P.E. Civil Exam Review: Geometric Design

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ASCE | KNOWLEDGE & LEARNING

Horizontal Curves







Horizontal Curves: EQUATIONS



Degree of curvature (D), chord basis:

$$R = \frac{50}{\sin\left(\frac{1}{2}D\right)}$$



Curve Equations: arc basis

$$R = \frac{5729.58}{D}$$



Degree of curvature (D), arc basis:

$$2\pi R = \left(\frac{360^{\circ}}{D}\right)(100) \longrightarrow R = \left(\frac{360^{\circ}}{D}\right)\left(\frac{100}{2\pi}\right) \longrightarrow R = \frac{5729.58}{D}$$



$$T = R \tan\left(\frac{1}{2}I\right)$$



$$LC = 2R\sin\left(\frac{1}{2}I\right)$$





$$M = R \left[1 - \cos\left(\frac{I}{2}\right) \right]$$



$$L = \frac{100I}{D}$$

Horizontal Curves Equations

$$R = \frac{5729.58}{D} \qquad \qquad L = \frac{100I}{D}$$



$$LC = 2R\sin\left(\frac{1}{2}I\right)$$

$$M = R \left[1 - \cos\left(\frac{I}{2}\right) \right]$$

$$E = \left(T\right) \tan\left(\frac{I}{4}\right)$$

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 $T = R \tan\left(\frac{1}{2}I\right)$

$$E = R \left[\frac{1}{\cos \frac{I}{2}} - 1 \right]$$

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Degree of curvature, chord basis. The degree of curvature is defined as the central angle subtended by a chord of 100 ft.



R = 50 / sin(1/2 D)

Degree of curvature, arc basis. The degree of curvature is defined as the central angle of a circle which will subtend an arc of 100 ft.



 $R = (360/D)(100/2\pi) = 5729.8 / D$

Example of a sharp curve vs. a flat curve.



A sharp curve has a small ${\bf D}$ and a large ${\bf R}.$

A flat curve has a large D and a small R.







- (a) What is the length of the curve?
- (b) What is the station of the PC?
- (c) What is the station of the PT?
- (d) What is the interior angle at the PI?
- (e) What is the tangent distance from the PI to the PC?
- (f) What is the long chord distance?
- (g) What is the external distance?
- (h) What is the degree of the curve (arc basis)?
- (i) What is the degree of the curve (chord basis)?
- (j) What is the chord length of a 100-ft arc (arc basis)?



The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

(a) What is the length of the curve?

$$L = \frac{100I}{D} \rightarrow L = \frac{100(95^{\circ})}{7.162} \rightarrow L = 1326.44 \ ft$$

(b) What is the station of the PC?

$$T = R \tan\left(\frac{I}{2}\right) \qquad T = 800 \tan(47.5^{\circ}) \qquad T = 873.05 \ ft$$
$$PC = (37 + 00) - (8 + 73.05) \qquad \rightarrow \qquad PC = 28 + 27 \ station$$

The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

(c) What is the station of the PT?

PC = 28 + 27 station

L = 1326.44 ft

 $PT = (28 + 27) + (13 + 26.44) \rightarrow PT = 41 + 53.44$ station

The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

(d) What is the interior angle at the PI?



The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

(e) What is the tangent distance from the PI to the PC?

$$T = R \tan\left(\frac{I}{2}\right) \rightarrow T = 873.05 \ ft$$

(f) What is the long chord distance?

$$LC = 2R\sin\left(\frac{I}{2}\right) = 2(800)\sin\left(\frac{95^{\circ}}{2}\right) \longrightarrow LC = 1179 \ ft$$

The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

(g) What is the external distance?

$$LC = R\left[\sec\left(\frac{I}{2}\right) - 1\right] = 800\left[\sec\left(\frac{95^{\circ}}{2}\right) - 1\right] \rightarrow E = 384 \ ft$$

$$E = (T) \tan\left(\frac{I}{4}\right) = 384 \ ft$$

$$E = R \left[\frac{1}{\cos \frac{I}{2}} - 1 \right] = 384 \ ft$$

The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

(h) What is the degree of the curve (arc basis)?

