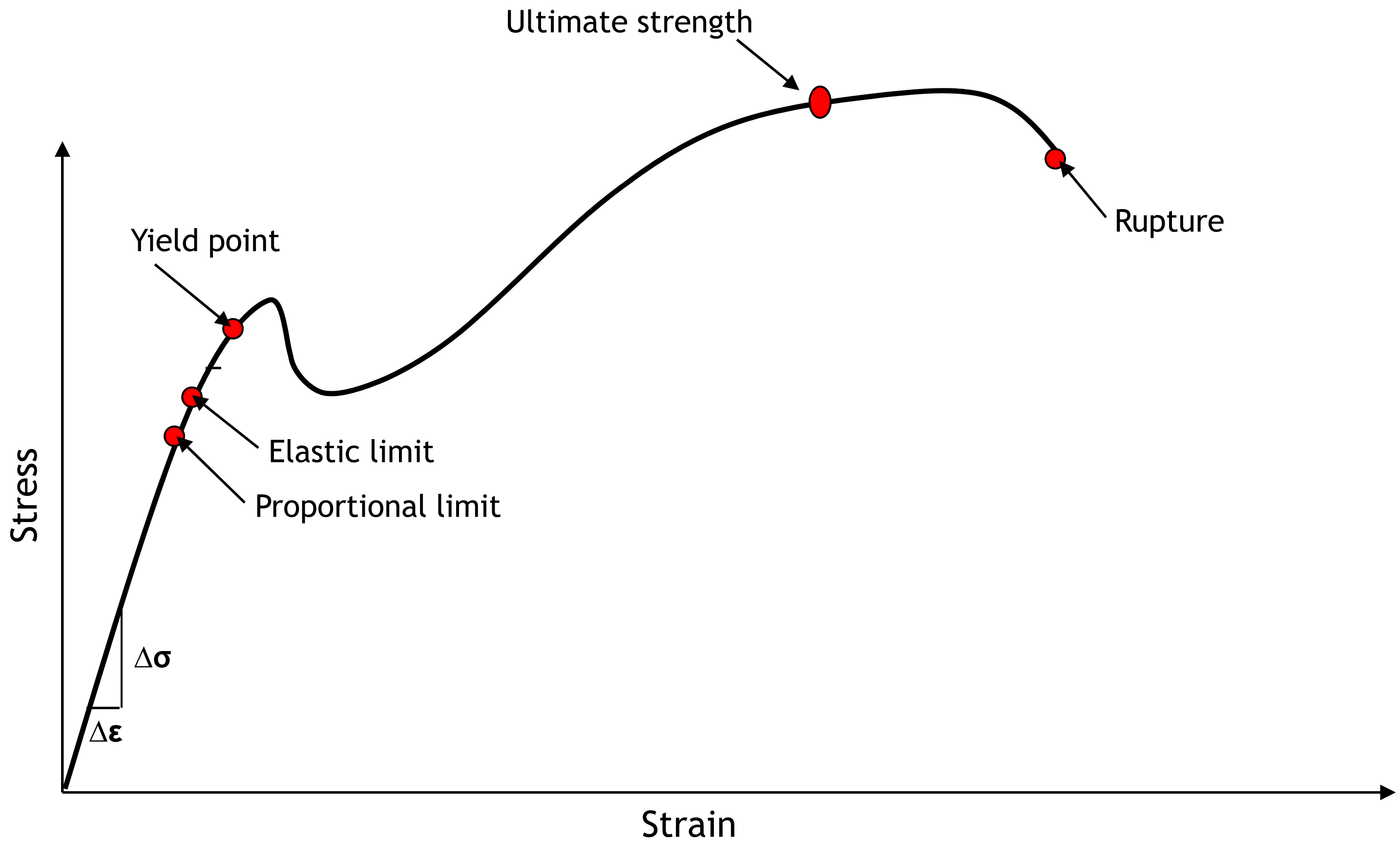
Stress – Strain Diagram



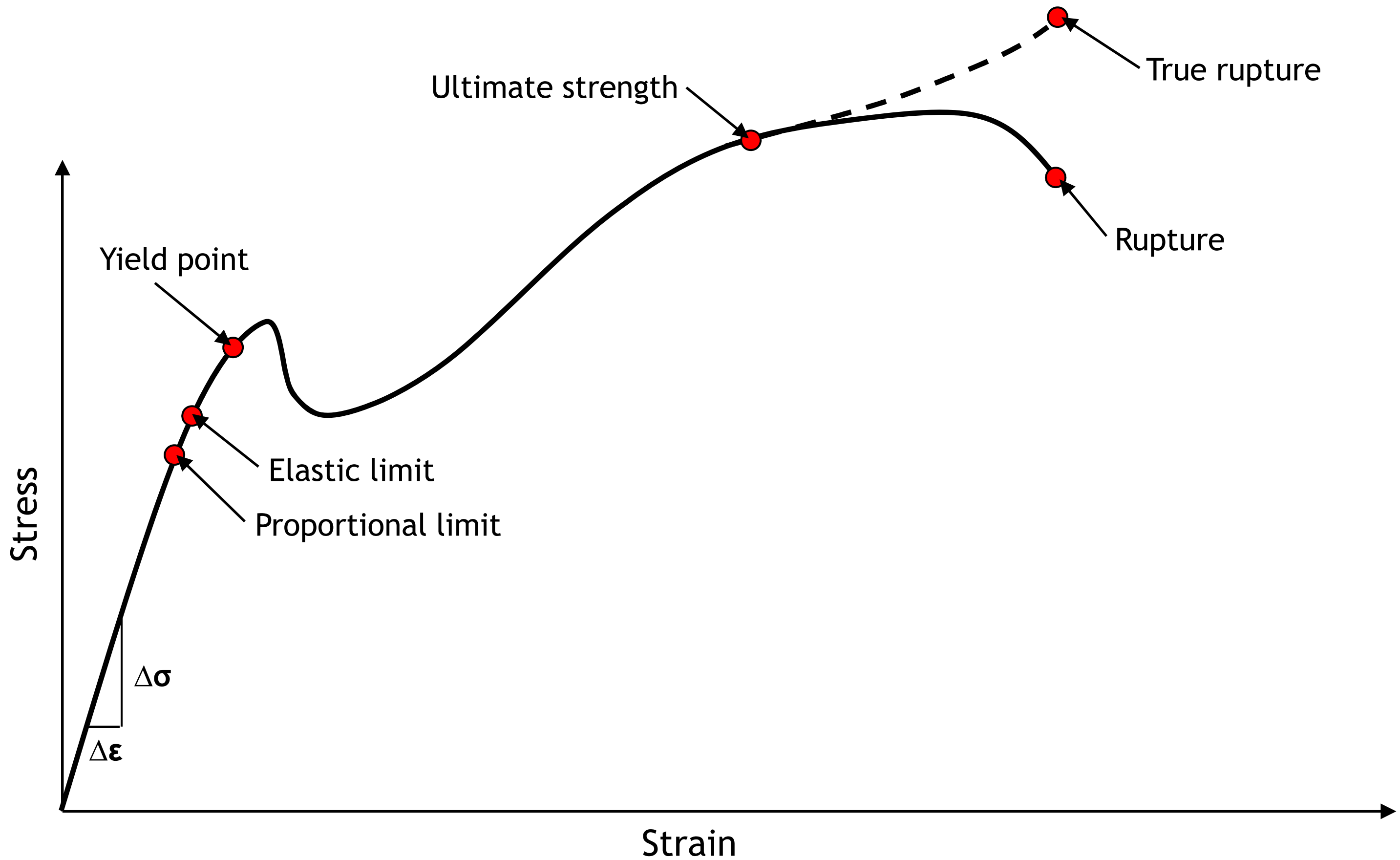
Stress – Strain Diagram



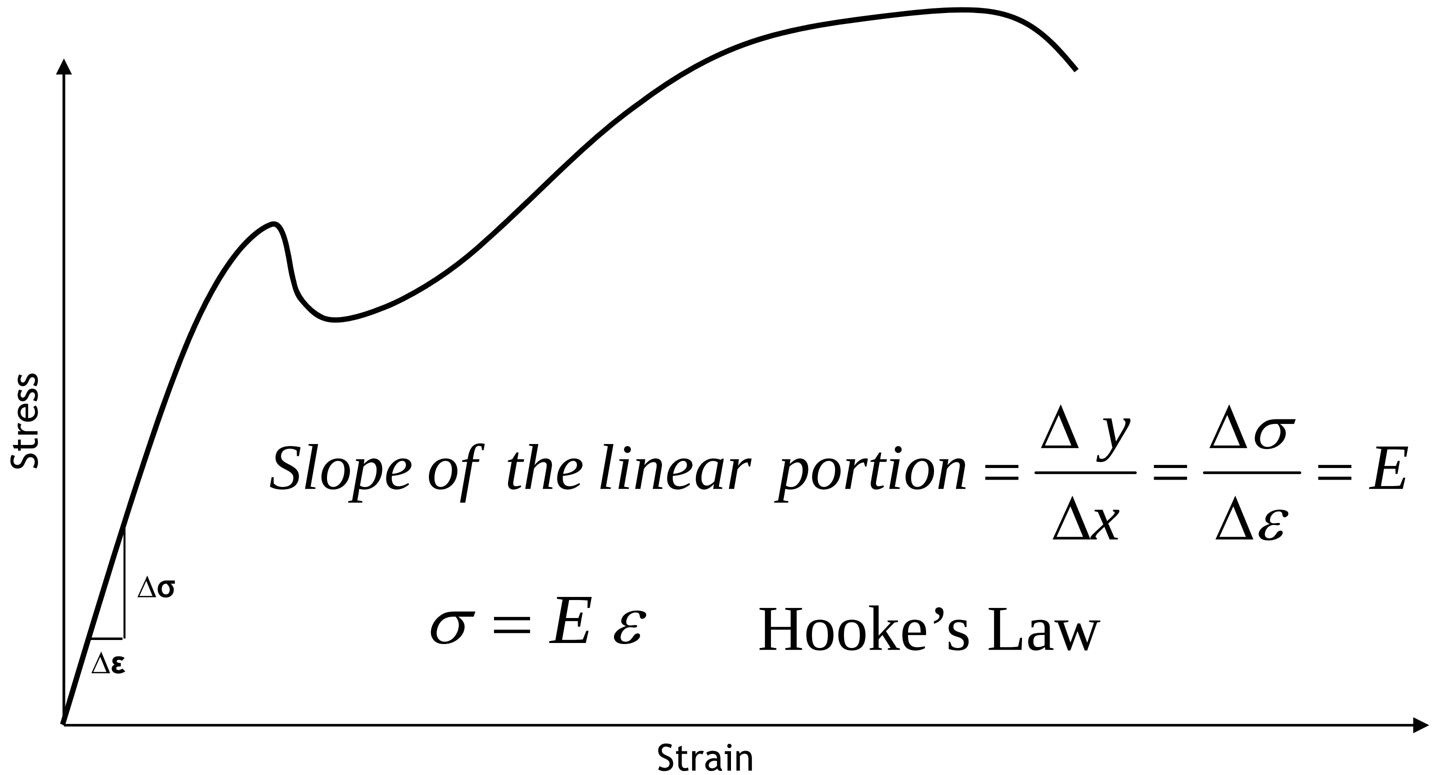
Stress – Strain Diagram



Stress Strain Diagram



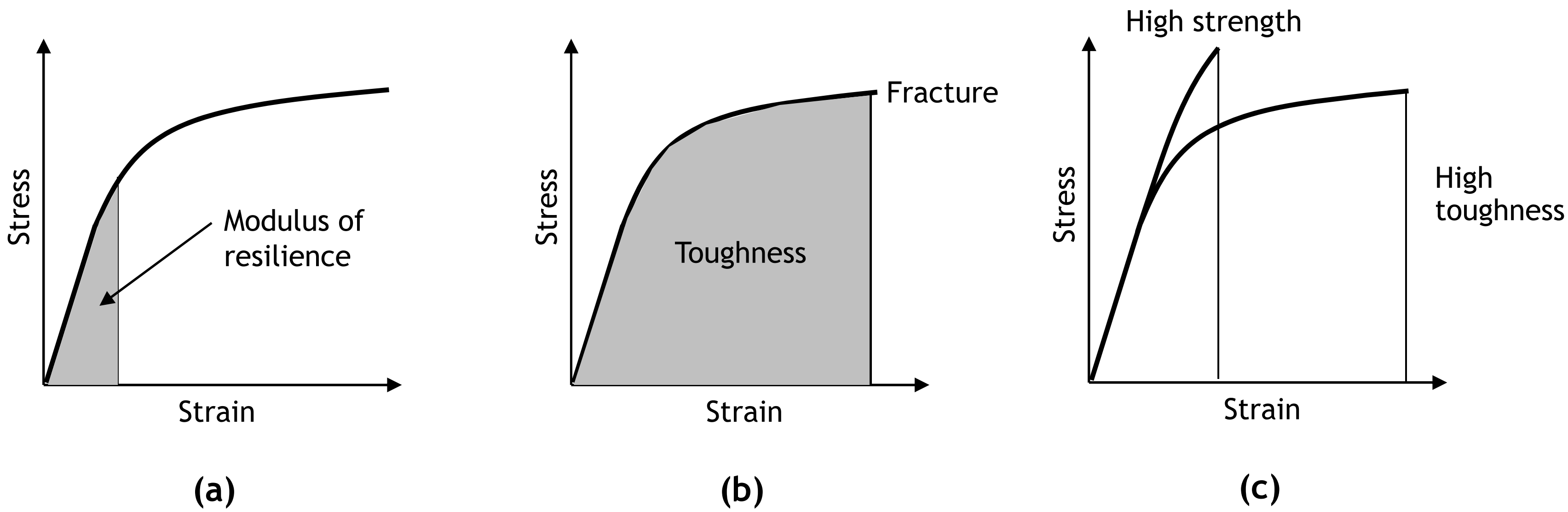
Stress- Strain Relationship



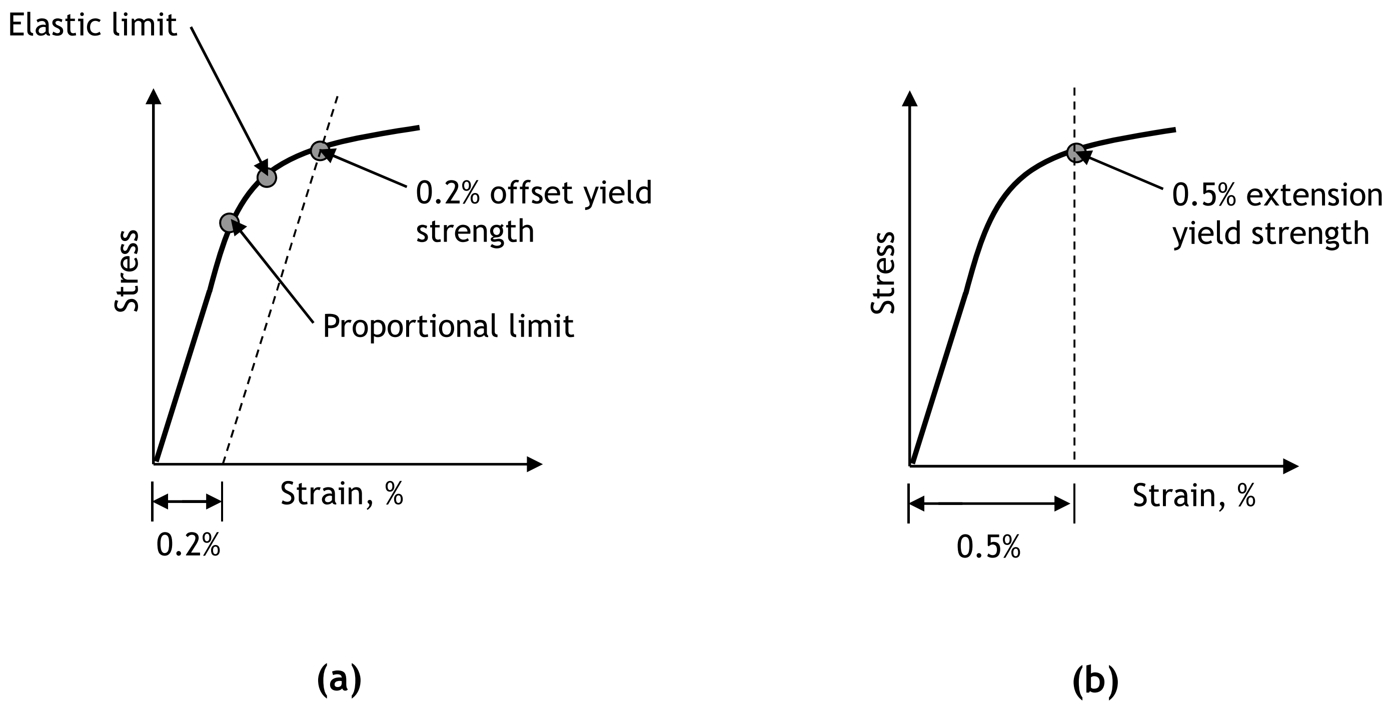
$$\text{Slope of the linear portion} = \frac{\Delta y}{\Delta x} = \frac{\Delta \sigma}{\Delta \varepsilon} = E$$

$$\sigma = E\,\varepsilon$$ Hooke's Law

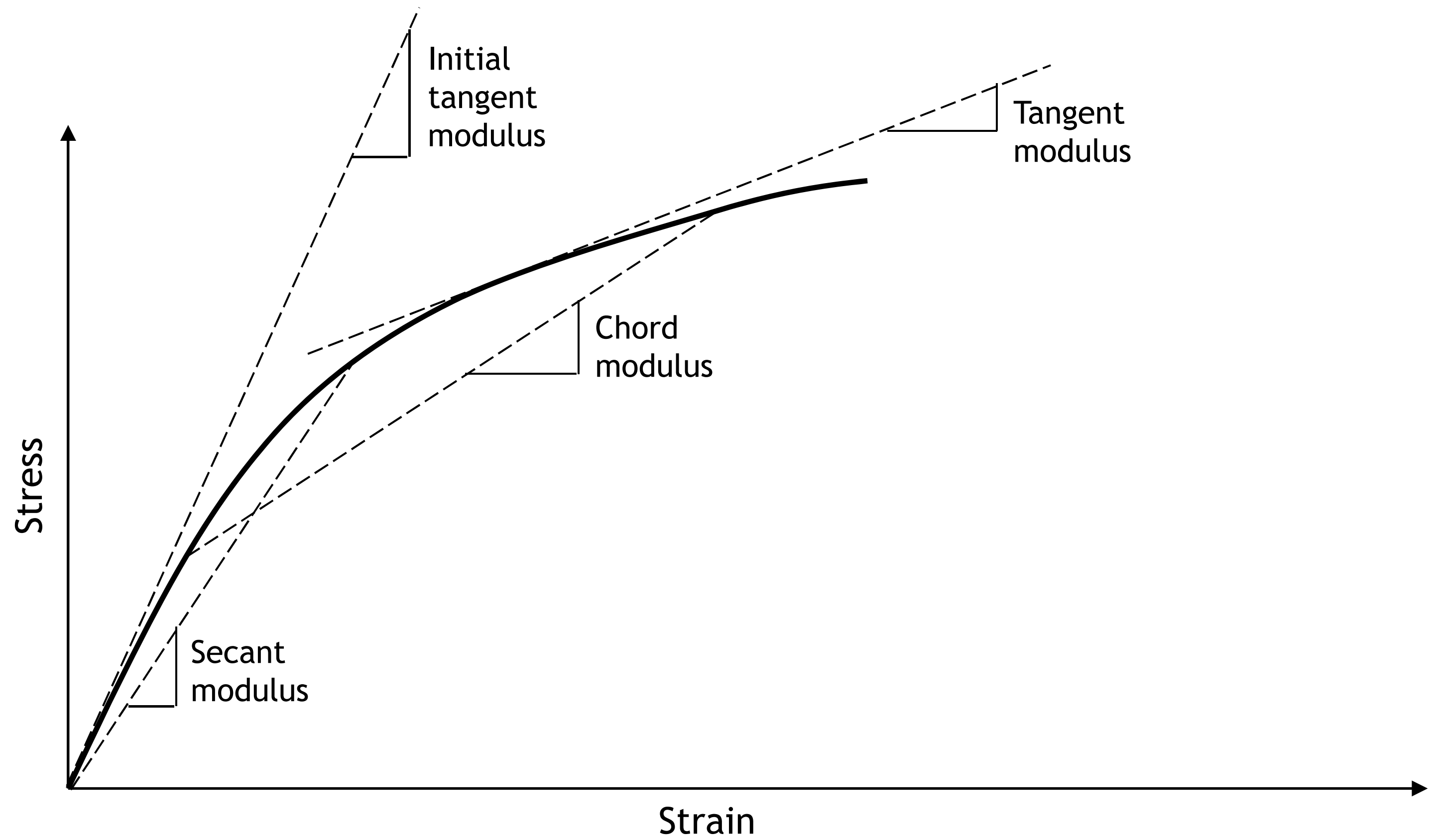
Stress – Strain Relationship Typical for Ductile Materials



