S.E. Exam Review: Structural Analysis

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NCEES Topics

- Moving loads
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- Shear diagrams
- Moment diagrams
- Deflections
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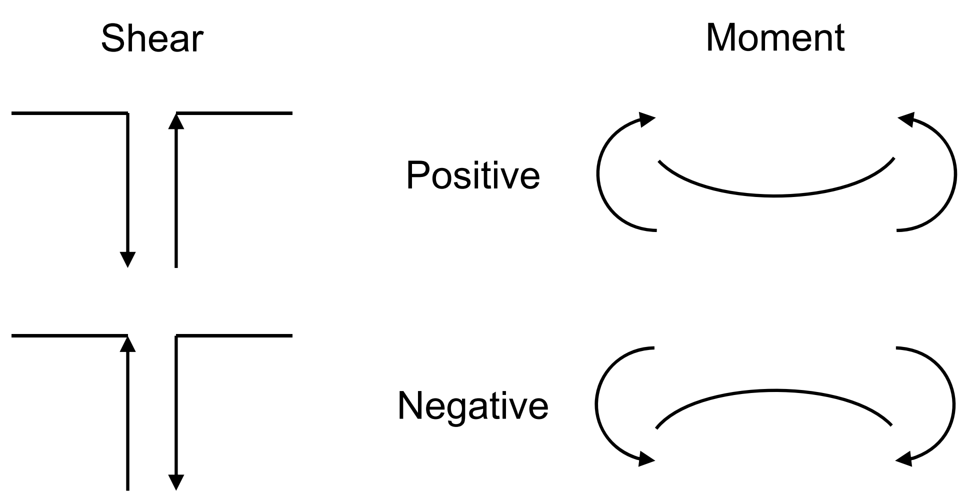
Beam Sign Convention (*BSC*)



Deflections

Deflections

- Double Integration
- Moment Area
- Conjugate Beam
- Virtual Work
- Castigliano's Theorem
- Tables

Conjugate Beam

- Theorem 1 – The slope at a point in the real beam is numerically equal to the shear at the corresponding point in the conjugate beam.
- Theorem 2 – The displacement at a point in the real beam is numerically equal to the moment at the corresponding point in the conjugate beam.

Conjugate Beam

- The conjugate beam has same length as the real beam
- Potentially different supports
- Loaded with the *M*/*EI* diagram of the real beam

Conjugate Beam

- Load on the conjugate beam is upward if the *M*/*EI* value is positive
- Positive shear (*BSC*) in the conjugate beam corresponds to counterclockwise rotation in the real beam
- Positive moment (*BSC*) in the conjugate beam corresponds to upward displacement in the real beam

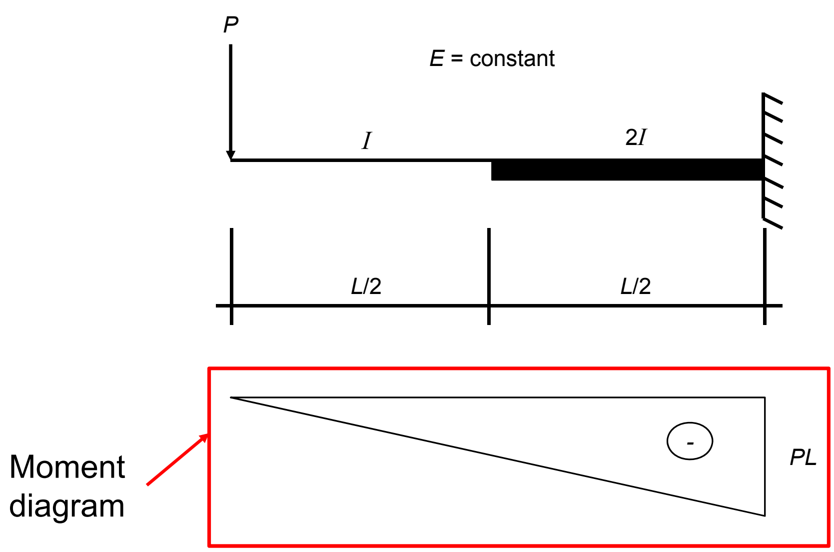
Conjugate Beam Supports

Real Beam	Conjugate Beam
$\theta \neq 0$, $\Delta = 0$ — Pin	$V \neq 0$, $M = 0$ — Pin
$\theta \neq 0$, $\Delta = 0$ — Roller	$V \neq 0$, $M = 0$ — Roller
$\theta = 0$, $\Delta = 0$ — Fixed	$V = 0$, $M = 0$ — Free
$\theta \neq 0$, $\Delta \neq 0$ — Free	$V \neq 0$, $M \neq 0$ — Fixed
$\theta \neq 0$, $\Delta = 0$ — Internal pin	$V \neq 0$, $M = 0$ — Internal hinge
$\theta \neq 0$, $\Delta = 0$ — Internal roller	$V \neq 0$, $M = 0$ — Internal hinge
$\theta \neq 0$, $\Delta \neq 0$ — Internal hinge	$V \neq 0$, $M \neq 0$ — Internal roller