$$D = \frac{5729.58}{R} = \frac{5729.58}{800} \quad \to \quad D = 7.16^{\circ}$$

(i) What is the degree of the curve (chord basis)?

$$R = \frac{50}{\sin\left(\frac{1}{2}D\right)} \qquad \sin\left(\frac{1}{2}D\right) = \frac{50}{800} \qquad D = 2\sin^{-1}(0.06)$$

 $D = 7.16^{\circ}$

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The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

(j) What is the chord length of a 100-ft arc (arc basis)?

chord length =
$$2R\sin\left(\frac{D}{2}\right) = 2(800)\sin\left(\frac{7.16^{\circ}}{2}\right) \rightarrow \text{ chord length} = 100 \text{ ft}$$

Example Problem 2

• Convert the following angle to degrees

39^{deg} 41^{min} 54^{sec}

Solving: 54/60 = 0.9 min

41.9/60=0.69 deg

Answer = 39.69 degrees

Example Problem 3

• Express the following angle in terms of degrees, minutes, and seconds

91.74 degrees Solving; 0.74 (60) = 44.4 minutes 0.4 (60) = 24.0 seconds Answer: $91^{deg} 44^{min} 24^{sec}$

The two tangents shown intersect 2000 ft beyond Station 10+00. The back tangent has a bearing of N 45°00'00" W and the forward tangent has a bearing of N15°00'00" E. The decision has been made to design a 3000 ft radius horizontal curve between the two tangents.

(a) What is the central angle of the curve?

(b)Compute the tangent distance and length of curve.

(c) What is the station of the PC?

(d) What is the station of the PT?

(e) What is the degree of curvature of this curve using the arc definition?



- 1. What is the central angle for this curve?
- (a) 32 degrees
- (b) 65 degrees
- (C) 45 degrees
- (d) 60 degrees
- (e) 15 degrees
- 2. What is the tangent distance for this curve?
- a) 1551.70 ft
- b) 1732.05 ft
- c) 1673.29 ft
- d) 2000.00 ft
- e) 3000.00 ft
- 3) What is the length of this curve?
- (a) 3000.00 ft
- (b) 3190.60 ft
- (c) 3141.60 ft
- (d) 2000.00 ft
- (e) 1047.20 ft
- 4) What is the station of the PC?
- a) 30 +00.00
- b) 20+00.00
- c) 17+32.05
- d) 19+32.05
- e) 12+67.95

5) What is the station of the PT?

- (a) 47+32.05
- (b) 44+09.55
- (c) 43+32.05
- (d) 46+32.05
- (e) 42+09.55
- 6) What is the degree of curvature of this curve using the arc definition?
- a) 60 degrees
- b) 3.5 degrees
- c) 2.9 degrees
- d) 1.9 degrees
- e) 3.9 degrees

The long chord of a circular curve is 600 feet, the intersection angle is 110°. Find the radius. The forward tangent of this curve needs to be moved in 5 feet due to a right of way dispute. What radius curve would you specify to hold the B.C. at the same location as in the original curve?



The long chord of a circular curve is 600 feet. The intersection angle is 110 degrees. What is the radius of this curve? The forward tangent of this curve needs to be moved in 5 feet due to a right of way dispute. What radius curve would you specify to hold the PC at the same location as in original first curve.

We know: L.C. = 600 ft
$$LC = 2R \sin \frac{I}{2}$$

Then

$$R = \frac{600}{2\sin(55^{\circ})} = 366.23 \ ft$$

$$T = R \tan \frac{I}{2} = 366.23 \tan (55^{\circ}) = 523.03 \ ft$$



The alignment of a proposed highway was adjusted in order to avoid a critical obstacle. This was accomplished by moving the forward tangent 120 ft forward.