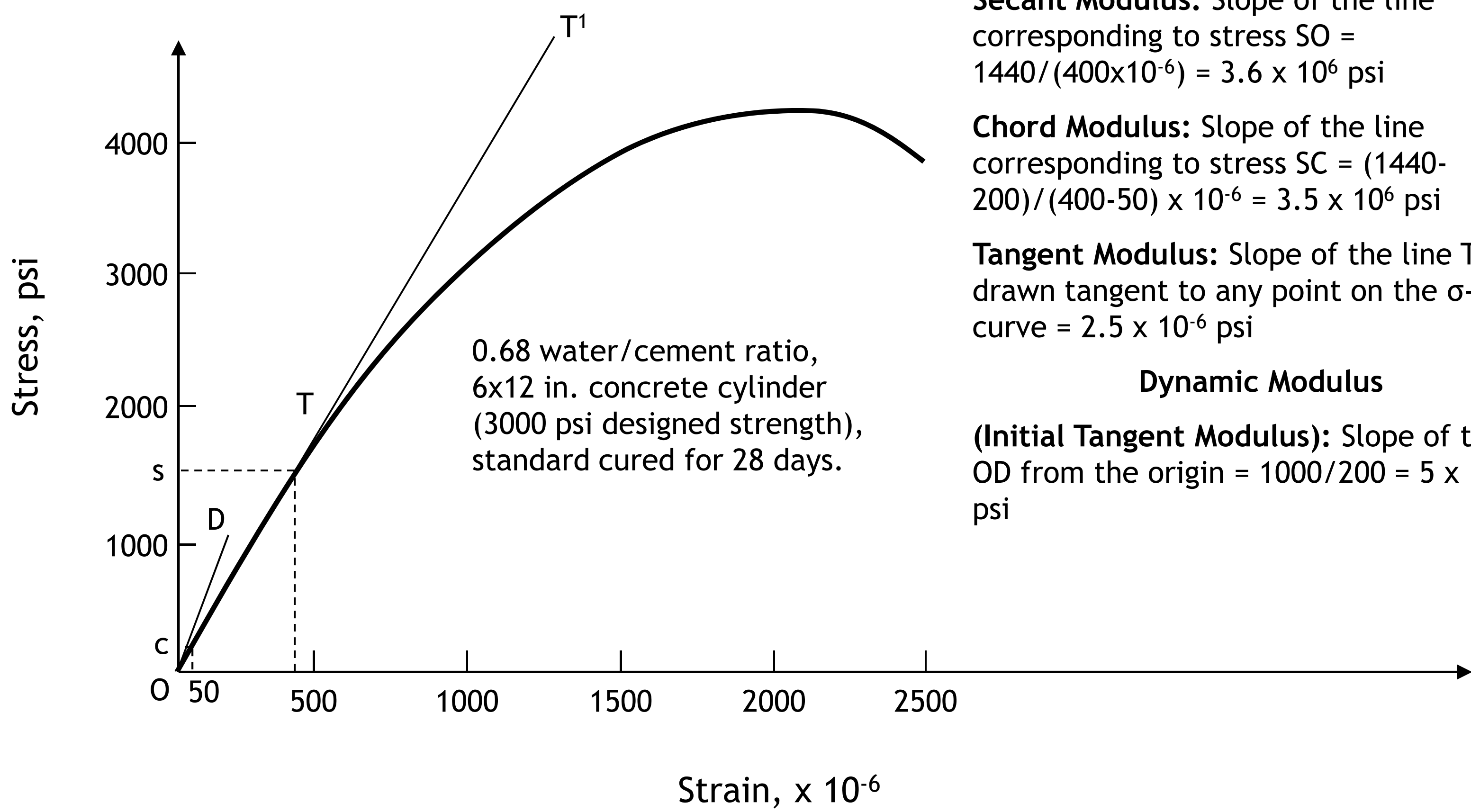
Areas under stress-strain curves: (a) modulus of resilience, (b) toughness, and (c) high-strength and high-toughness materials.



Methods for estimating yield stress: (a) offset method and (b) extension method



Methods for approximating modulus of Elasticity



CALCULATING THE ELASTIC MODULI

σ_{ULT} = 3600 psi; 40% σ_{ULT} = 1440 psi = SO

Secant Modulus: Slope of the line corresponding to stress SO = 1440/(400x10^{-6}) = 3.6 x 10^6 psi

Chord Modulus: Slope of the line corresponding to stress SC = (1440-200)/(400-50) x 10^{-6} = 3.5 x 10^6 psi

Tangent Modulus: Slope of the line TT^1 drawn tangent to any point on the σ-ε curve = 2.5 x 10^{-6} psi

Dynamic Modulus

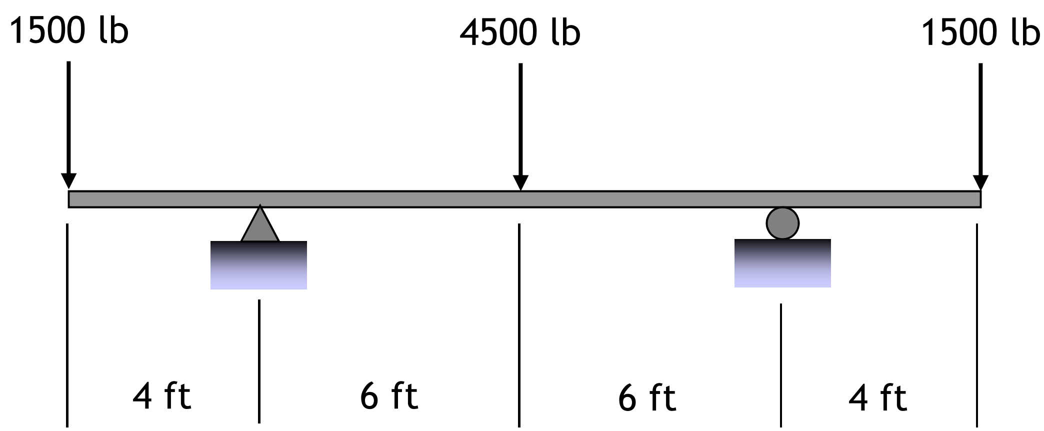
(Initial Tangent Modulus): Slope of the OD from the origin = 1000/200 = 5 x 10^6 psi

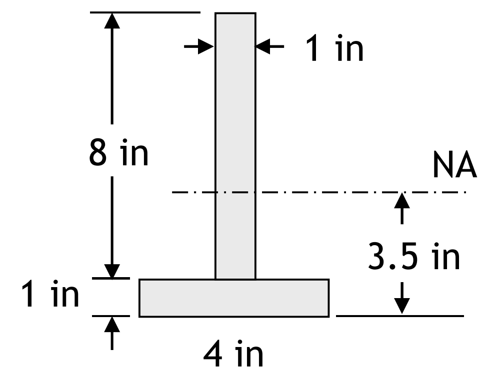
Different types of elastic moduli and the method by which these are determined for Portland Cement Concrete

TYPICAL for Brittle Materials

Problem 1)

For the beam shown, compute the maximum bending stress in compression and maximum bending stress in tension. Also compute the shear stress at the neutral axis as well as at a section 6 in. from the bottom of the cross-section. Calculate the shear stresses at a point along the beam where the maximum shear force occurs.

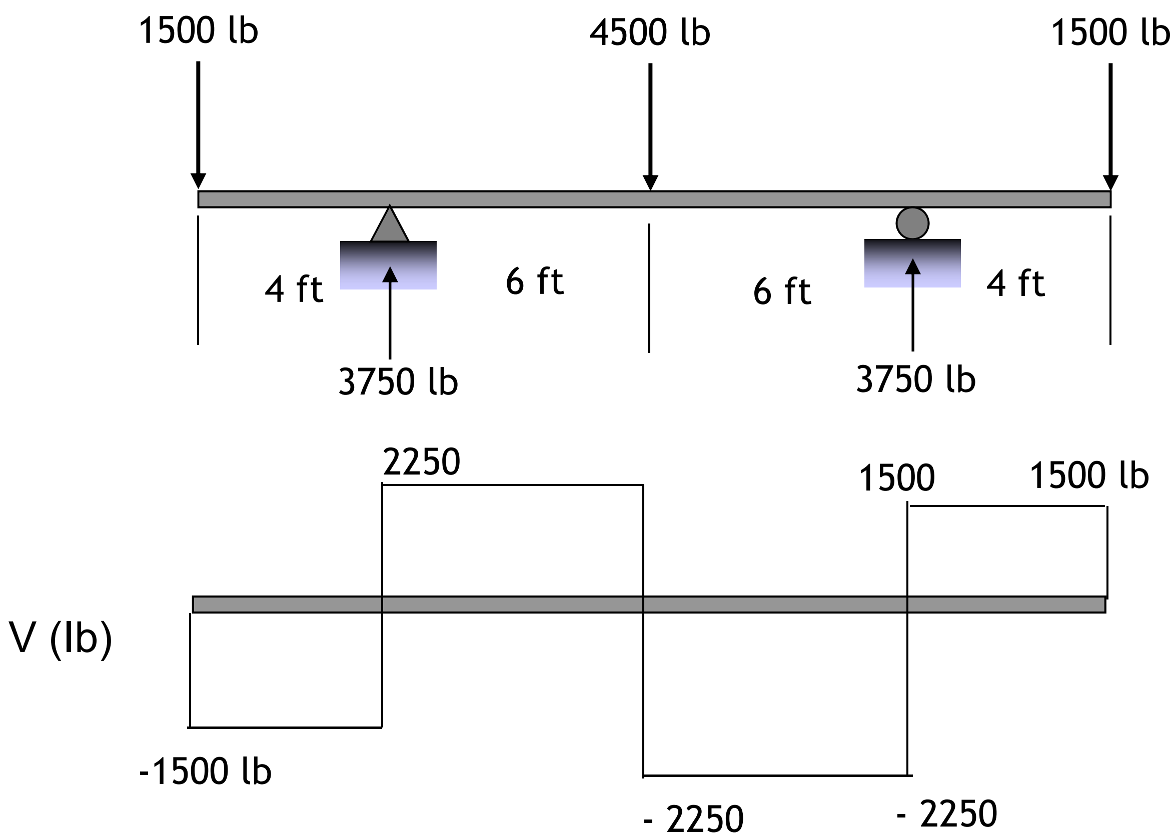


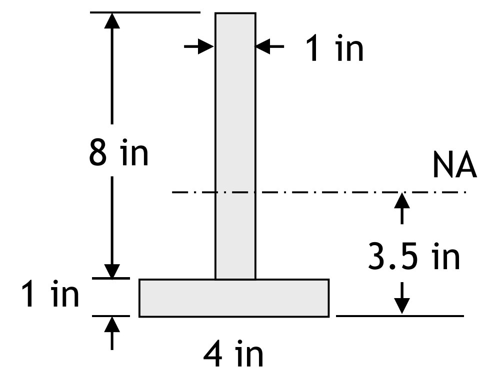


I_{NA} = 97.0 in^4

Problem 1 (continued)

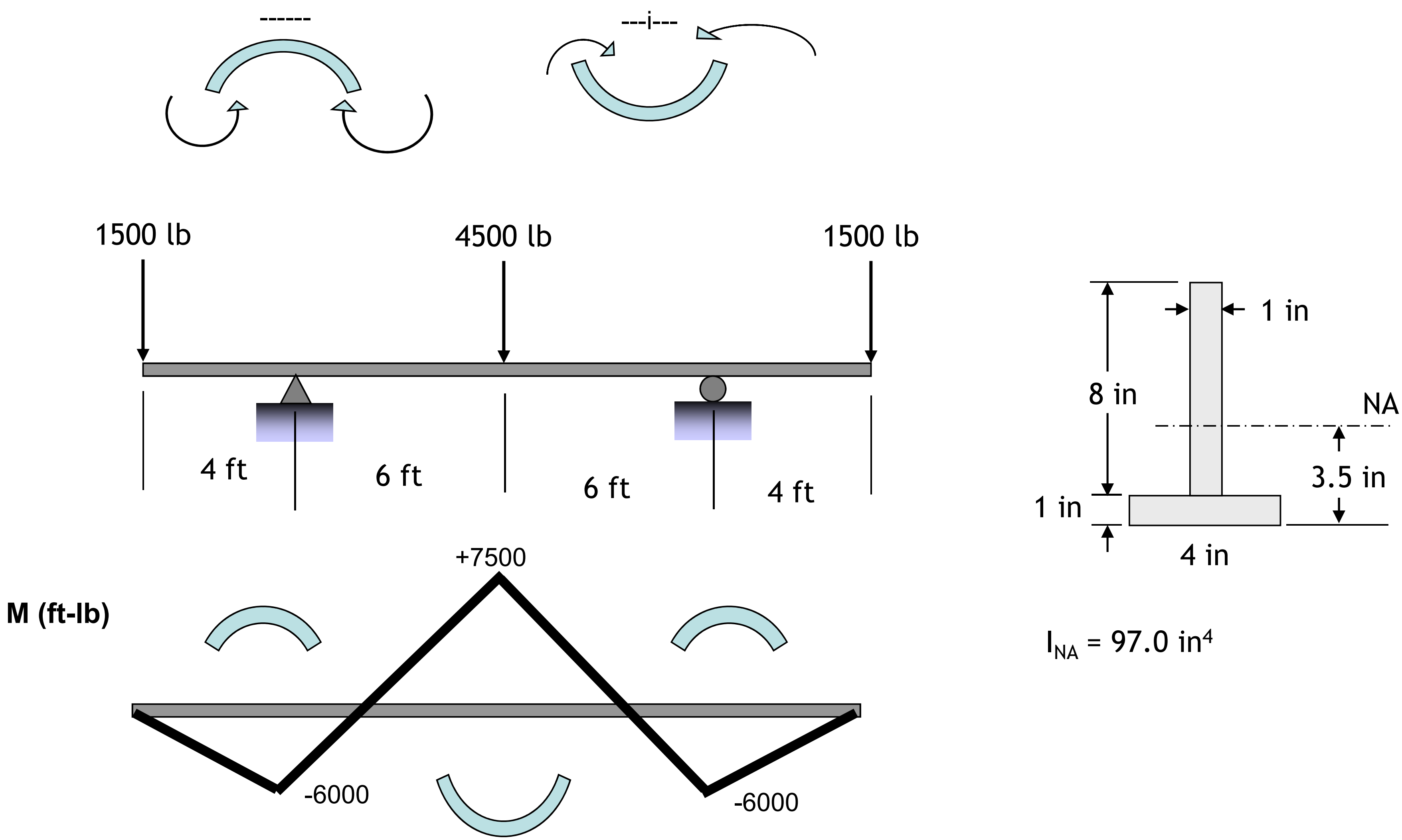
For the beam shown, compute the shear stress at the NA and at 6 in. from the bottom of the cross section at the point where the maximum shear force occurs.





I_{NA} = 97.0 in^4

Problem 1 (continued) - For the beam shown, compute the max. bending stress in tension and in compression.