Conjugate Beam Example

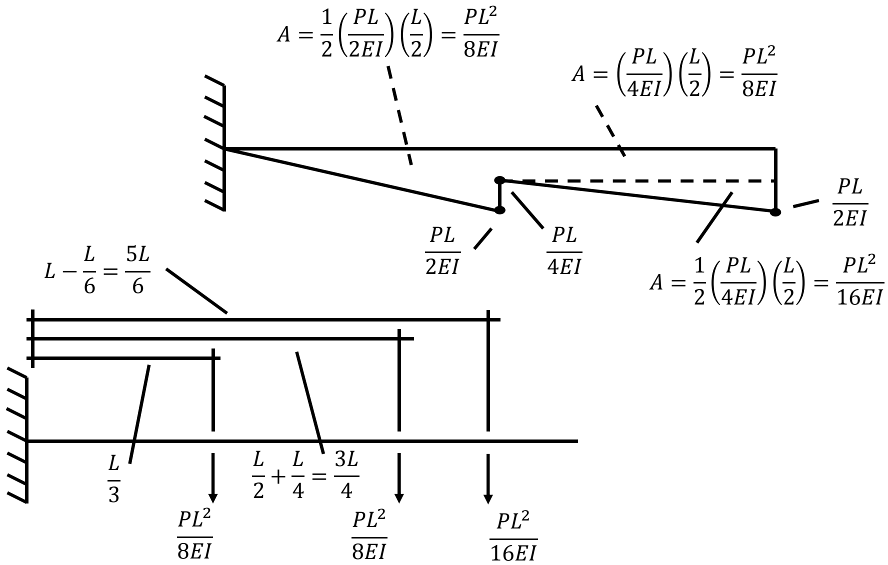
Conjugate Beam Example

Calculate the deflection and rotation at the free end of the beam



Conjugate Beam Example

Construct the *M/EI* diagram and compute the equivalent concentrated loads



Conjugate Beam

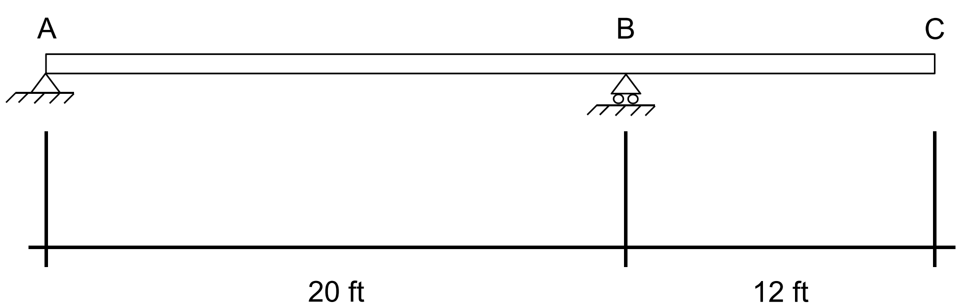
Compute shear in conjugate beam (slope in real beam) and moment if conjugate beam (deflection in real beam). Values are calculated at fixed end of conjugate beam (free end of real beam).

$$\theta_{real} = V_{conjugate} = \frac{PL^2}{8EI} + \frac{PL^2}{8EI} + \frac{PL^2}{16EI} = \frac{5PL^2}{16EI}\, CCW$$

$$\Delta_{real} = M_{conjugate} = \left(\frac{PL^2}{8EI}\right)\left(\frac{L}{3}\right) + \left(\frac{PL^2}{8EI}\right)\left(\frac{3L}{4}\right) + \left(\frac{PL^2}{16EI}\right)\left(\frac{5L}{6}\right) = \frac{3PL^3}{16EI} \downarrow$$

Influence Lines

Beam for Quantitative Influence Lines

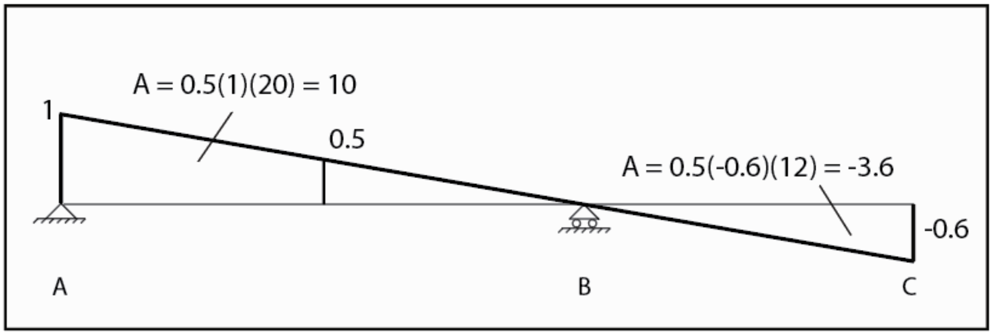


Influence Lines – Examples Using Mueller-Breslau Principle

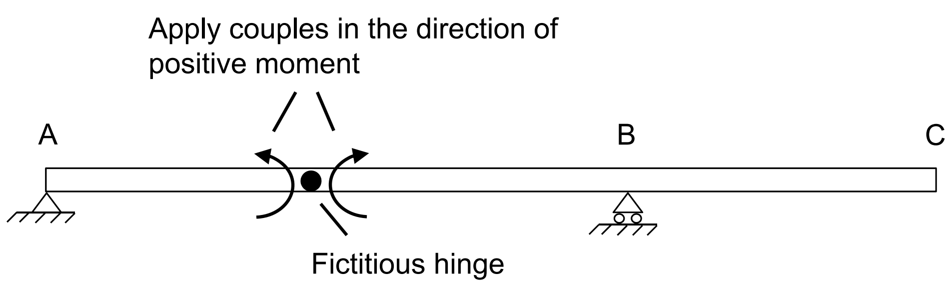
Mueller-Breslau Principle

If some force component in a structure is considered to act through a small distance (or angle), the shape of the displaced structure will be, to some scale, the influence line for that force component.