You are asked to find the radius of the new curve if it begins at the same location as the PC of the initial curve.

You need to also determine the stations for PC and PT for the new curve alignment.







For the original curve, find R and T :

$$R = \frac{5729.58}{D} = \frac{5729.58}{6} = 954.93 \ ft$$
$$T = R \tan \frac{I}{2} = (954.93)(0.637) = 608.36 \ ft$$

PC station for the original curve is:

(85 + 71.92) - (6 + 8.36) = 79 + 63.56



$$x = \frac{132.40 \, f}{\sin 65^0}$$

 $T_{new} = 608.36 + 132.40 = 740.76$ ft

$$R_{newcurve} = \frac{T_{new}}{\tan(65^{\circ}/2)} = \frac{740.76}{0.637} = 1162.76 \ ft$$

$$D_{new} = \frac{5729.58}{1162.76} = 4.93^{\circ}$$
$$L_{new \, curve} = \frac{100 \, I}{D_{new}} = \frac{100 \, (65^{\circ})}{4.93^{\circ}} = 1319.10 \, ft$$

PT Station = Station PC + L

New PT Station = (79 + 63.56) + (13 + 19.10) = <u>92 + 82.66</u>

Vertical Curves







PROPERTIES OF A PARABOLA: Applied to Vertical Curve Analysis

PROPERTIES OF A PARABOLA:

1. The curve elevation at its midpoint is halfway from the elevation at the P.V.I. to the elevation at the midpoint of the long chord.

$$E = \frac{1}{2} \left(PVI \ elevation - \frac{PVC \ elevation + PVT \ elevation}{2} \right)$$

The curve lies midway between the point of intersection of the grade lines and the middle point
of the chord joining the BVC (beginning of vertical curve) and the EVC (end of vertical curve).

2. The tangent offsets vary as the square of the distance from the point of tangency

$$\frac{Y}{x^2} = \frac{E}{T^2}$$

PROPERTIES OF A PARABOLA:



The Algebraic Signs of "r"





Curve Length 800 ft





Stations	61	62	63	64	65	66	67	68	69
Elevations on grade lines	248.20	252.20	256.20	260.20	264.20	261.20	258.20	255.20	252.20



Calculations for example 7:

Elevation of PVI = 264.20 ft

Elevation of C = $\frac{1}{2}$ (248.20+252.20) = 250.2 ft

Elevation of B = $\frac{1}{2}$ (264.20+250.20) = 257.20 ft

$$E = \frac{(g1 - g2)L}{8} = \frac{(4 - (-3))(8)}{8} = 7$$

$$r = \frac{\left(g2 - g1\right)}{L} = \frac{-3 - 4}{8} = -\frac{7}{8}$$

rate of grade change



The tangent offsets vary as the square of the distance from the point of tangency.







Stations	61	62	63	64	65	66	67	68	69
Elevations on grade lines	248.20	252.20	256.20	260.20	264.20	261.20	258.20	255.20	252.20

$$y = 2^2 \left[\frac{7}{4^2} \right] = 1.75$$



Stations	61	62	63	64	65	66	67	68	69
Elevations on grade lines	248.20	252.20	256.20	260.20	264.20	261.20	258.20	255.20	252.20



Stations	61	62	63	64	65	66	67	68	69
Elevations on grade lines	248.20	252.20	256.20	260.20	264.20	261.20	258.20	255.20	252.20



Stations	61	62	63	64	65	66	67	68	69
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Stations	61	62	63	64	65	66	67	68	69
Elevations on grade lines	248.20	252.20	256.20	260.20	264.20	261.20	258.20	255.20	252.20



Highest and Lowest Points on Vertical Curves:

X is the distance between PVC and station of high or Low point

$$X = \frac{-g_1}{r}$$

For the vertical curve in example problem 5:

$$r = \frac{g_2}{L} - \frac{g_1}{R} = -\frac{3-4}{8} = -\frac{7}{8}$$
$$X = \frac{-g_1}{r} = -\frac{4}{\frac{-7}{8}} = +4.57 \text{ Stations from PVC}$$

Elevation at highest point Station 65+57 is equal to:

$$264.20 - (0.03)(57) - \frac{(3.43)^2}{(4)^2}(7.00) = 257.34 \ ft$$

Calculations for the Elevation at highest point on the vertical curve which is at station 65+57:

Elevation of tangent line at station 65+57 is, 264.20-(.03)(57)= 264.20-1.7 = 262.49 ft

- Offset at station 65+57 is calculated by 7 / [(4)(4)] = y / [(3.43)(3.43)]
- Offset at station 65+57 is y=5.147 ft
- Elevation at station 65+57 on the vertical curve is 262.49 5.147 which is equal to 257.34 ft








- Station of BVC = 50 4 =station 46
- Elevation of BVC = 550.97 + (4 x 6) = 574.97 ft
- Station of EVC = 50 + 4 =station 54
- Elevation of EVC = 550.97 + (4 x 2) = 558.97 ft

- Elevation of middle point of chord = $\frac{574.97 + 558.97}{2} = 566.97 \text{ ft}$
- Offset to curve at intersection = $\frac{566.97 550.97}{2} = 8.00 \text{ ft}$

Vertical Curve Problem #8 Solutions • Offset at A and F = $\binom{1}{4}^{2} \times 8.00 = 0.50$ ft • Offset at B and E = $\binom{2}{4}$ x 8.00 = 2.00 ft • Offset at C and D = $\binom{3}{4}$ x 8.00 = 4.50 ft

- Elevation of A = 574.97 6.00 = 568.97 ft
- Elevation of B = 568.97 6.00 = 562.97 ft
- Elevation of C = 562.97 6.00 = 556.97 ft
- Elevation of V = 556.97 6.00 = 550.97 ft
- Elevation of F = 558.97 2.00 = 556.97 ft
- Elevation of E = 556.97 2.00 = 554.97 ft
- Elevation of D = 554.97 2.00 = 552.97 ft
- Elevation of V = 552.97 2.00 = 550.97 ft*

- *checks!

• Elevation of a = station 47= 568.97+ 0.50 = 569.47 Elevation of b = station 48 = 562.97 + 2.00 = 564.97 Elevation of c = station 49 = 556.97 + 4.50 = 561.47 Elevation of o = station 50 = 550.97 + 8.00 = 558.97 Elevation of d = station 51 = 552.97 + 4.50 = 557.47 Elevation of e = station 52 = 554.97 + 2.00 = 556.97 Elevation of f = station 53 = 556.97 + 0.50 = 557.47

STATION		TANGENT ELEVATION	OFFSET FROM TANGENT	ELEVATION CURVE
BVC =	46	574.97	0	574.97
	47	568.97	+0.50	569.47
	48	562.97	+2.00	564.97
	49	556.97	+4.50	561.47
	50	550.97	+8.00	558.97
	51	552.97	+4.50	557.47
	52	554.97	+2.00	556.97
	53	556.97	+0.50	557.47
EVC =	54	558.97	0	558.97

SIGHT DISTANCE:

Minimum sight distance (s), where sight distance is less than the curve length (L) in stations and the height of driver's eye (h) is 3.5 ft to 3.75 ft above pavement.

$$s^2 = \frac{8Lh}{g_1 - g_2}$$

If the calculated sight distance (s) is greater than the curve length (L) then use the following equation:

$$s = \frac{L}{2} + \frac{4h}{g_1 - g_2}$$

EXAMPLE PROBLEM 9:

Sight distance calculations for the vertical curve in problem 5:

$$L = 800 \ ft$$
 $g_1 = 4\%$ $g_2 = -3\%$ $h = 3.75 \ ft$

$$s = \sqrt{\frac{8(8.00)(3.75)}{4 - (-3)}} = 5.8554 \text{ stations} \rightarrow s = 585.54 \text{ ft}$$