Bending Stress in beams : $$\sigma = \frac{M\ C}{I}$$

$$\sigma_c = \frac{M\ C_c}{I}$$

$$\sigma_t = \frac{M\ C_t}{I}$$

In the negative moment region

↓

$$C_c = 3.5\ \ in$$

$$C_t = 5.5\ \ in$$

$$\sigma_t = \frac{(6000)(12)(5.5)}{97} = 4082\ psi$$

$$\sigma_c = \frac{(6000)(12)(3.5)}{97} = 2598\ psi$$

In the positive moment region

↓

$$C_c = 5.5\ \ in$$

$$C_t = 3.5\ \ in$$

$$\sigma_t = \frac{(7500)(12)(3.5)}{97} = 3247\ psi$$

$$\sigma_c = \frac{(7500)(12)(5.5)}{97} = 5103\ psi$$

Section Modulus $S = \frac{I}{C}$

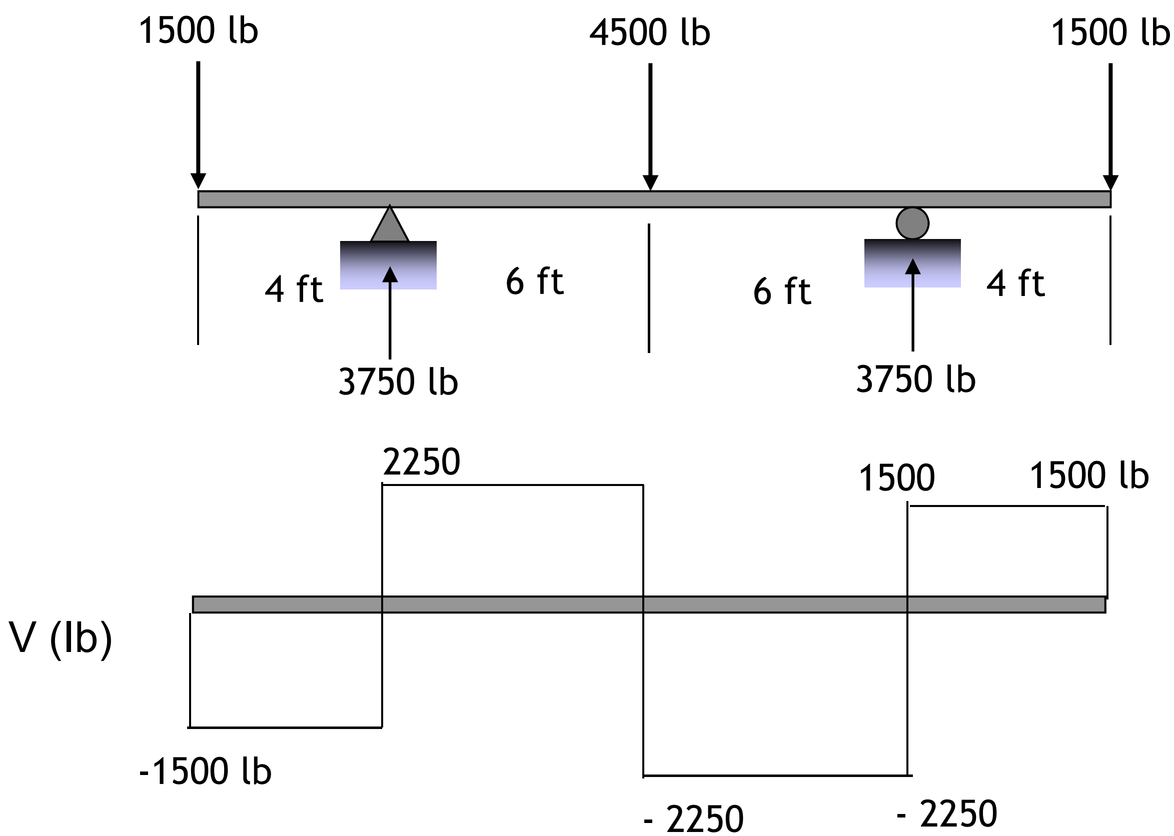
$$\sigma = \frac{M\,C}{I}$$

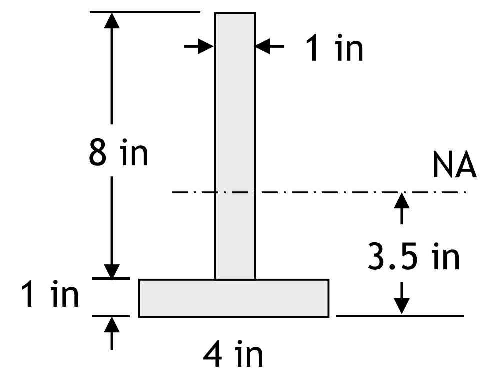
$$\Rightarrow \left.\right\} \quad \sigma = \frac{M}{S}$$

$$S = \frac{I}{C} = \frac{97}{5.5} = 17.6 \quad in^3$$

Problem 1 (continued)

For the beam shown, compute the shear stress at the NA and at 6 in. from the bottom of the cross section at the point where the maximum shear force occurs.





I_{NA} = 97.0 in^4

Shear stress in bending

$$\tau = \frac{V\,Q}{I\,b}$$

$$Q_{6\,in\ \ from\,bottom\,of\ \ cross-section} = (3)\,(1)\,[\,4] = 12\ \ in^3$$

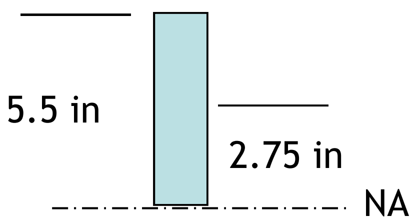
$$\tau_{\ 6\,in\ \ from\,bottom\,of\ \ cross-section} = \frac{(2250)\,(12)}{(97)(\,1)} = 278.3\ psi$$

$$Q_{\ at\ netural\ axis} = \ (5.5)\,(1)\,[\,2.75] = 15.12\ \ in^3$$

$$\tau_{\ at\ netural\ axis} = \frac{(2250)\,(15.12)}{(97)(\,1)} = 350.7\ \ psi$$

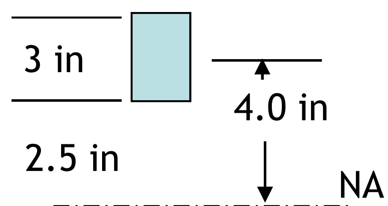
What is the value of Q if shear stress is being calculated at:

Neutral axis

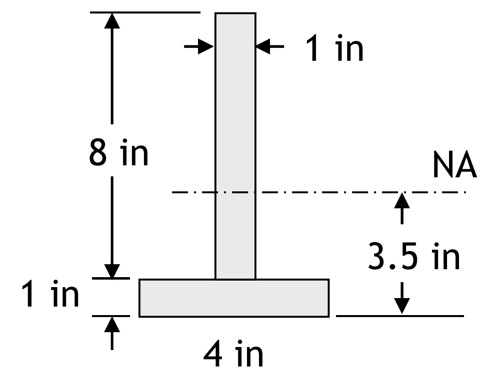


$$Q = (5.5)(1)[2.75] = 15.12 \ in^3$$

a section 6 in. above the base



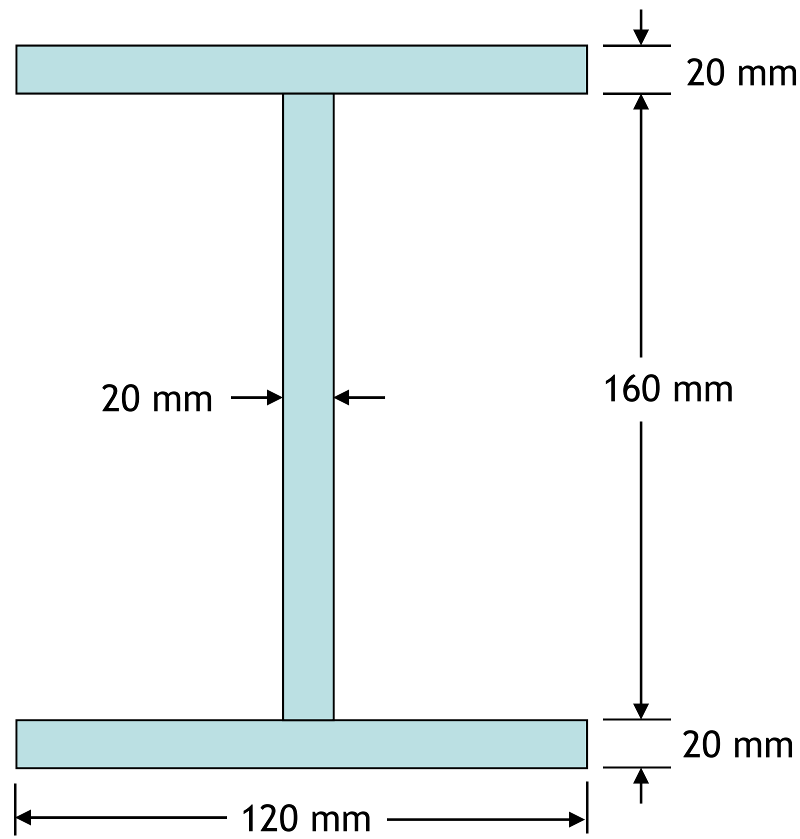
$$Q = (3)(1)[4] = 12 \ in^3$$



$I_{NA} = 97.0 \ in^4$

Problem 2.

The vertical shear force acting on the I-section shown is 100 kN. Compute the maximum shear stress in bending acting on the this beam section.



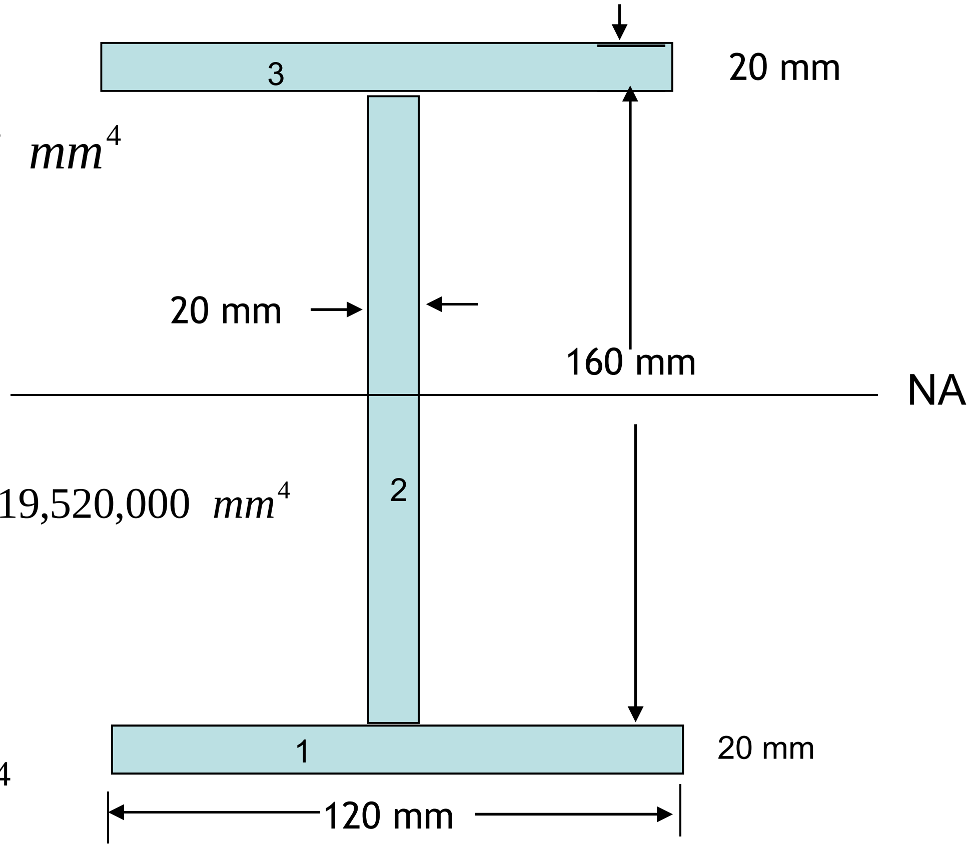
$$I = \frac{bh^3}{12} + Ad^2 \quad \textit{Parallel Axis Theorem}$$

$$I_{1\,about\;NA} = \frac{(120)(20)^3}{12} + (120)(20)[90]^2 = 19{,}520{,}000 \;\; mm^4$$

$$I_{2\,about\;NA} = \frac{(20)(160)^3}{12} = 6{,}826{,}667 \;\; mm^4$$

$$I_{3\,about\;NA} = \frac{(120)(20)^3}{12} + (120)(20)[90]^2 = 19{,}520{,}000 \;\; mm^4$$

$$I_{total\;about\;NA} = 45{,}866{,}666\; mm^4$$



Problem 2 (continued)
Maximum shearing stress occurs at the neutral axis.

$$Q\ at\ NA = (20)(80)[40] + (120)(20)[90] = 280{,}000\ mm^3$$

$$V = 100\ KN$$

$$b = 20\ mm \quad at\ NA$$

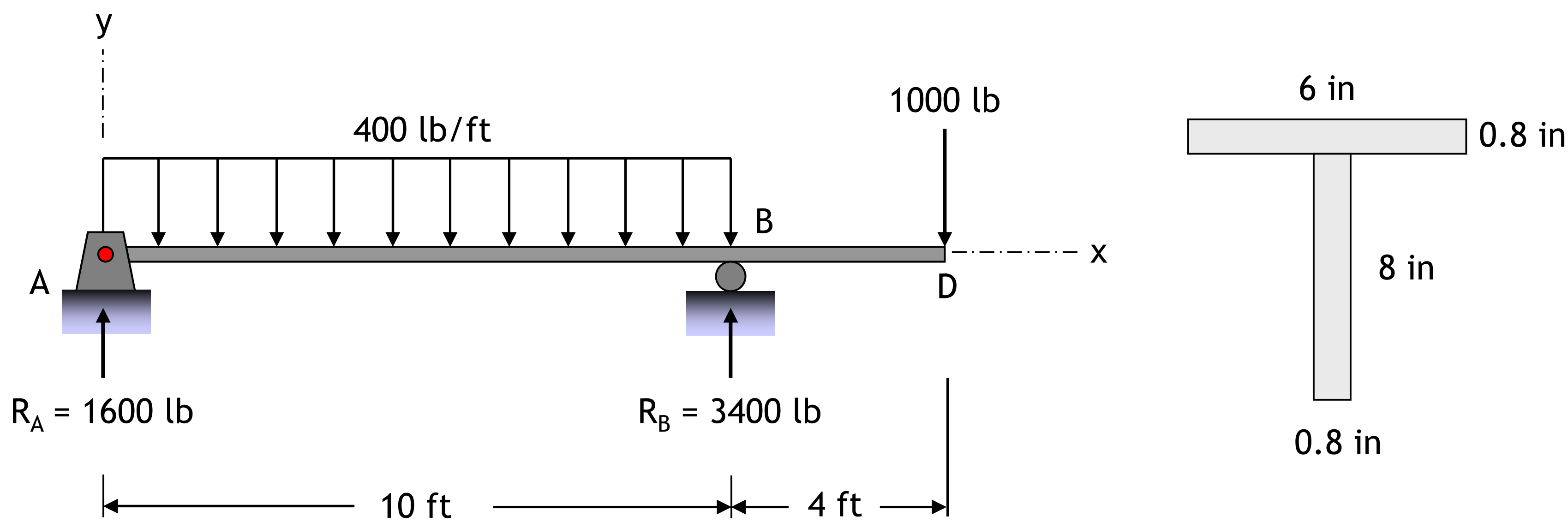
$$\tau = \frac{V\,Q}{I\,b}$$

$$I_{total\ about\ NA} = 45{,}866{,}666\ mm^4$$

$$\tau = \frac{(100)\,(280000)}{(45866666)\,(20)} = 0.03\ \ KN / mm^2$$

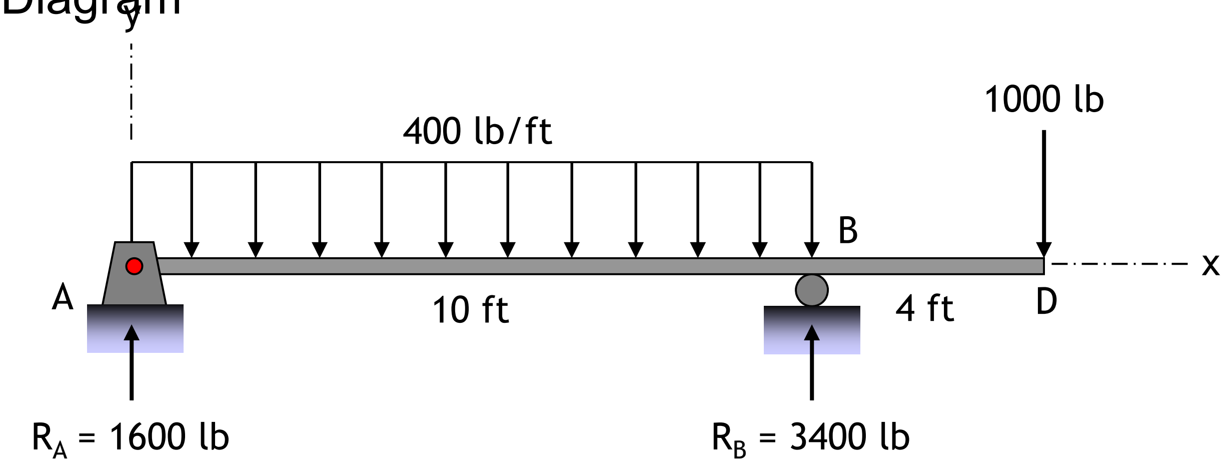
Problem 3.

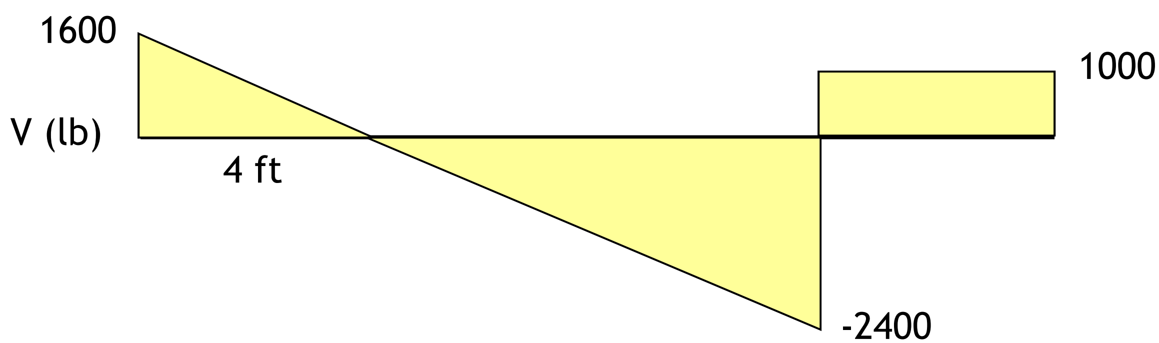
The simply supported beam has the T-shaped cross section shown. Determine the values and locations of the maximum tensile and compressive bending stresses.