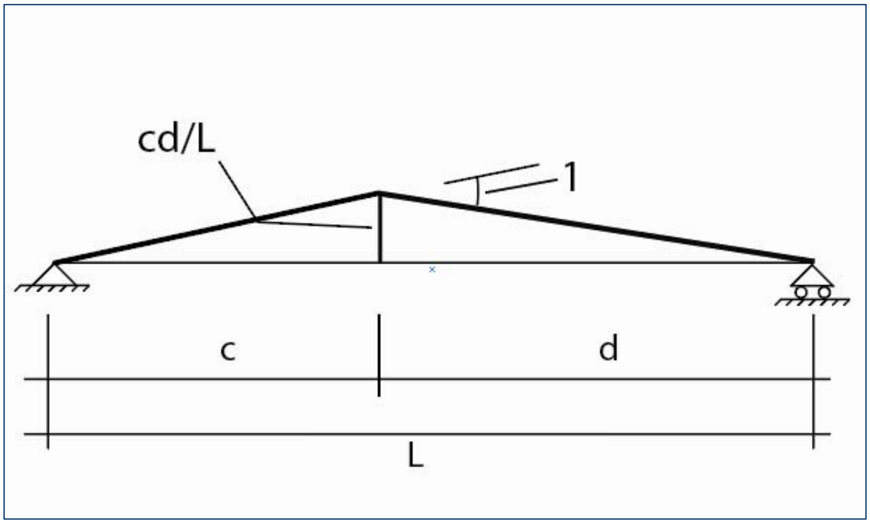
Influence Line for Reaction at *A*



Influence Line for Moment



Influence Line for Moment



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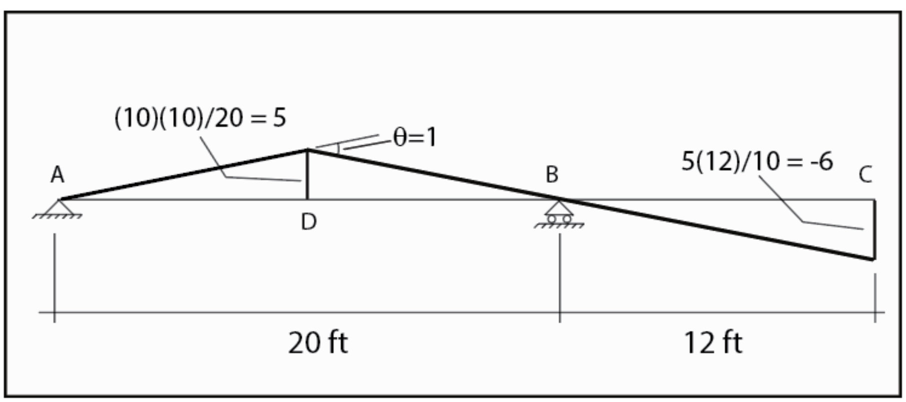
Influence Line for Moment at *D*



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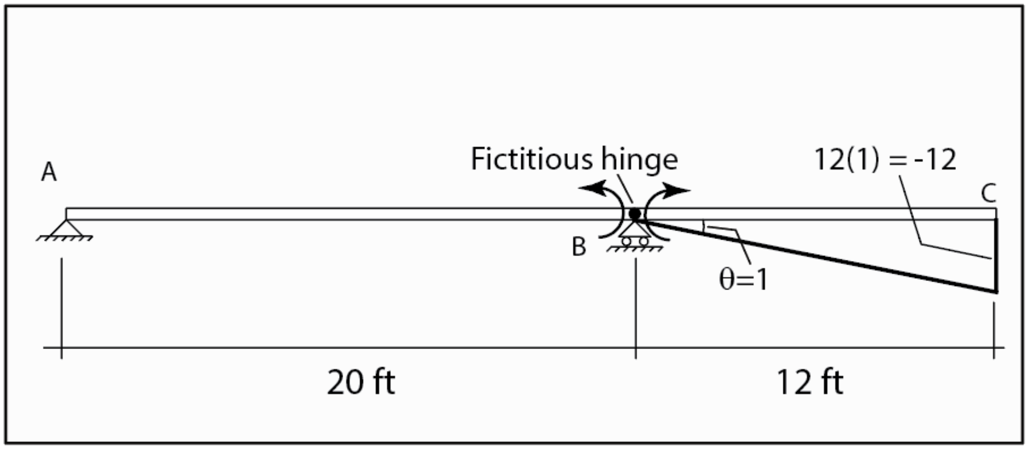
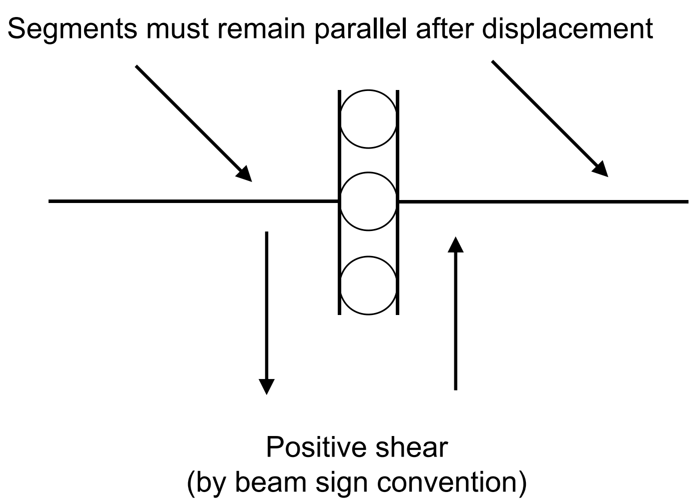
Influence Line for Moment at *B*