If the calculated sight distance (s) is greater than the curve length (L) then use the following equation:

$$s = \frac{L}{2} + \frac{4h}{g_1 - g_2}$$

EXAMPLE PROBLEM 10:

A proposed 2-lane highway has a vertical alignment that is +3% grade intersecting a -2% grade at station 26+00 at an elevation of 228.00. The proposed alignment must bridge over an existing railroad track which crosses the proposed alignment at station 28+50. The elevation of the railroad track at the point of intersection is 195.00. The proposed highway alignment must have a vertical elevation difference of 26.0 ft at the point of intersection of the two alignment in order to satisfy vertical clearance requirements.

You are to determine the longest length of vertical curve, rounded to the nearest 100 ft, that will fulfill these criteria. Determine the station of the high point on the vertical curve.



EXAMPLE PROBLEM 10:

For vertical curve to go through a fixed point, use the following formula :

$$L = 2\left(A + \frac{2(offset)}{g1 - g2}\right) + 4\sqrt{\frac{A(offset)}{g1 - g2}} + \left(\frac{offset}{g1 - g2}\right)^2$$

L = 10.91 stations therefore use <u>L= 1000</u> ft to maintain 26 ft minimum clearance

Station of high point

$$x = -\frac{g1}{r} = -\frac{-3}{\frac{-2-3}{10}} = 6$$
 Stations from PVC

EXAMPLE PRBLEM 10:

Using design curve length L=1000 ft

Location of high point

$$x = -\frac{g1}{r} = -\frac{-3}{\frac{-2-3}{10}} = 6$$
 Stations from PVC

Station at PVC = (26+00)– (5+00) = 21+00

Station of high point = (21+00) + (6+00) = 27+00

EXAMPLE PROBLEM 11:

The proposed highway is to cross another highway at right angles. The elevation of the proposed crossing has been established and a minimum vertical clearance of 25 ft. will be required between the proposed highway and the existing highway.

A. Determine the location and elevation of the low point on the existing vertical curve.

B. Determine the minimum station at which the crossing may be located.

C. Determine the maximum station at which the crossing may be located.



EXAMPLE PROBLEM 11 :

Calculate Low point on existing vertical curve

$$x = -\frac{g1}{r} = -\frac{-4}{\frac{+3 - (-4)}{12}} = 6.8571$$
 Stations from PVC

Station at low point = (82+00) + (6+85.71) = 88+85.71

Elevation of low point is calculated to be 746.28 ft

Proposed highway has an elevation of 777 ft and required clearance is 25 ft, therefore, there exists two locations on the curve with elevation of 777 - 25 = 752 ft

By trial and error, we need to find minimum and maximum station where curve elevation is 752 ft



Trial and error for maximum station:

Station	Tangent elevation	offset	Curve elevation
93+00	751.00	0.29	751.29
94+00	754.00	0.00	754.00
93+30	751.90	0.14	752.04
93+29	751.87	0.15	752.02
93+28	751.84	0.15	751.99
93+28.5	751.855	0.15	752.00

Maximum Station turns out to be <u>93+28.5</u>

Trial and error for minimum station:

Station	Tangent elevation	offset	Curve elevation
84+00	752.00	1.17	753.17
85+00	748.00	2.62	750.62
84+50	750.00	1.82	751.82
84+40	750.40	1.68	752.08
84+42	750.32	1.71	752.03
84+43	750.28	1.72	752.00

Minimum Station turns out to be <u>84+43</u>



References:

M^cCormack, Jack, Surveying Fundamentals, Prentice Hall, 1983

Brinker, Russell, Wolf, Paul, Elementary Surveying, Sixth Edition, Harper and Row, 1977

Thank You.

Any Questions?

Good Luck !