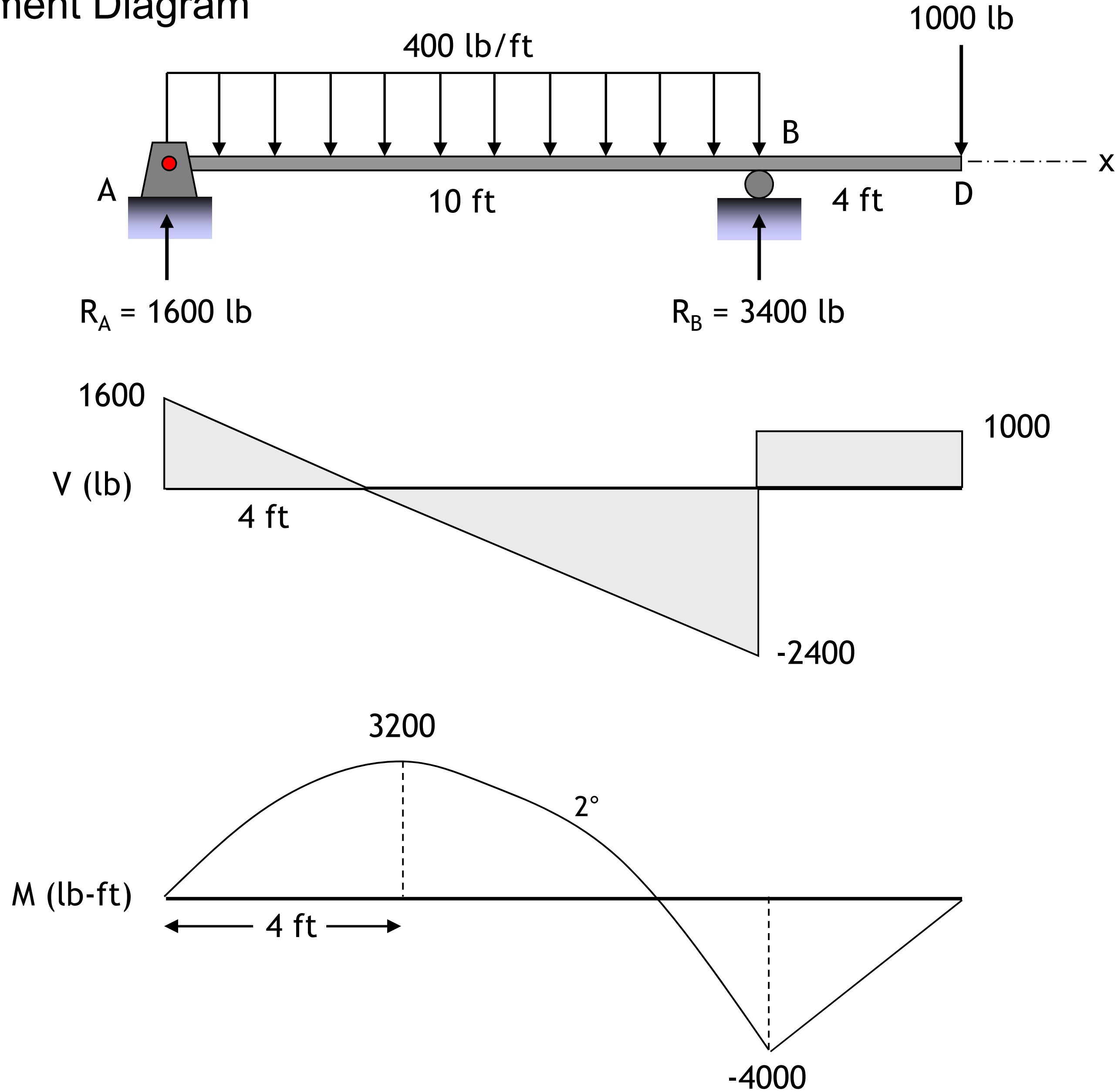
Problem 3 (continued)

Shear Diagram

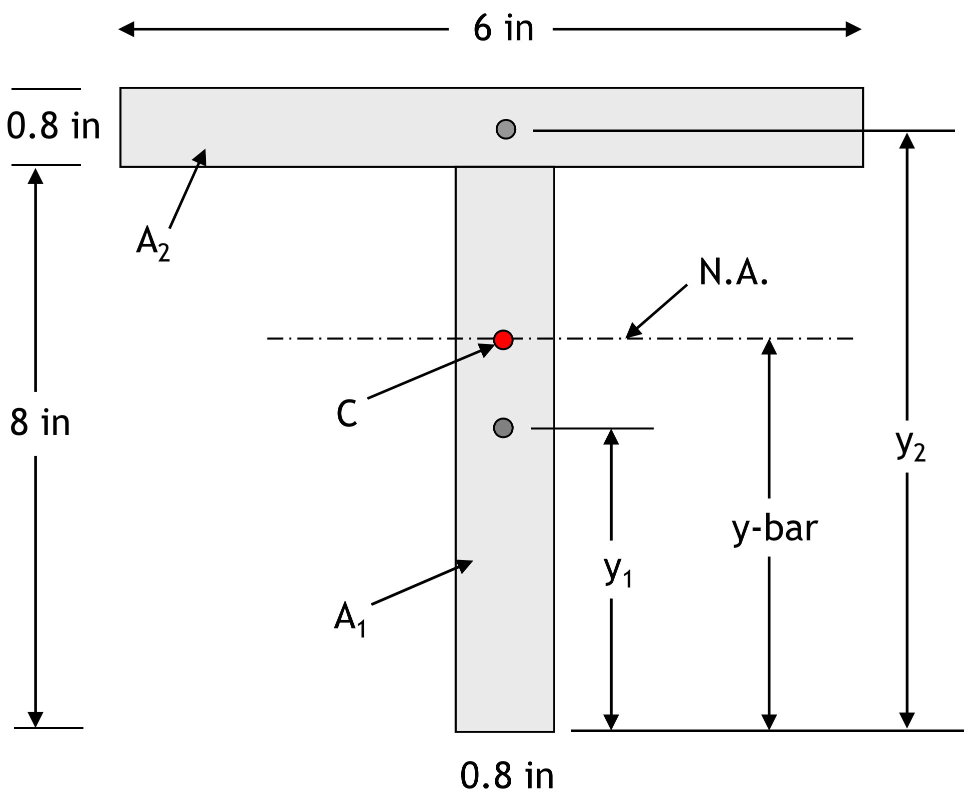




Moment Diagram



Find y-bar.

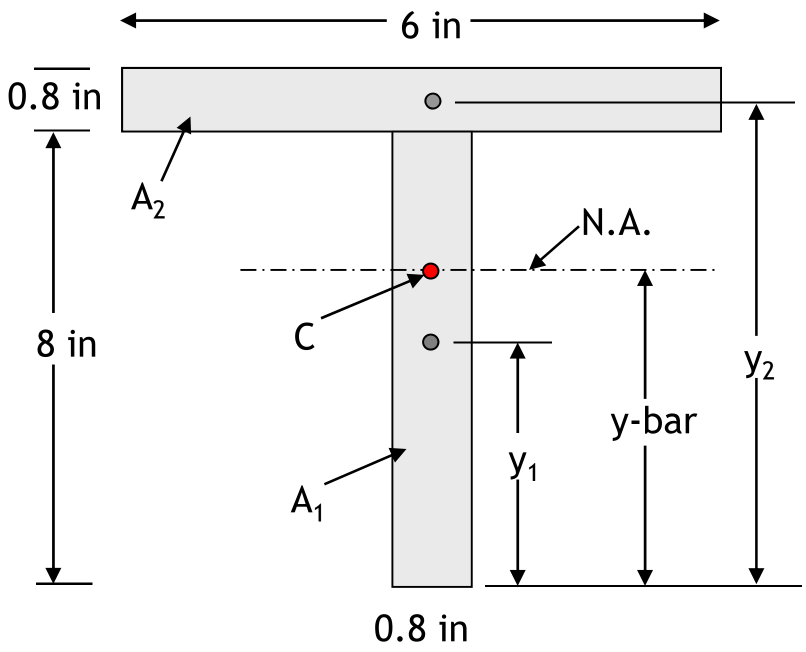


$$y - bar = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{6.4(4) + 4.8(8.4)}{6.4 + 4.8}$$

$$= 5.886\ in$$

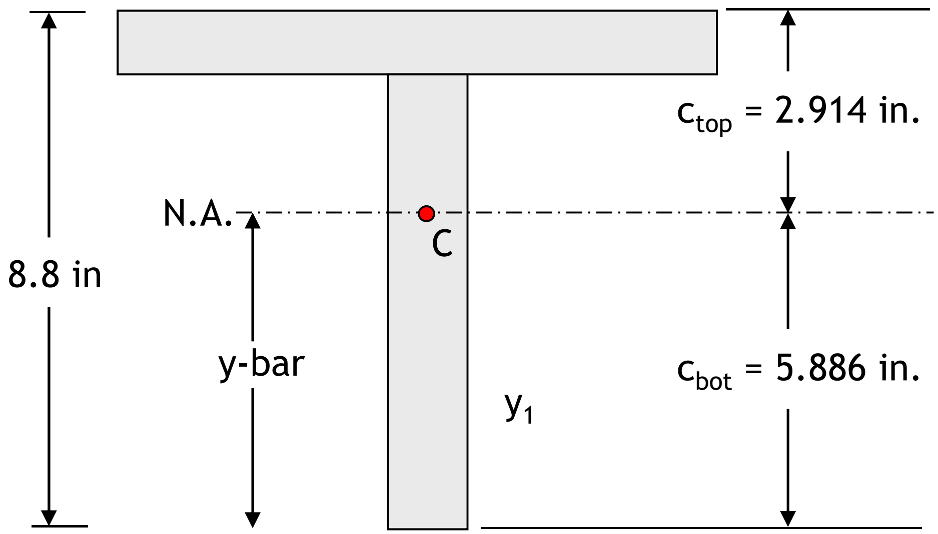
Compute Moment of Inertia, I.

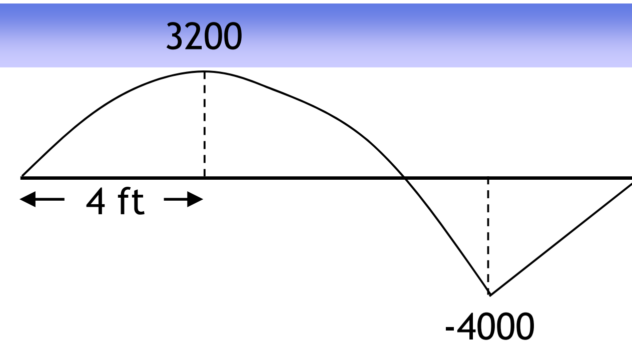


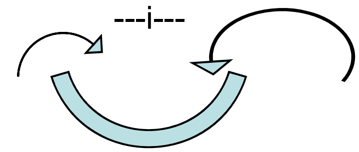
$$I = \left[\frac{0.8(8)^3}{12} + 6.4(4 - 5.886)^2\right] + \left[\frac{6(0.8)^3}{12} + 4.8(8.4 - 5.886)^2\right]$$

$$I = 87.49\ in^4$$

Stresses at x = 4 ft







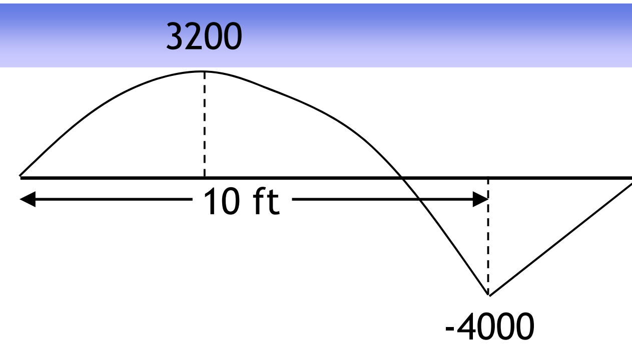
$$\sigma_{top} = \frac{Mc_{top}}{I} = \frac{(3200 \times 12)(2.914)}{87.49} = 1279\ psi$$

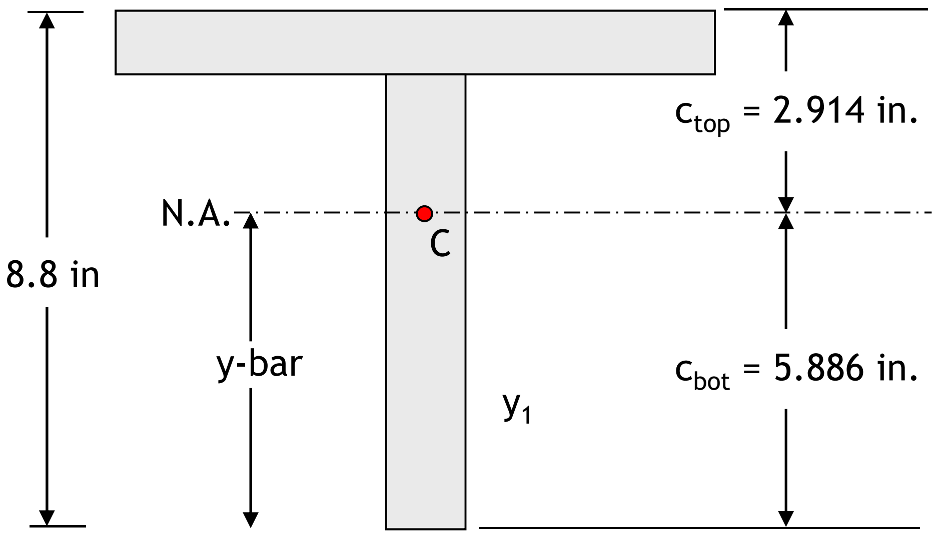
Compression on top

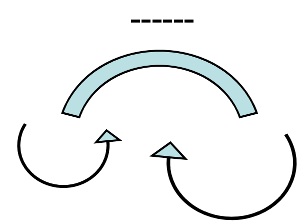
$$\sigma_{bot} = \frac{Mc_{bot}}{I} = \frac{(3200 \times 12)(5.886)}{87.49} = 2580\ psi$$

Tension in bottom

Stresses at x = 10 ft







$$\sigma_{top} = \frac{Mc_{top}}{I} = \frac{(4000 \times 12)(2.914)}{87.49} = 1599\ psi$$

Tension on top

$$\sigma_{bot} = \frac{Mc_{bot}}{I} = \frac{(4000 \times 12)(5.886)}{87.49} = 3230\ psi$$

Compression in bottom

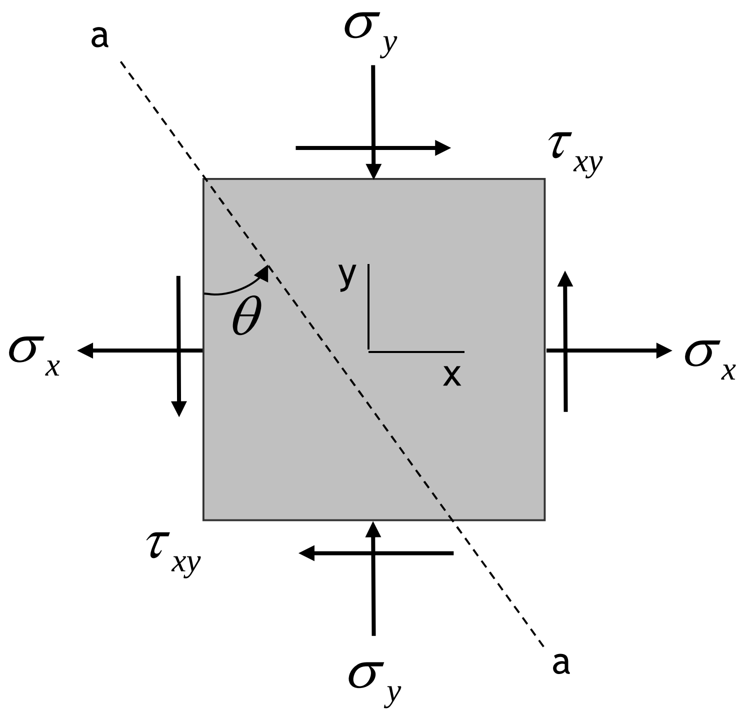
Problem 3 results.

Identify maximum tensile and compressive stresses in the beam.

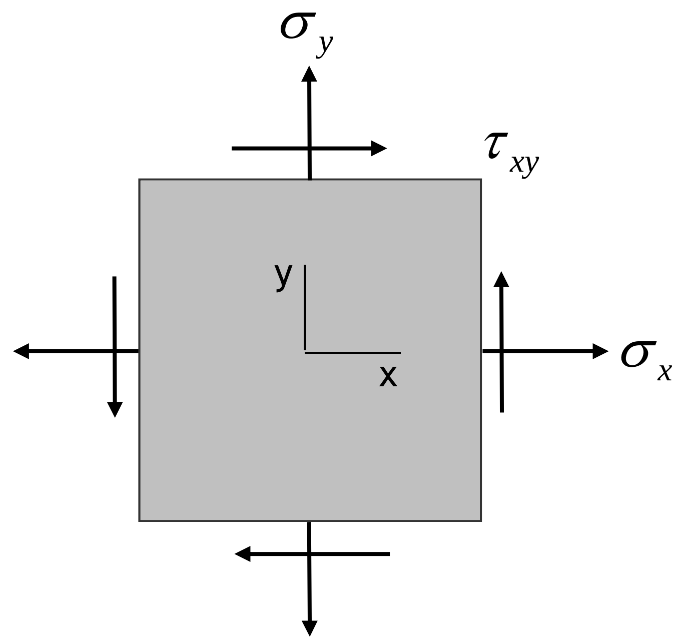
$$\left(\sigma_T\right)_{\max} = 2580\ psi$$ (bottom of the section at x = 4 ft)

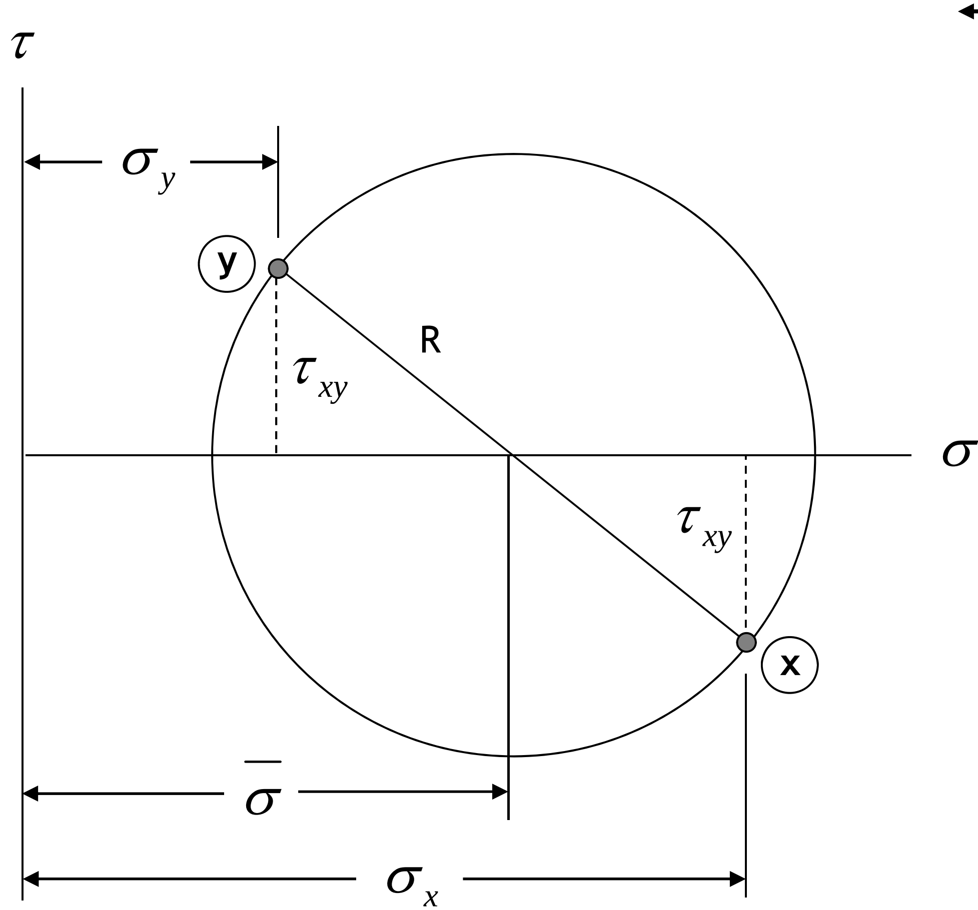
$$\left(\sigma_C\right)_{\max} = 3230\ psi$$ (bottom of the section at x = 10 ft)

Original State of Stress

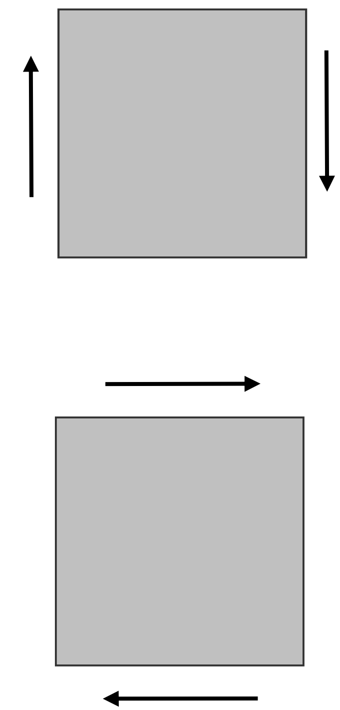


Construction of Mohr's circle from given stress components.