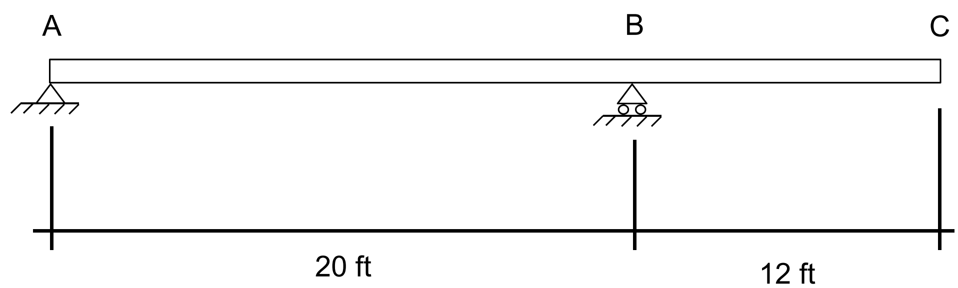


Influence Line Shear Discontinuity

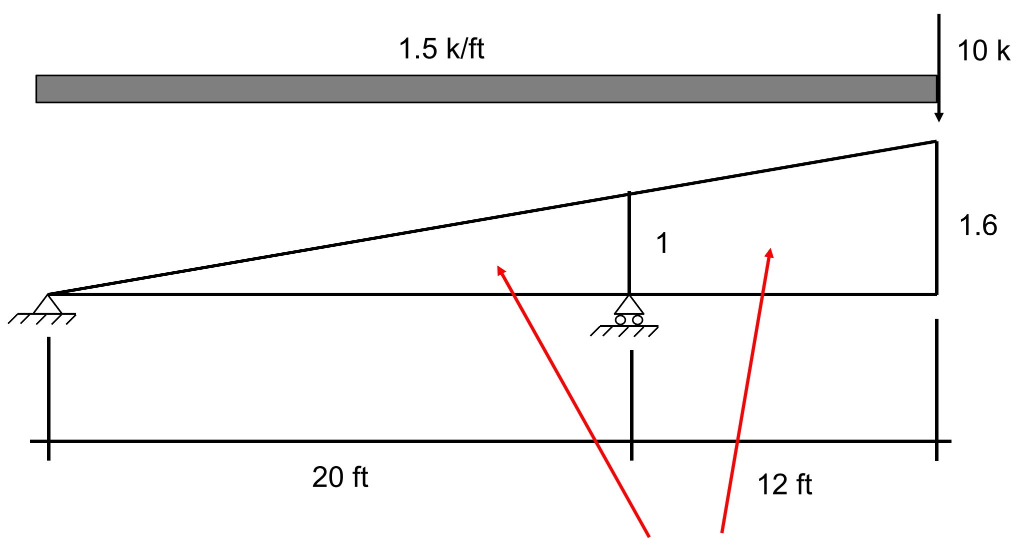


Influence Lines Example 1

The beam shown is subjected to a single concentrated load of 10 k and a uniform load of 1.5 k/ft of arbitrary length. The concentrated load may positioned at any point and the uniform load may occupy any length on the beam. Determine the maximum upward reaction at B.