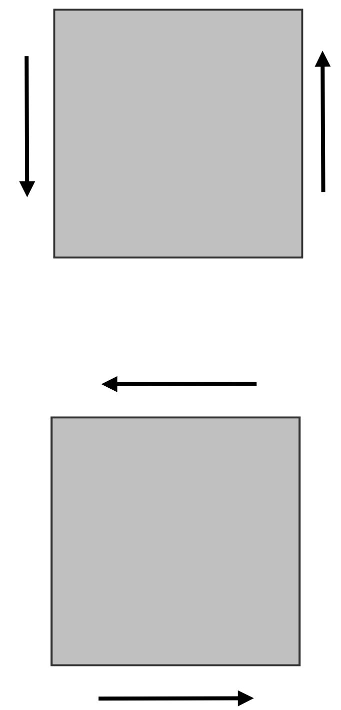




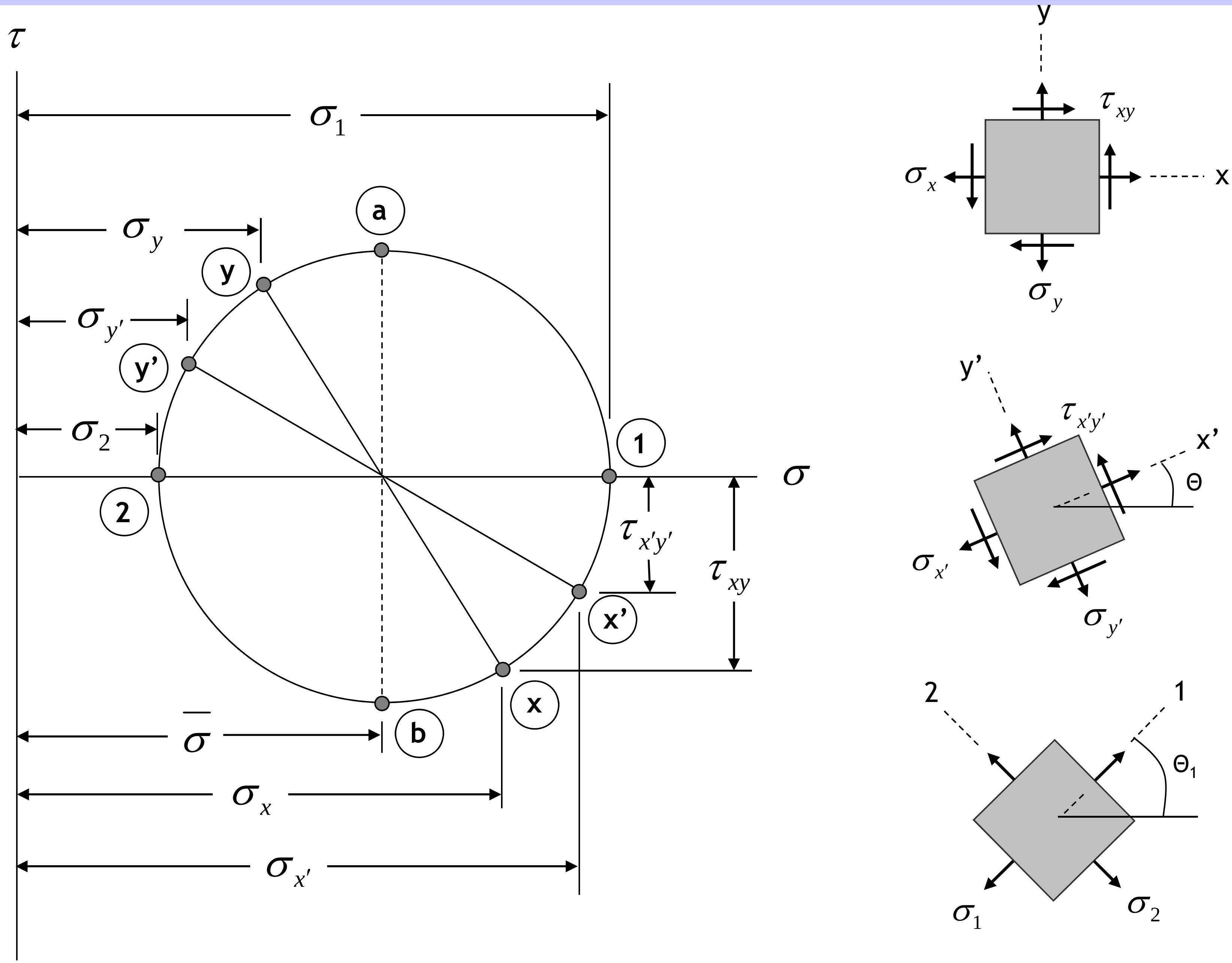
Convention for plotting shear stress on Mohr's circle.



Shear plotted up

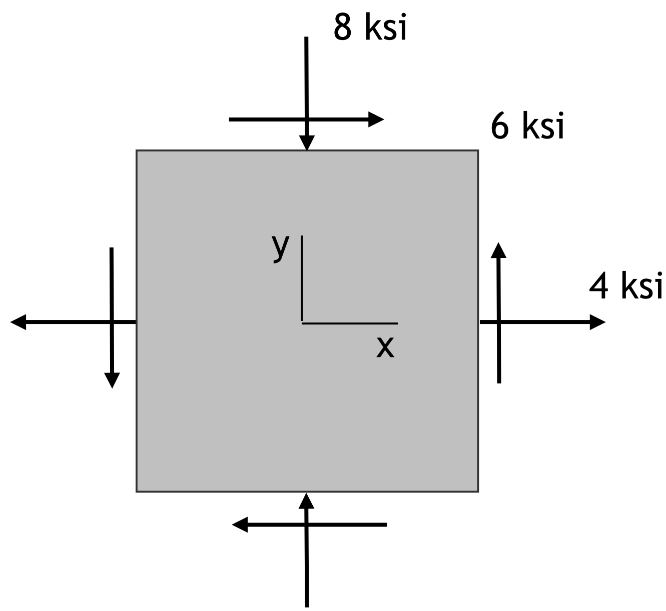


Shear plotted down

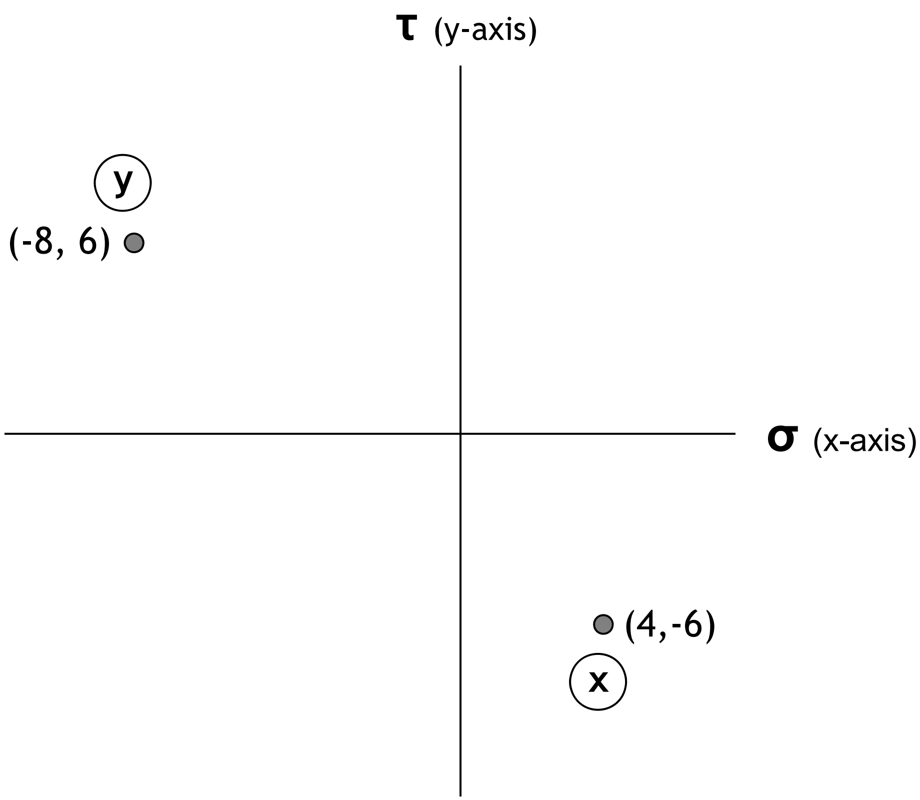

τ
σ₁
σ_y
σ_y'
σ₂
σ
a
y
y'
1
2
x'
x
b
τ_x'y'
τ_xy
σ̄
σ_x
σ_x'
y
x
τ_xy
σ_x
σ_y
y'
x'
τ_x'y'
Θ
σ_x'
σ_y'
2
1
Θ₁
σ₁
σ₂

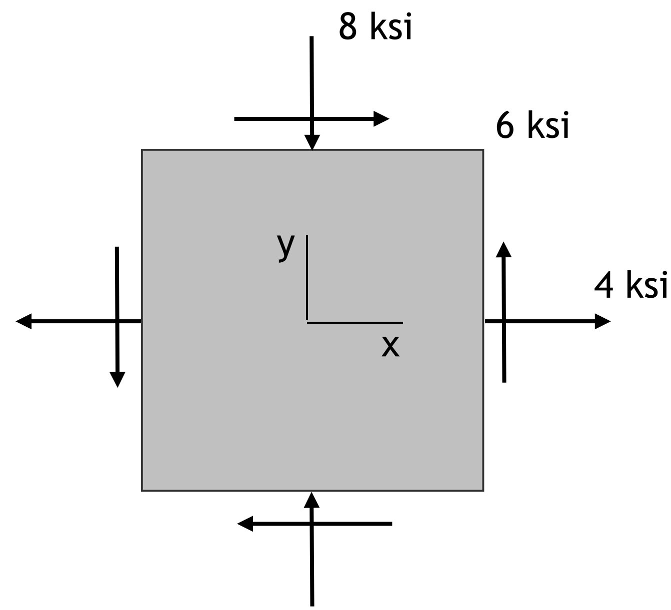
Problem 4)

For the state of stress shown, determine (a) the principal stresses; and (b) the maximum in-plane shear stress. Show the results on properly oriented elements.

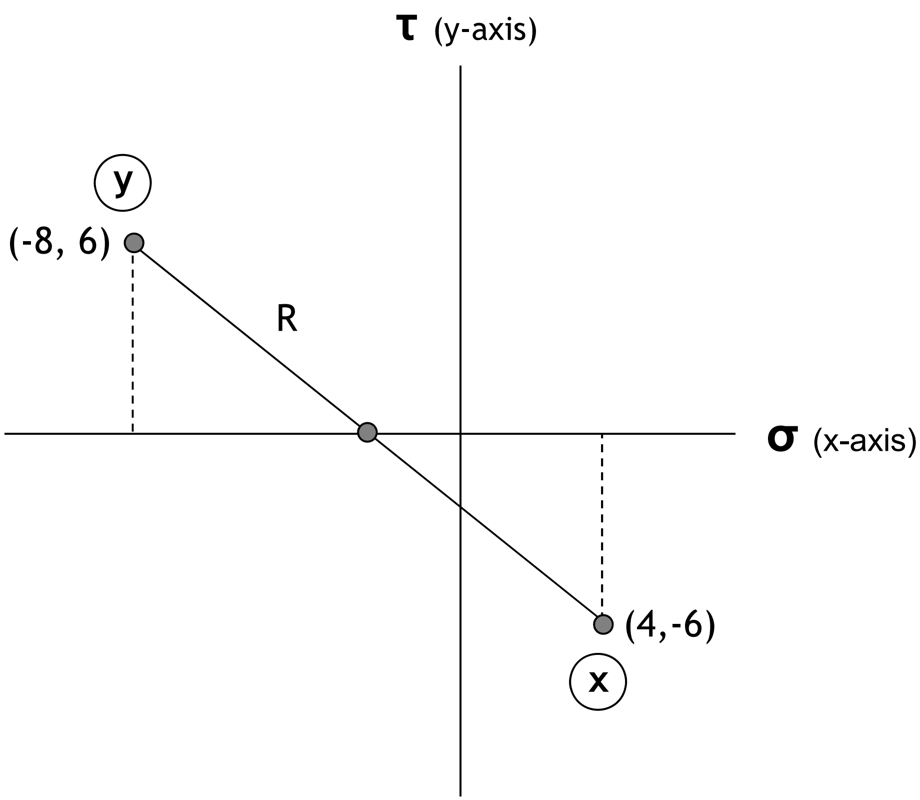


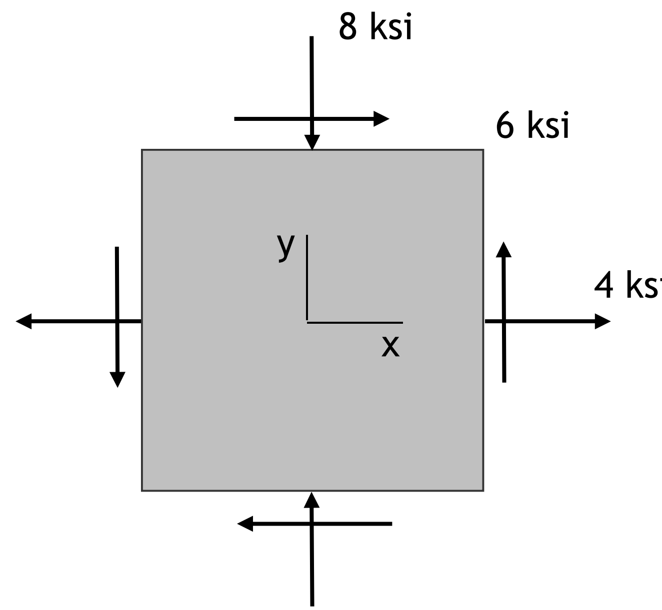
Problem 4 (continued)



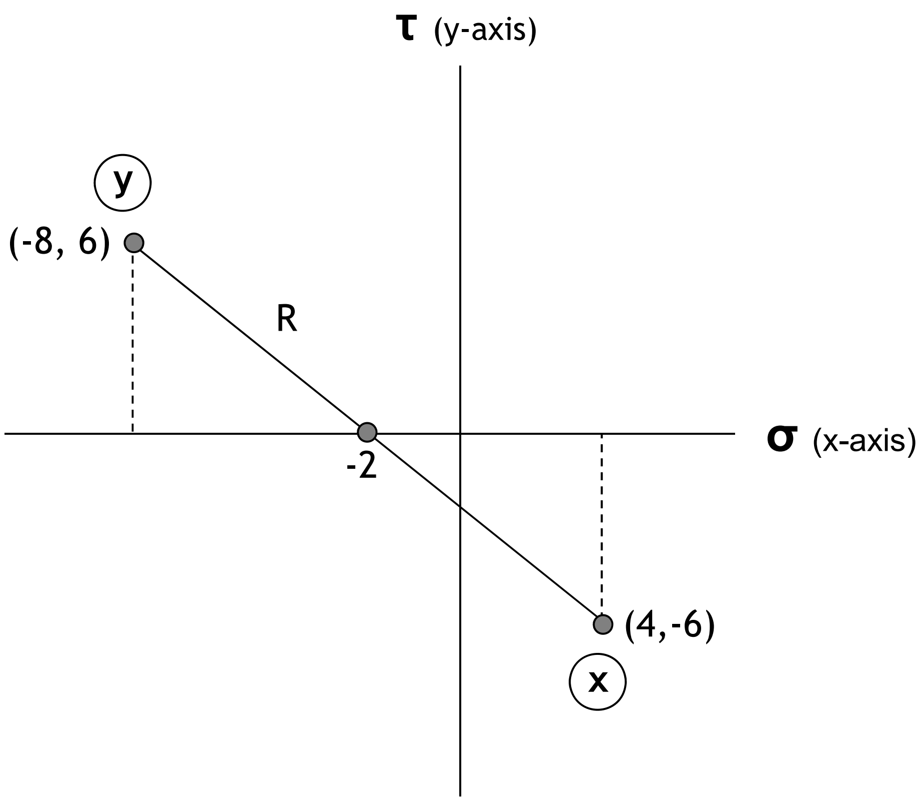


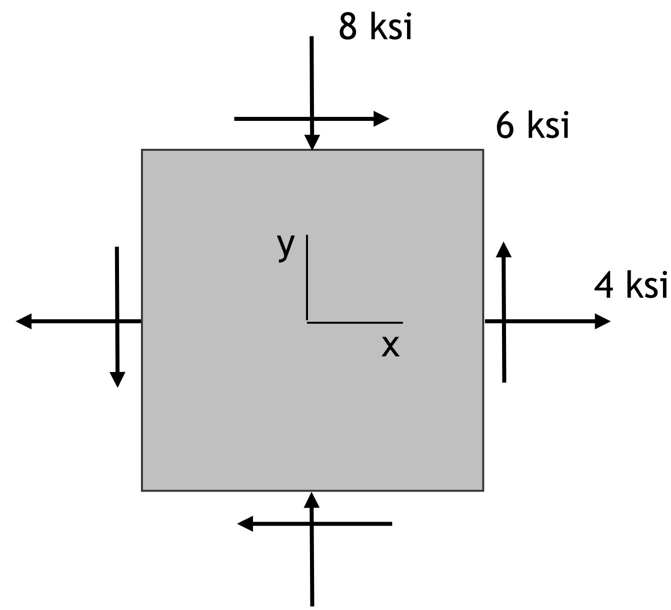
Problem 4 (continued)