Influence Lines Example 1

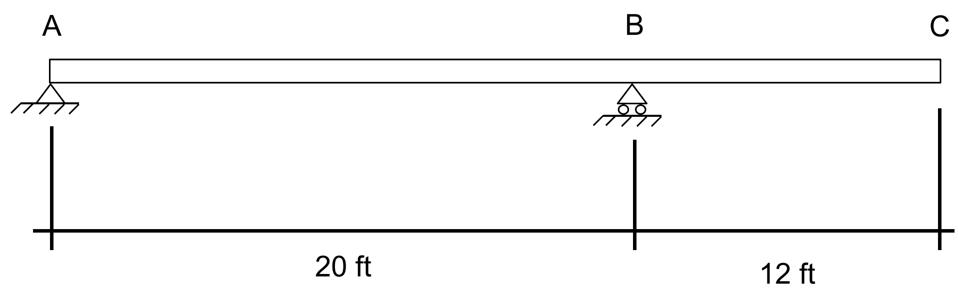


$$A = \frac{1}{2}(1.6)(32\text{ ft}) = 25.6$$

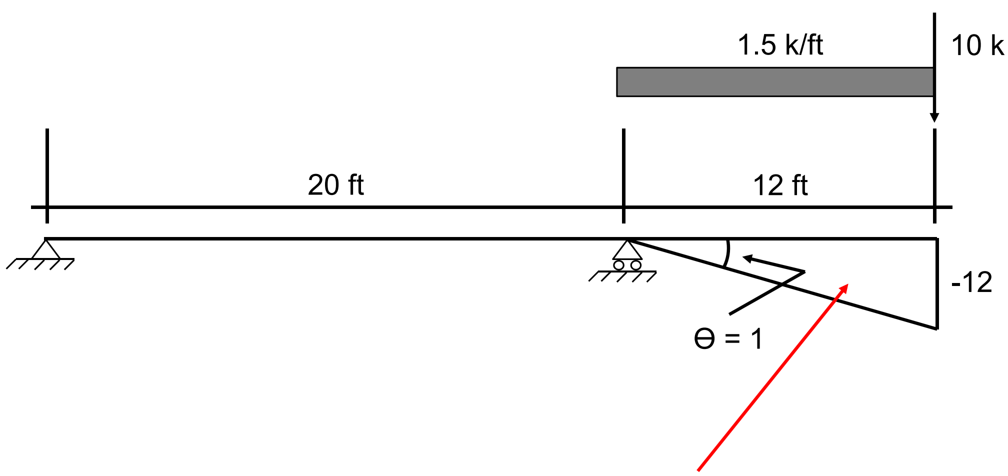
$$(10\text{ k})(1.6) + (1.5\text{ k/ft})(25.6) = 54.4\text{ k}$$

Influence Lines Example 2

The beam shown is subjected to a single concentrated load of 10 k and a uniform load of 1.5 k/ft of arbitrary length. The concentrated load may positioned at any point and the uniform load may occupy any length on the beam. Determine the maximum negative moment at B.



Influence Lines Example 2

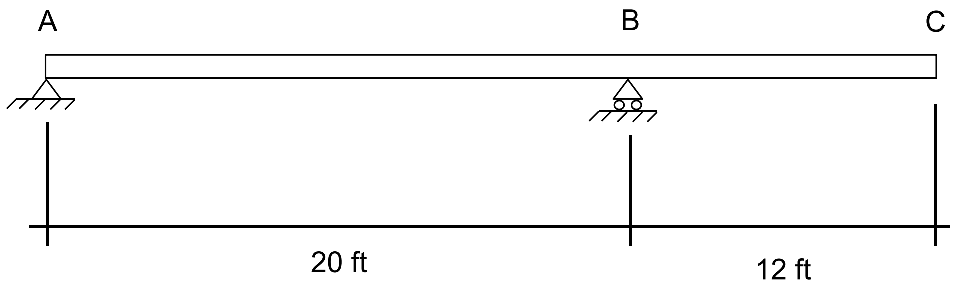


$$A = \frac{1}{2}(12)(12\ \text{ft}) = 72$$

$$(10\ \text{k})(-12) - \left(1.5\frac{\text{k}}{\text{ft}}\right)(72) = -228\ \text{k}\cdot\text{ft}$$

Influence Lines Example 3

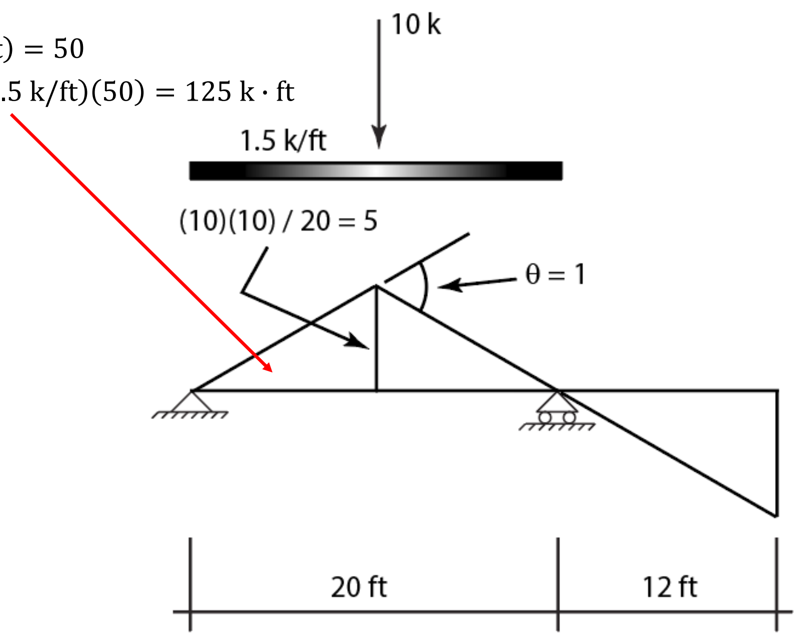
The beam shown is subjected to a single concentrated load of 10 k and a uniform load of 1.5 k/ft of arbitrary length. The concentrated load may positioned at any point and the uniform load may occupy any length on the beam. Determine the maximum positive moment at a point half way between *A* and *B*.