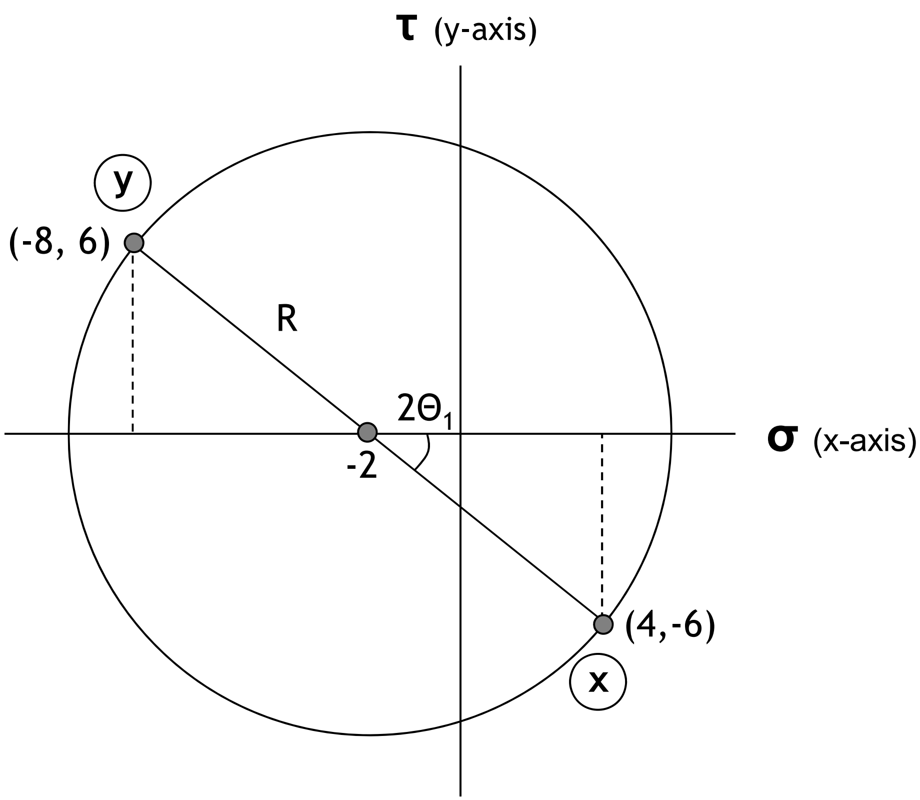


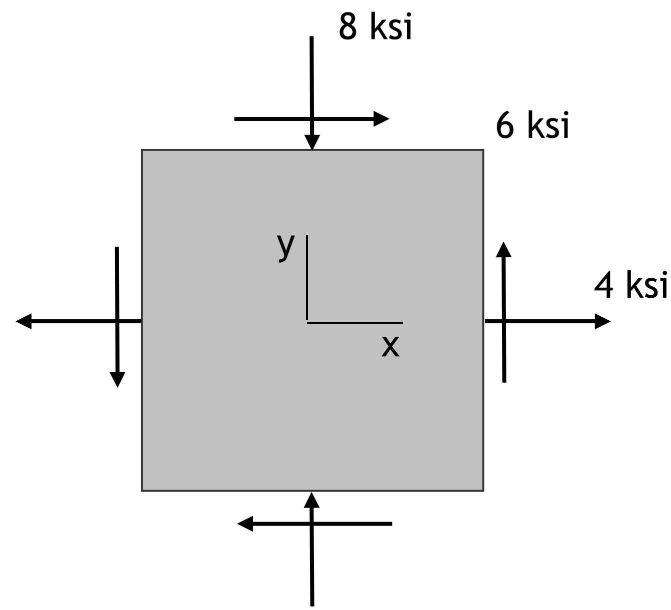
Problem 4 (continued)



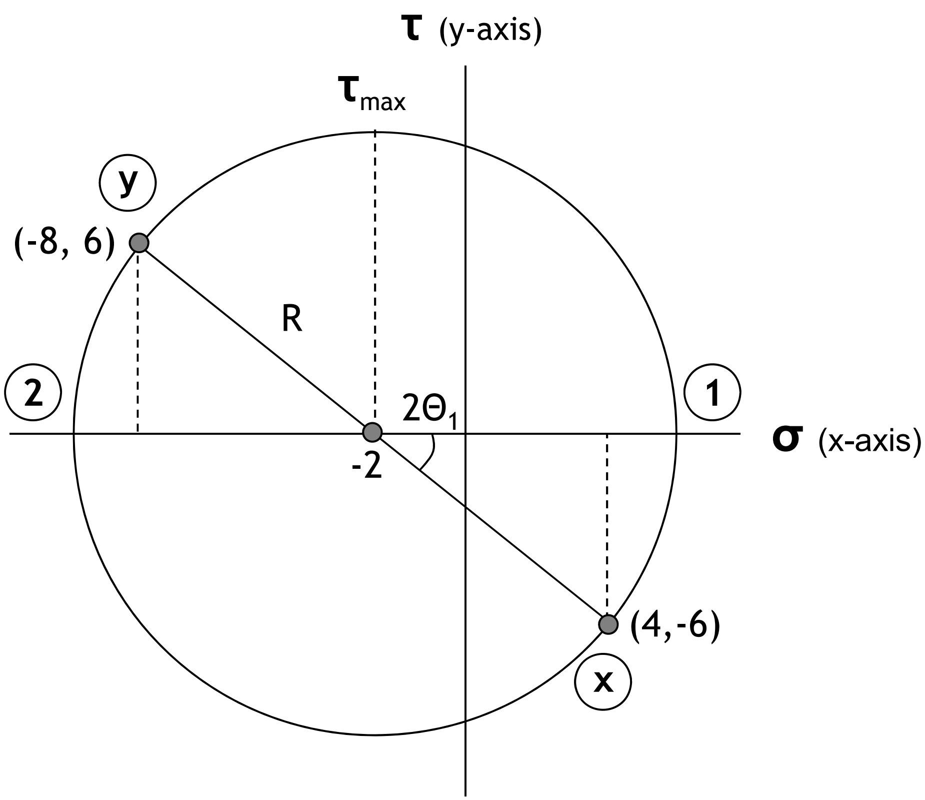


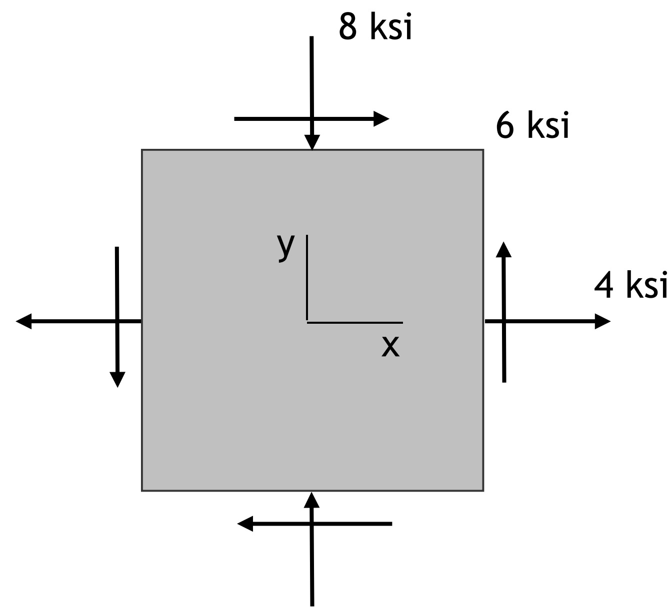
Problem 4 (continued)



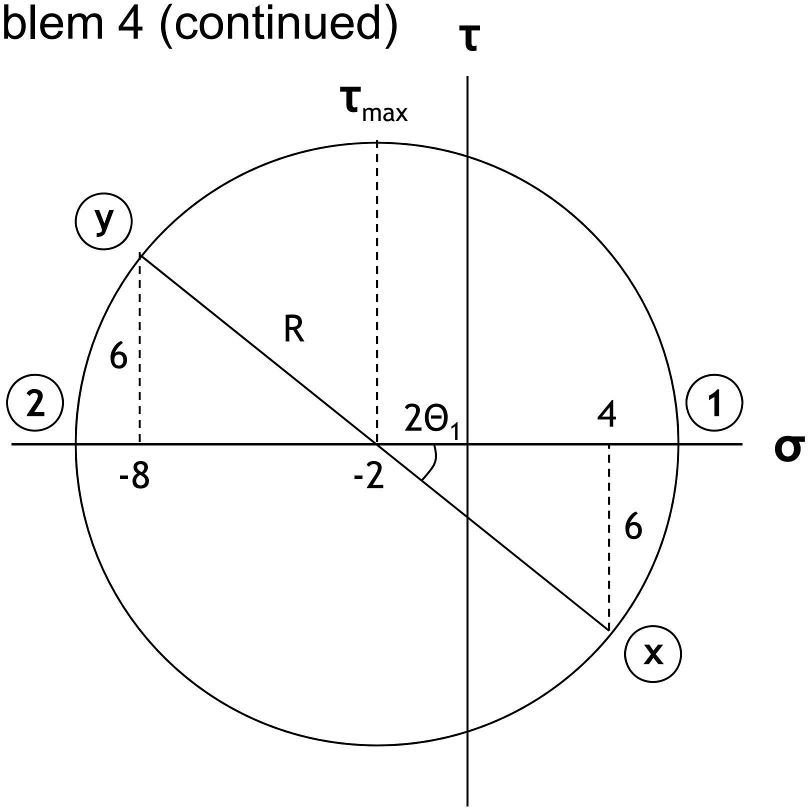


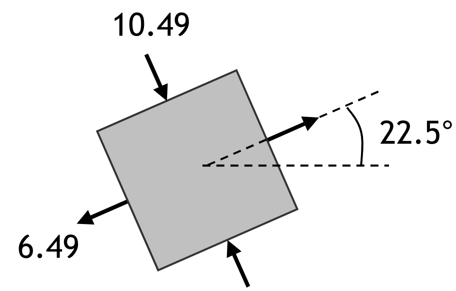
Problem 4 (continued)

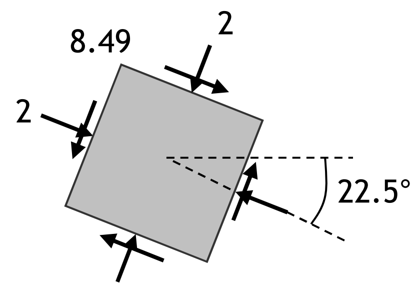




Problem 4 (continued)







(a) $R = \sqrt{6^2 + 6^2} = \sqrt{72}\ ksi$

$2\Theta_1 = \tan^{-1}(6/6) = 45° \Rightarrow \Theta_1 = 22.5°$

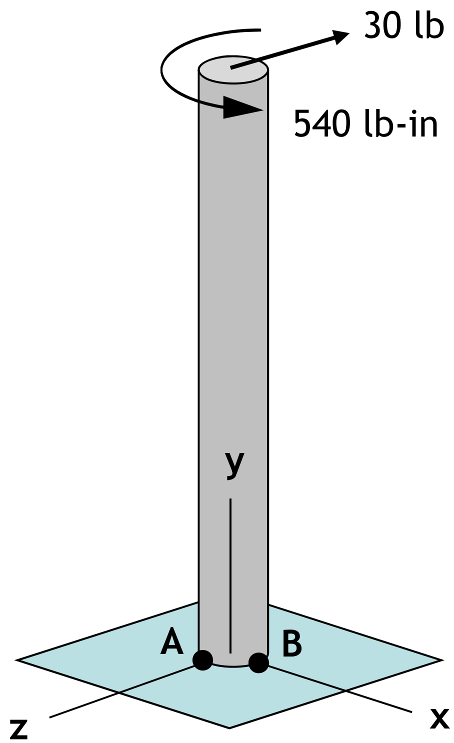
$\sigma_1 = -2 + \sqrt{72} = 6.49\ ksi$

$\sigma_2 = -2 - \sqrt{72} = -10.49\ ksi$

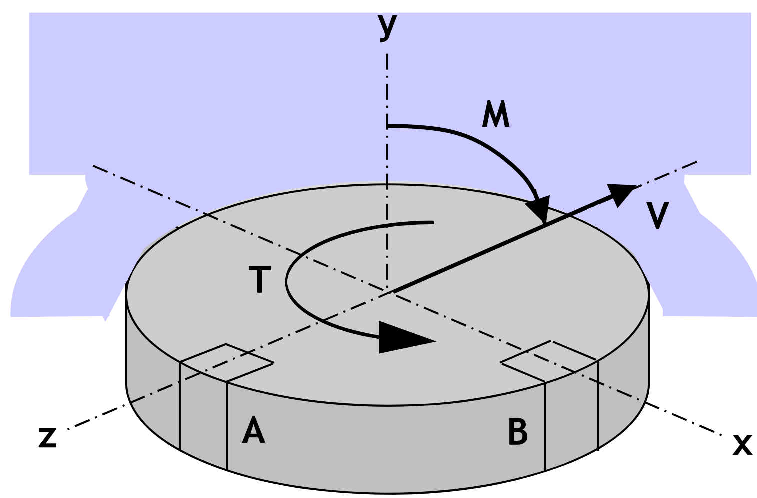
(b) $\tau_{max} = R = \sqrt{72} = 8.49\ ksi$

Problem 5)

The radius of the 15-in. long bar in Fig. (a) is 3/8 in. Determine the maximum normal stress in the bar at (1) point A; and (2) point B.



(a)



(b)

Problem 5 (continued)

The internal force system acting on the cross section at the base of the rod is shown in Fig. (b). It consists of the torque T = 540 lb-in, the bending moment M = 15P = 15(30) = 450 lb-in (acting about the x-axis), and the transverse shear force V = P = 30 lb.

The cross-sectional properties of the bar are:

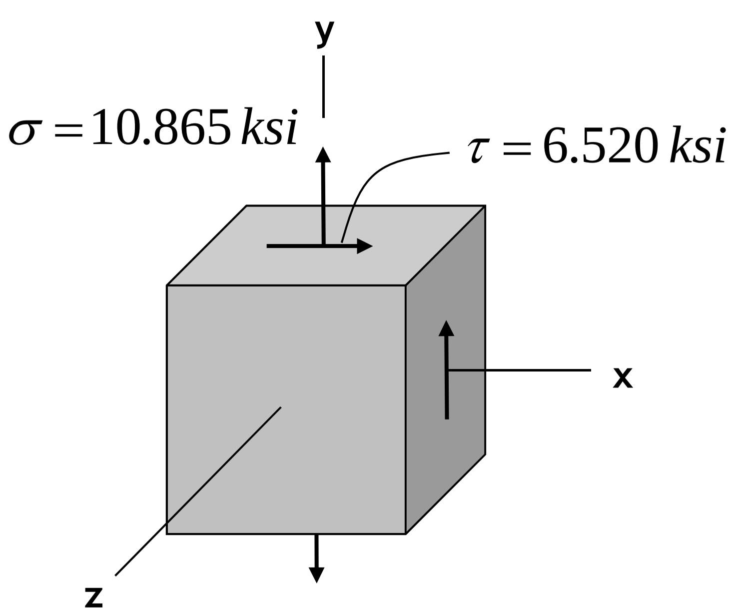
$$I = \frac{\pi r^4}{4} = \frac{\pi (3/8)^4}{4} = 15.532 \times 10^{-3}\ in^4$$

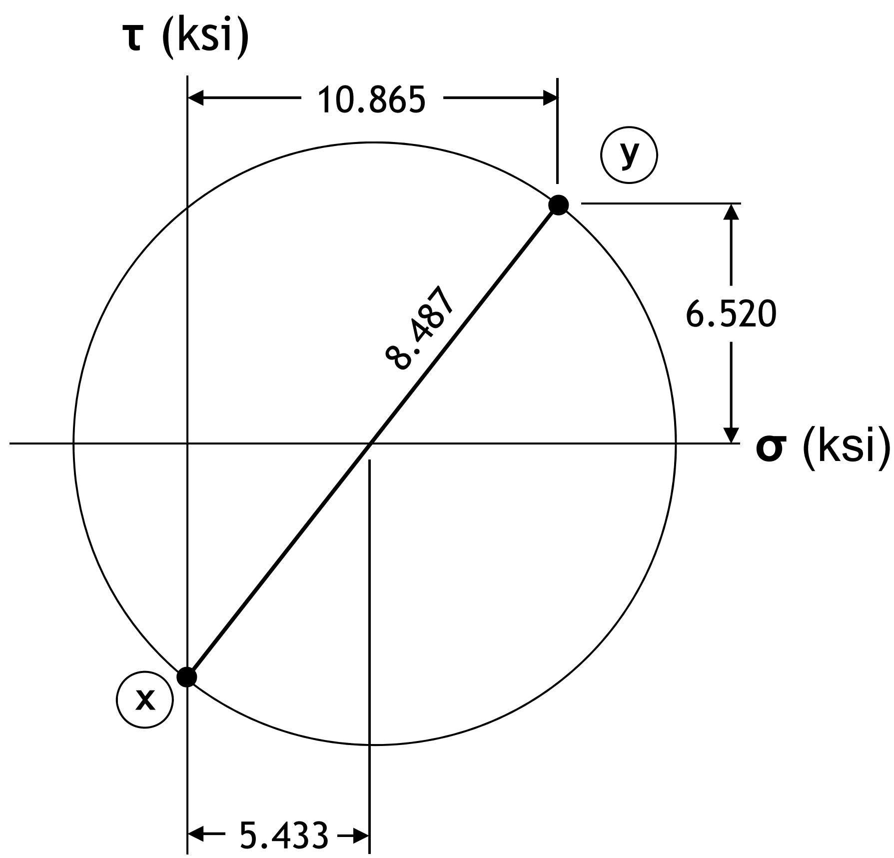
$$Polar\ moment\ of\ Inertia \qquad J = \frac{\pi r^4}{2}$$

$$J = 2I = 2\left(15.532 \times 10^{-3}\right) = 31.06 \times 10^{-3}\ in^4$$

Problem 5 (continued) **PART 1**

State of stress at point **A** with the corresponding Mohr's circle.