Influence Lines Example 3

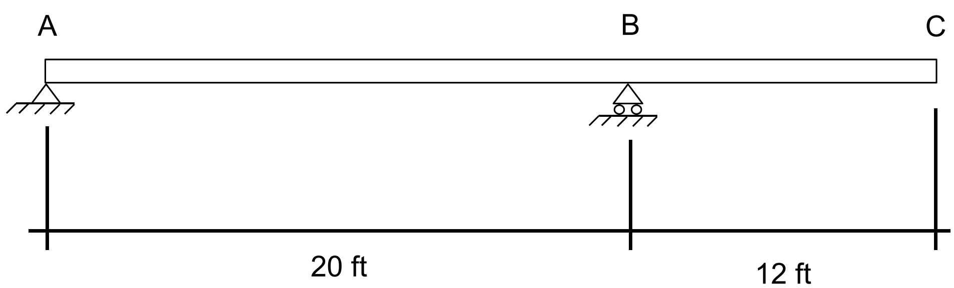
$A = \frac{1}{2}(5)(20 \text{ ft}) = 50$

$(10 \text{ k})(5) + (1.5 \text{ k/ft})(50) = 125 \text{ k} \cdot \text{ft}$



Influence Lines Example 4

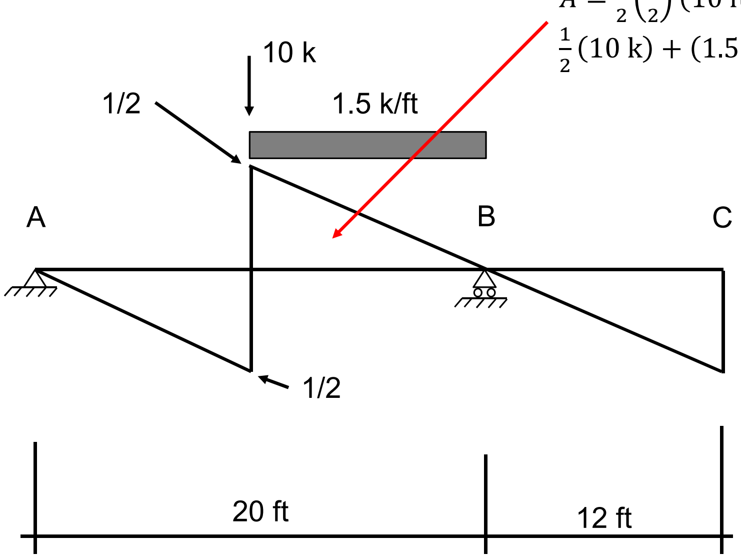
The beam shown is subjected to a single concentrated load of 10 k and a uniform load of 1.5 k/ft of arbitrary length. The concentrated load may positioned at any point and the uniform load may occupy any length on the beam. Determine the maximum positive shear at a point half way between *A* and *B*.



Influence Lines Example 4

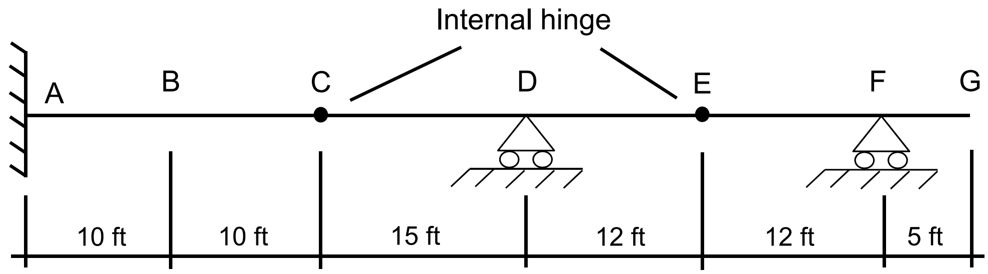
$A = \frac{1}{2}\left(\frac{1}{2}\right)(10\text{ ft}) = 2.5$

$\frac{1}{2}(10\text{ k}) + (1.5\text{ k/ft})(2.5) = 8.75\text{ k}$



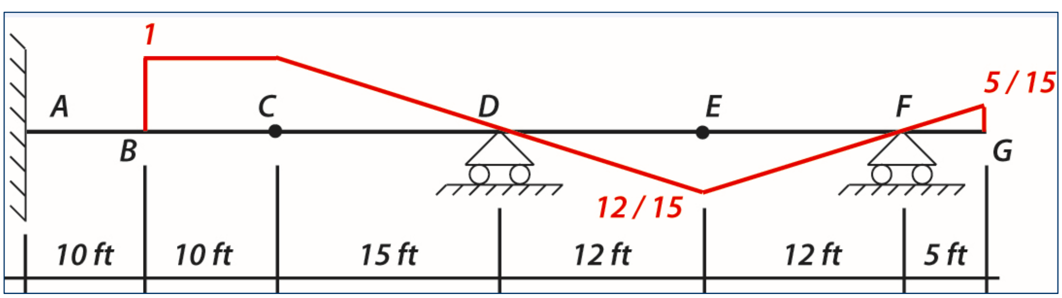
Influence Lines Example 5

Draw the influence lines for the shear at *B* and the moment at *B*. Note that the beam is statically determinate.