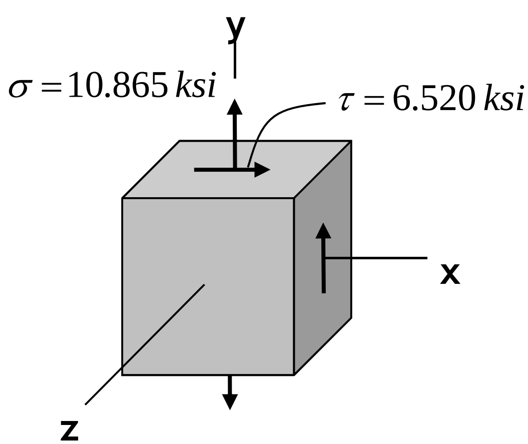


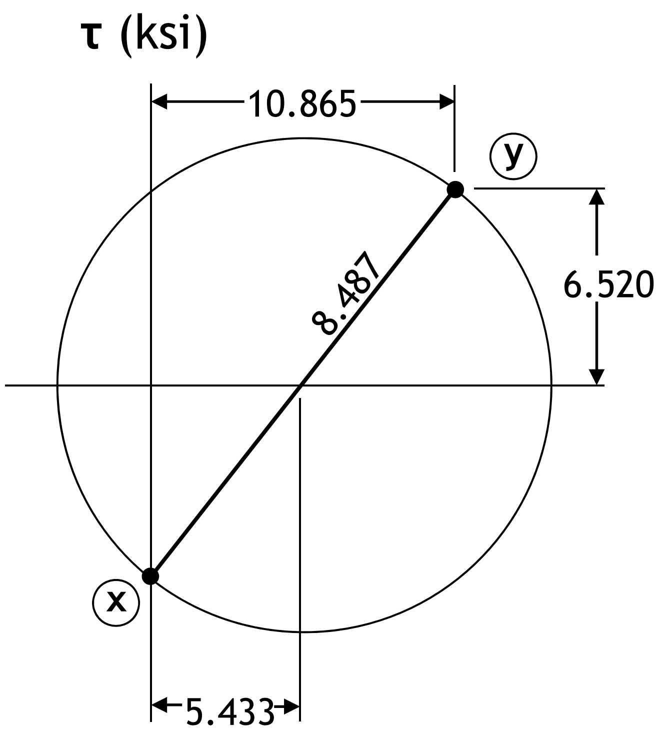


Problem 5 (continued)

The bending stress is calculated:

$$\sigma = \frac{Mr}{I} = \frac{(450)(3/8)}{15.532\times10^{-3}} = 10\,865\ psi = 10.865\ ksi$$



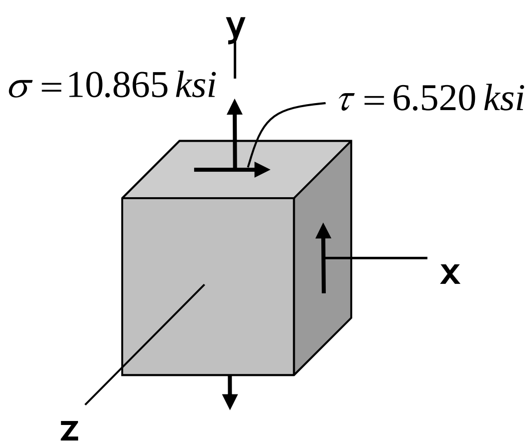


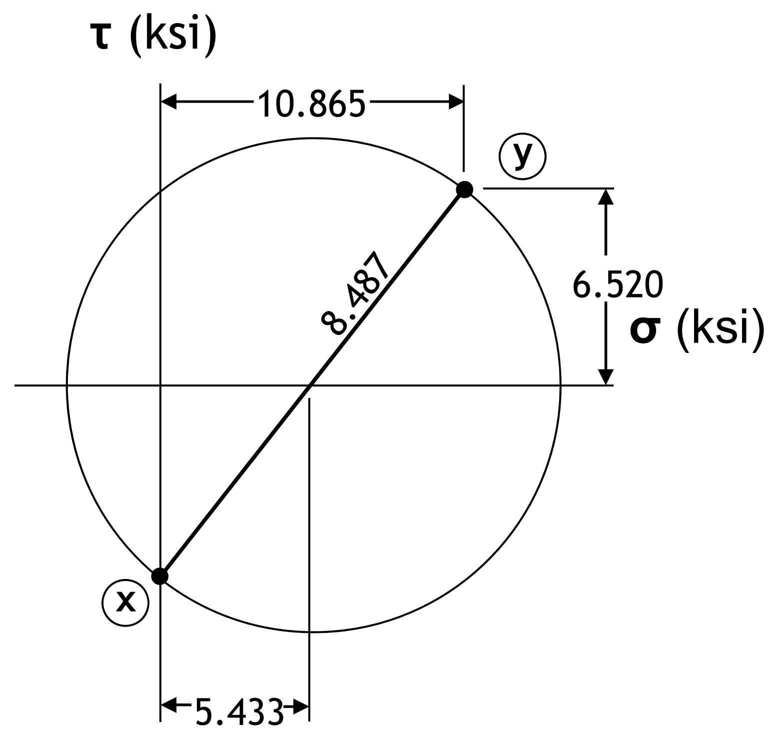
Problem 5 (continued)

And the shear stress is calculated:

$$\tau_T = \frac{Tr}{J} = \frac{540(3/8)}{31.06\times10^{-3}} = 6520\ psi = 6.520\ ksi$$

The shear stress due to the transverse shear force **V** is zero at **A**.

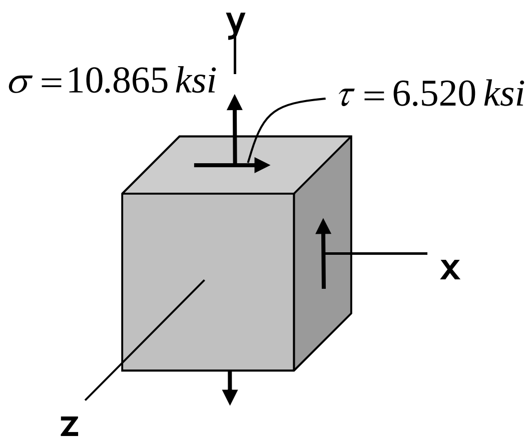


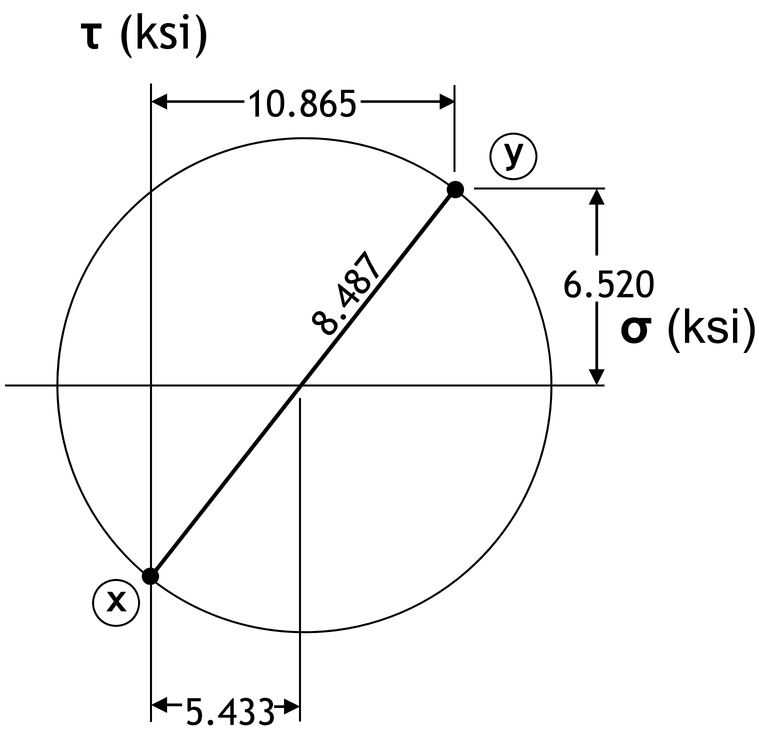


Problem 5 (continued)

The maximum normal stress at point **A** is:

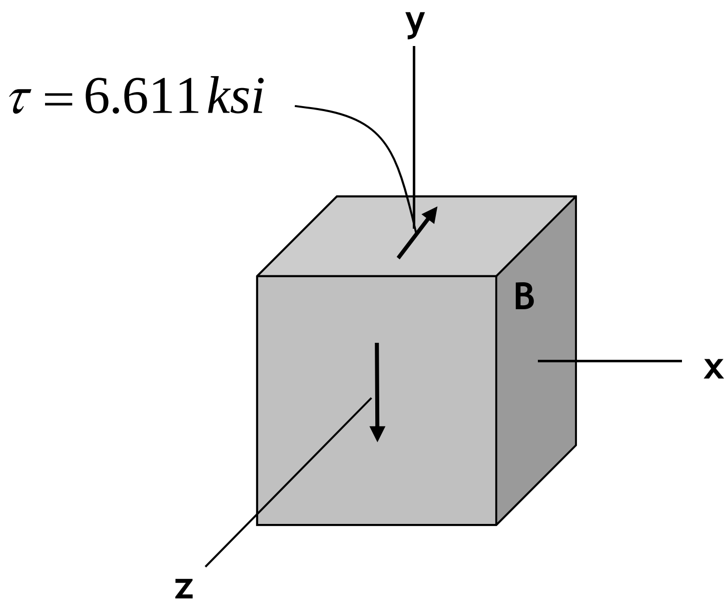
$$\sigma_{\max} = 5.433 + 8.487 = 13.92\ ksi$$

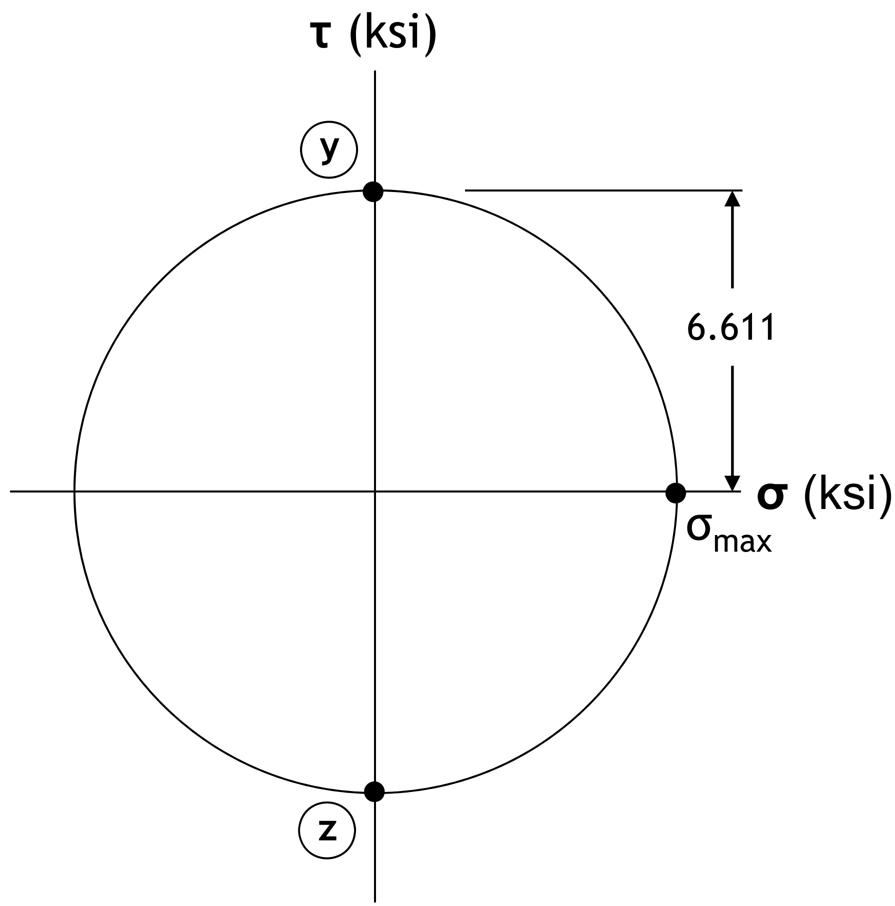




Problem 5 (continued) **PART 2**

State of stress at point **B** with the corresponding Mohr's circle.





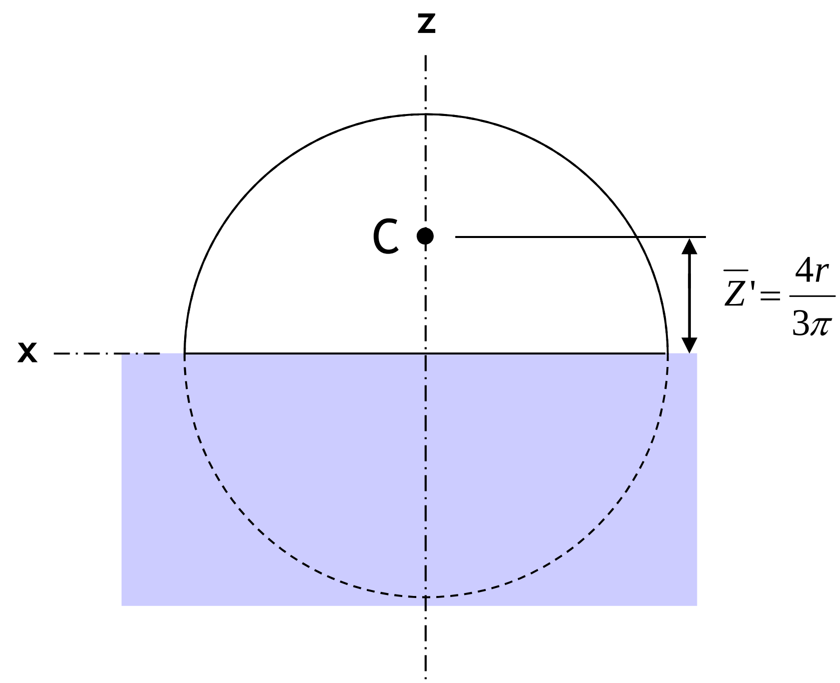
Problem 5 (continued)

The shear stress due to torque is τ_T = 6.520 ksi, as before. But because the point lies on the neutral axis, the bending stress is zero. There is, however, an additional shear stress caused by the transverse shear force V. The magnitude of this shear stress is τ_V = VQ/(Ib), where b = 2r = ¾ in. and Q is the first moment of half the cross-sectional area about the neutral axis. Referring to the figure below, Q is calculated:

$$Q = A'\overline{Z}' = \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)$$

$$= \left(\frac{2r^3}{3}\right) = \frac{2(3/8)^3}{3}$$

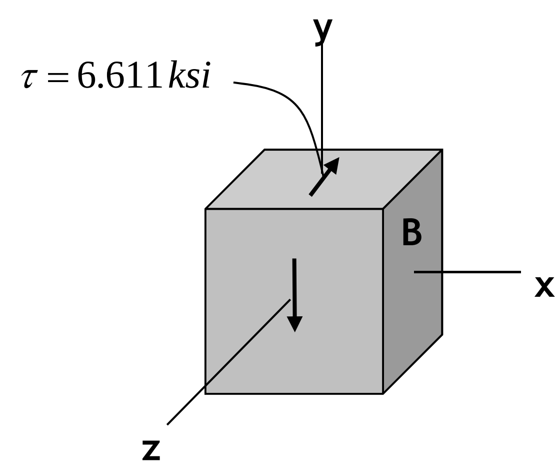
$$= 35.16 \times 10^{-3}\ in^3$$

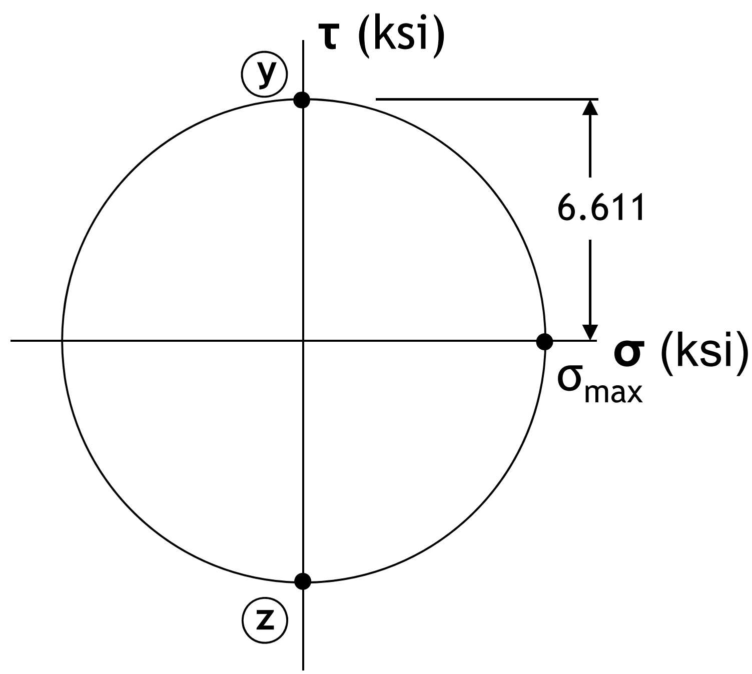


Problem 5 (continued)

Therefore:

$$\tau_V = \frac{VQ}{Ib} = \frac{30\left(35.16\times10^{-3}\right)}{\left(15.532\times10^{-3}\right)\left(3/4\right)} = 90.5\ psi = 0.091\,ksi$$

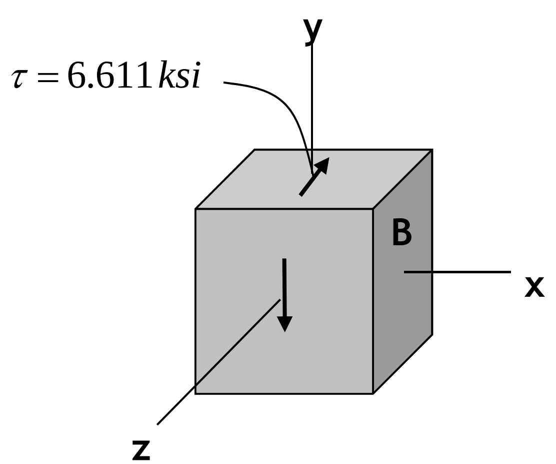


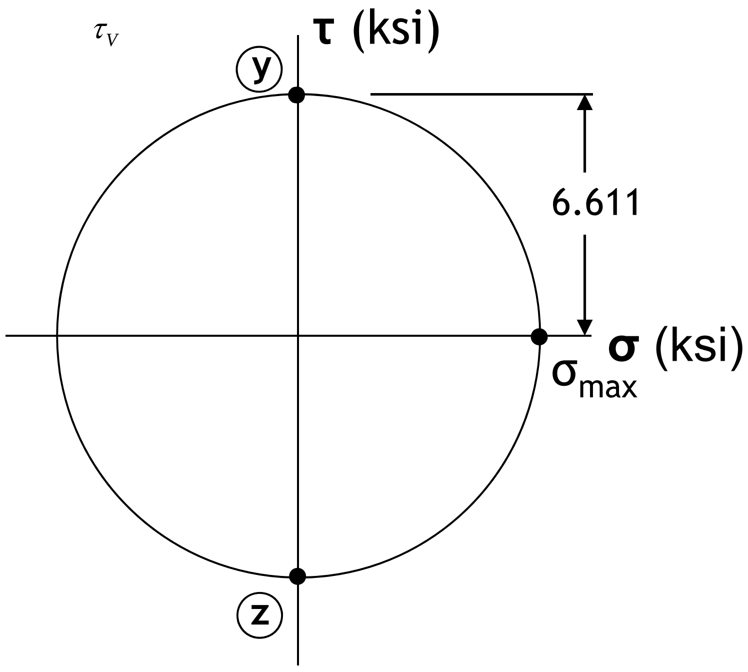


Problem 5 (continued)

Since τ_T and τ_V act on the same planes they can be added. So the total shear stress is:

$$\tau = \tau_T + \tau_V = 6.520 + 0.091 = 6.611\, ksi$$

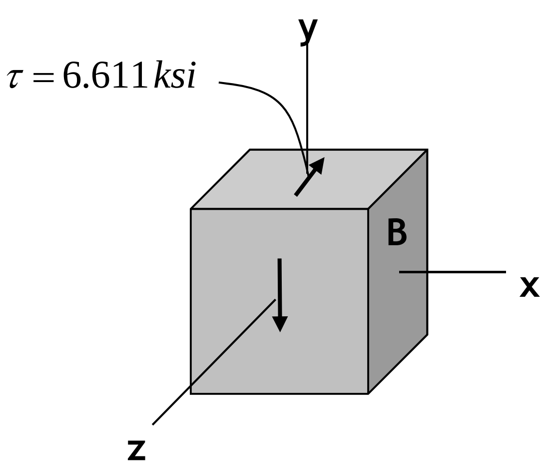


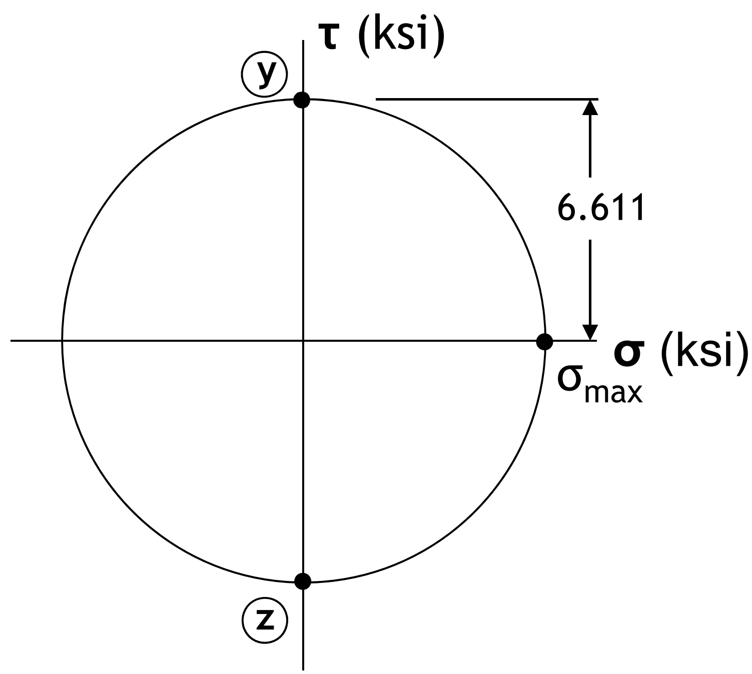


Problem 5 (continued)

The Mohr's circle for this state of pure shear yields for the maximum normal stress at **B**:

$$\sigma_{\max} = 6.61\, ksi$$





Poisson's ratio

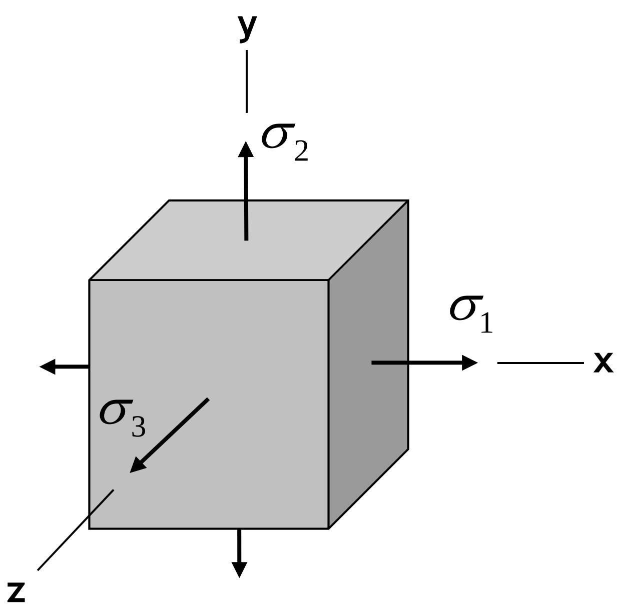
If a structural member is subjected to axial tension, the material elongates and there is a reduction in its cross-section. When the member is under compression, the opposite happens. The ratio of the lateral strain to axial strain is Poisson's ratio.

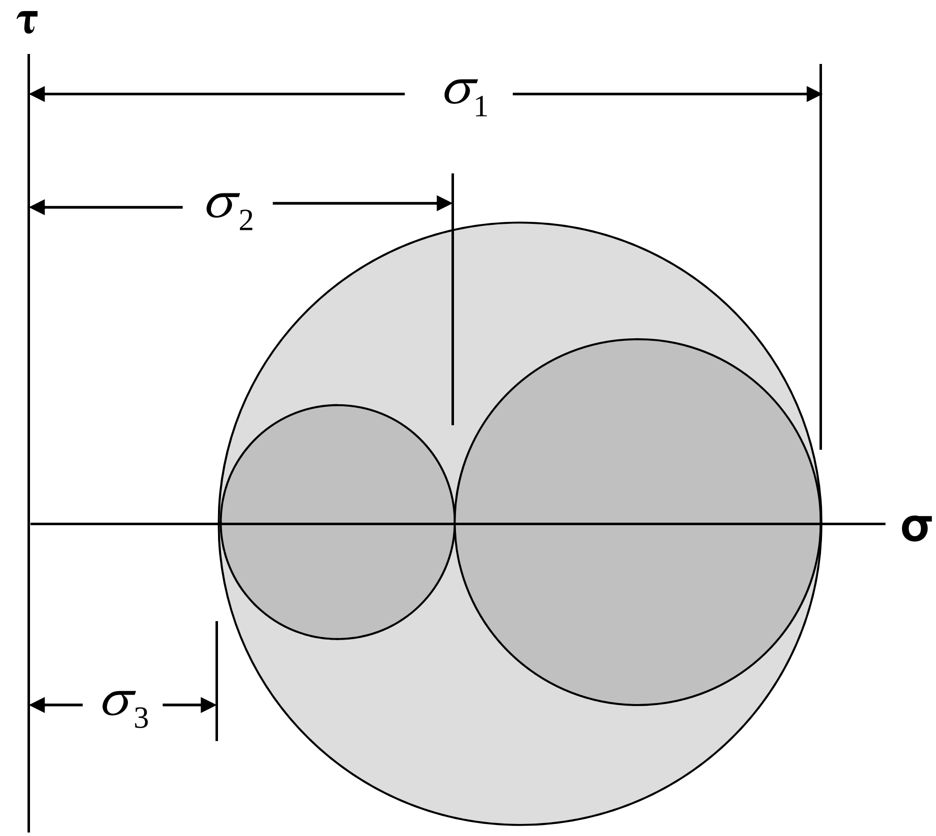
$$\nu = \frac{-\varepsilon_{lateral}}{\varepsilon_{axial}}$$

Most solids have a Poisson's ratio between 0.10 and 0.45

General State of Stress

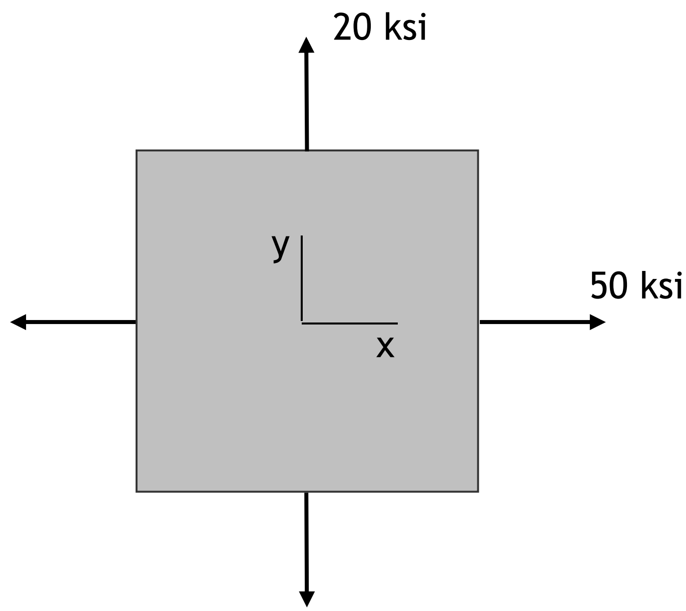
$$\tau_{abs} = \max\left(\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_3 - \sigma_1|}{2} \right)$$





Problem 6)

For the state of stress shown, determine the maximum in-plane shear stress and the absolute maximum shear stress.

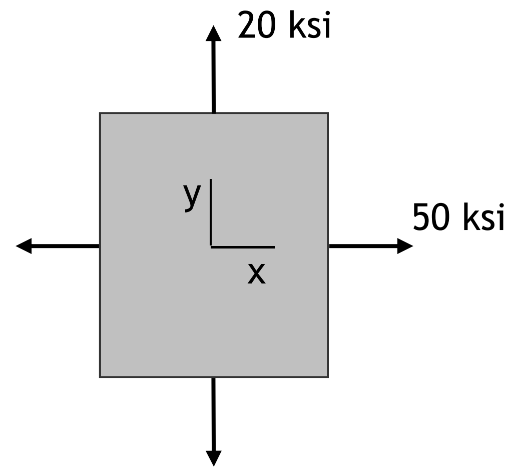


Problem 6 (continued)

The given stresses are:

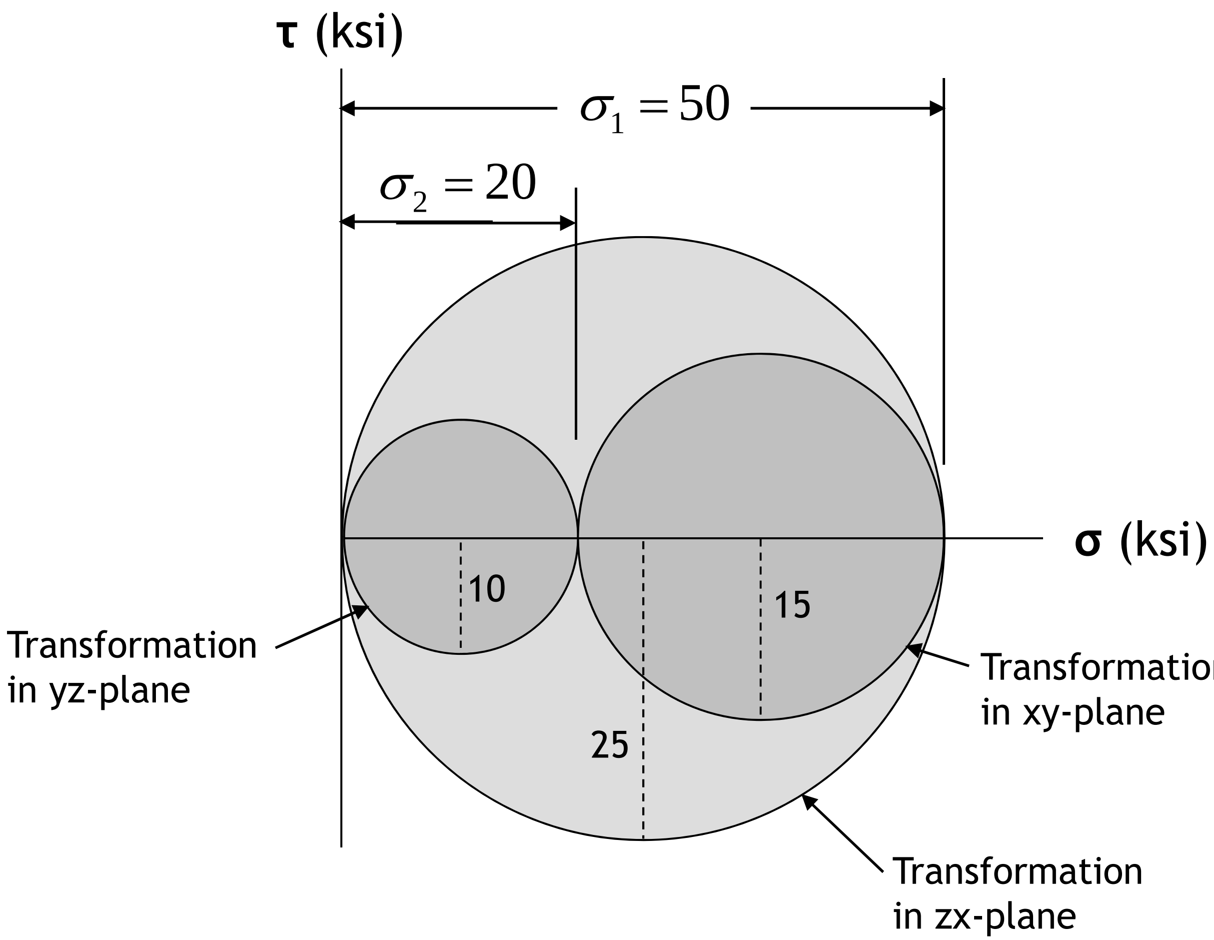
$$\sigma_1 = \sigma_x = 50$$

$$\sigma_2 = \sigma_y = 20$$



Problem 6 (continued)

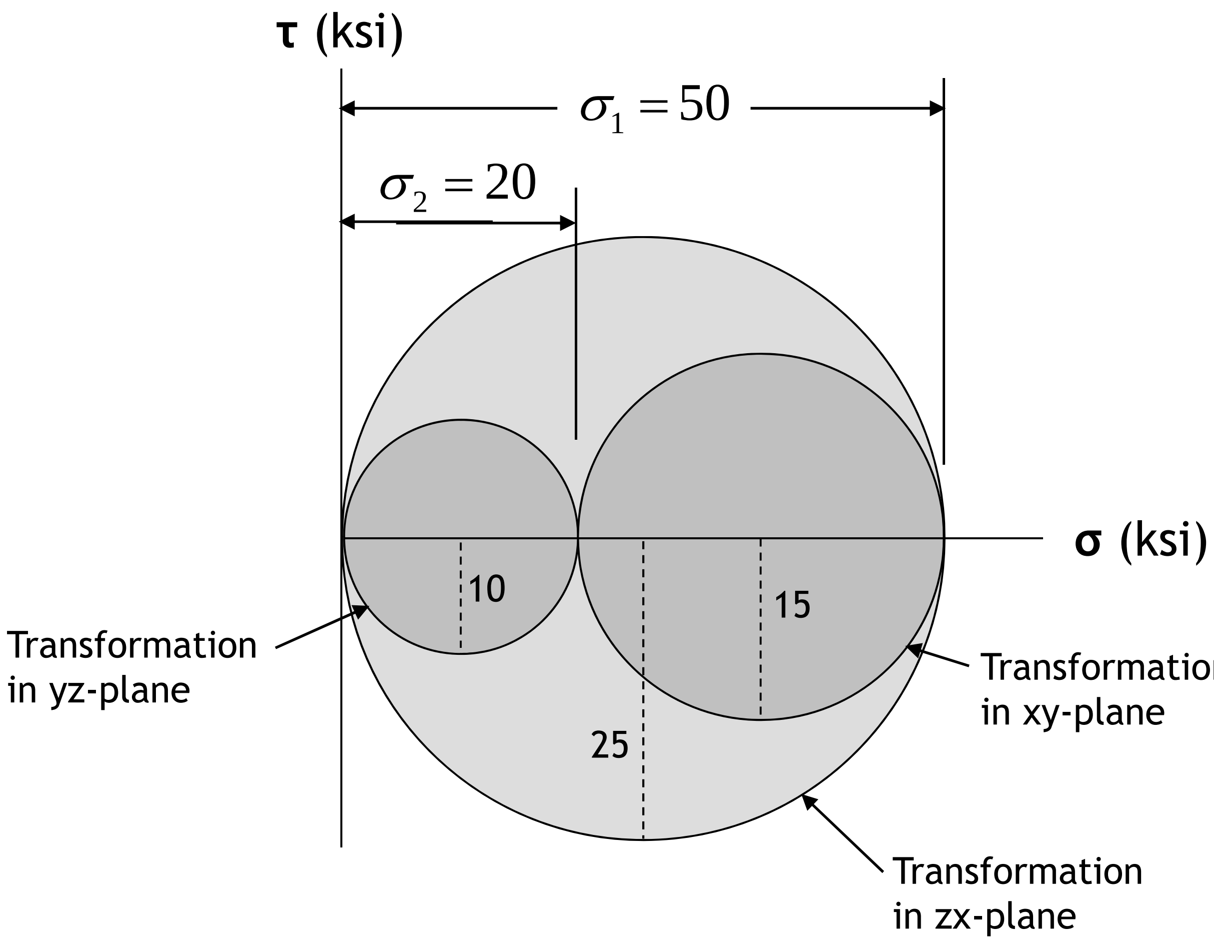
The maximum in-plane shear stress τ_{max} is equal to the radius of the circle that represents transformation in the xy-plane.



$$\tau_{max} = 15\, ksi$$

Problem 6 (continued)

The absolute maximum shear stress equals the radius of the largest circle, which represents the transformation in the zx-plane.



$$\tau_{abs} = 25\,ksi$$

References:

Pytel and Kiusalaas, "Mechanics of Materials," Thomson, 2003.

McCormac and Nelson, "Design of Reinforced Concrete," Wiley, 2003.

Thank You

and

GOOD LUCK!!!