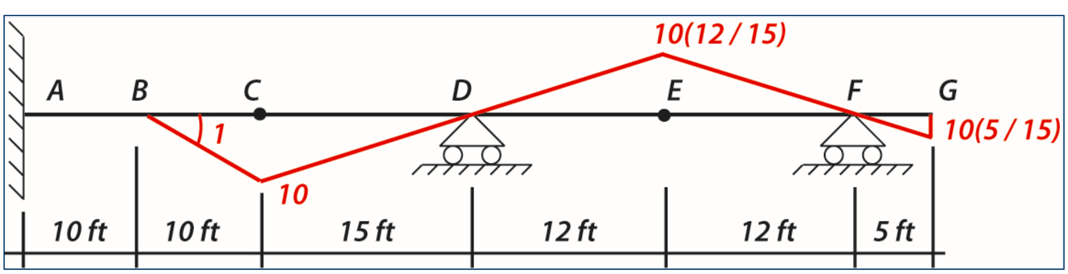


Influence Lines Example 5

Influence line for shear at *B*.

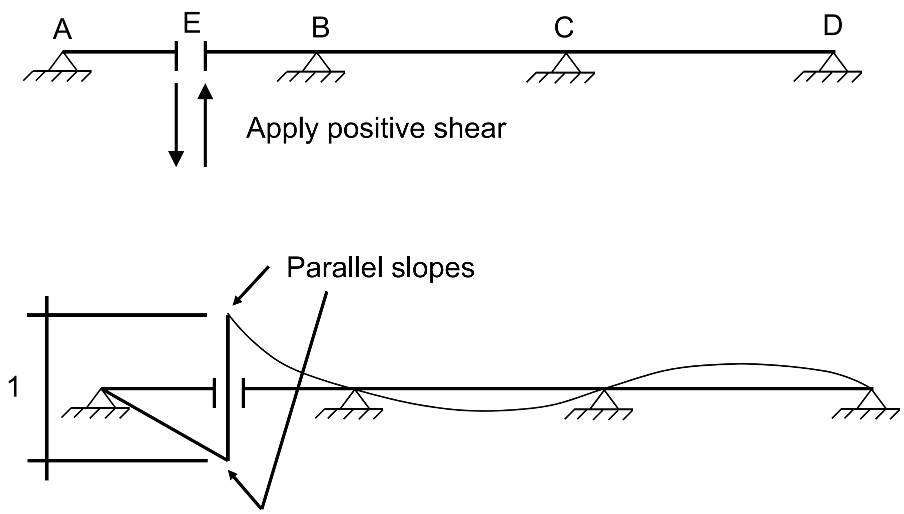


Influence line for moment at *B*.

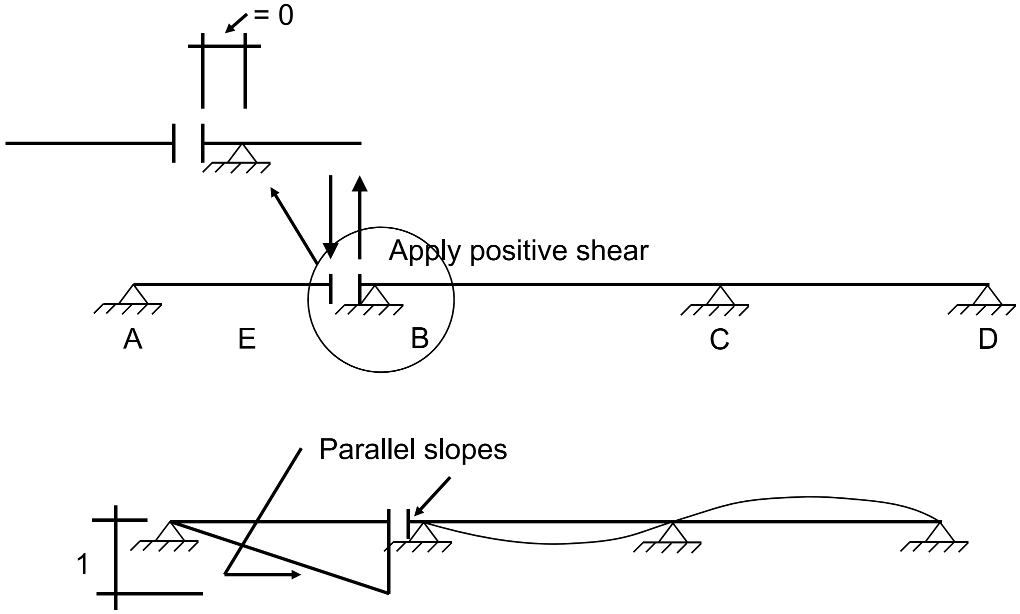


Qualitative Influence Lines

Mueller-Breslau Principle for Indeterminate Structures

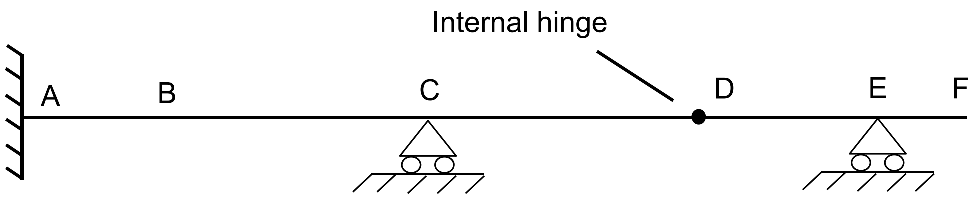


Mueller-Breslau Principle for Indeterminate Structures



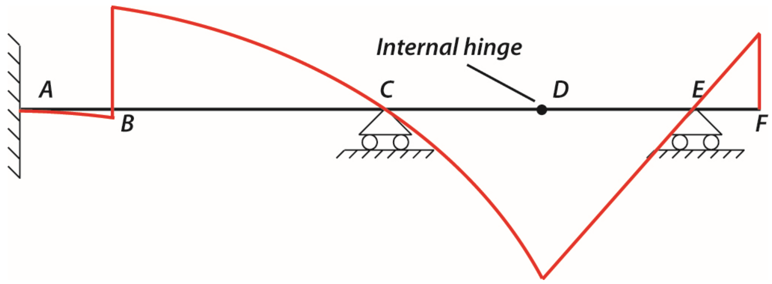
Influence Lines Example 6

Draw the influence lines for the shear at *B* and the moment at *B*. Note that the beam is statically determinate.

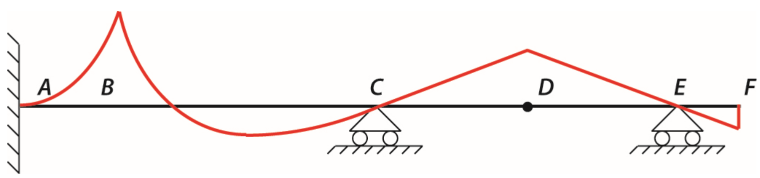


Influence Lines Example 6

Influence line for shear at *B*.

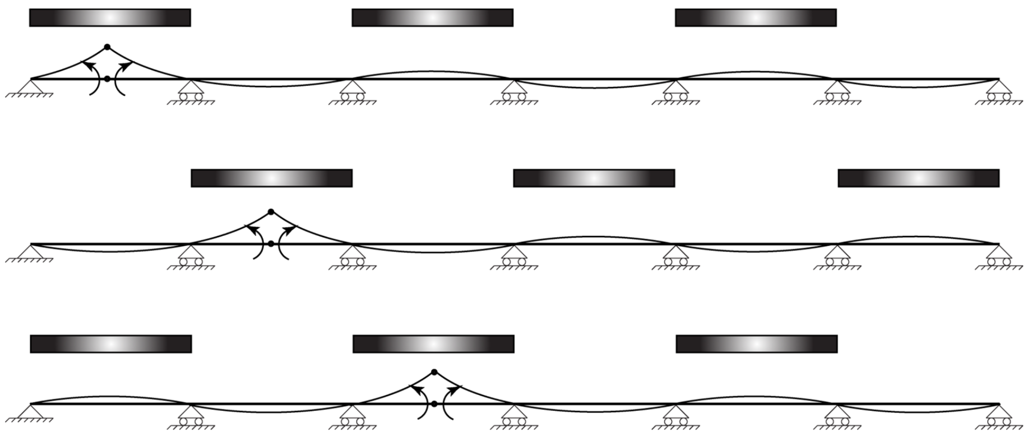


Influence line for moment at *B*.

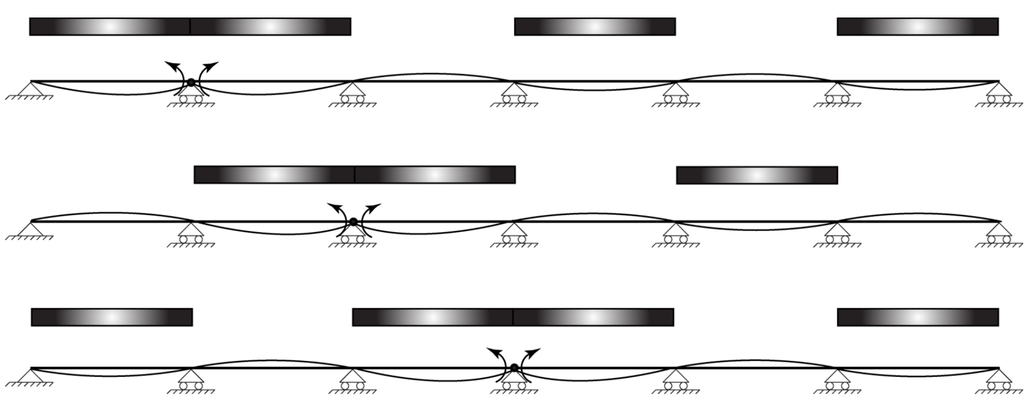


Loading for Maximum Effect

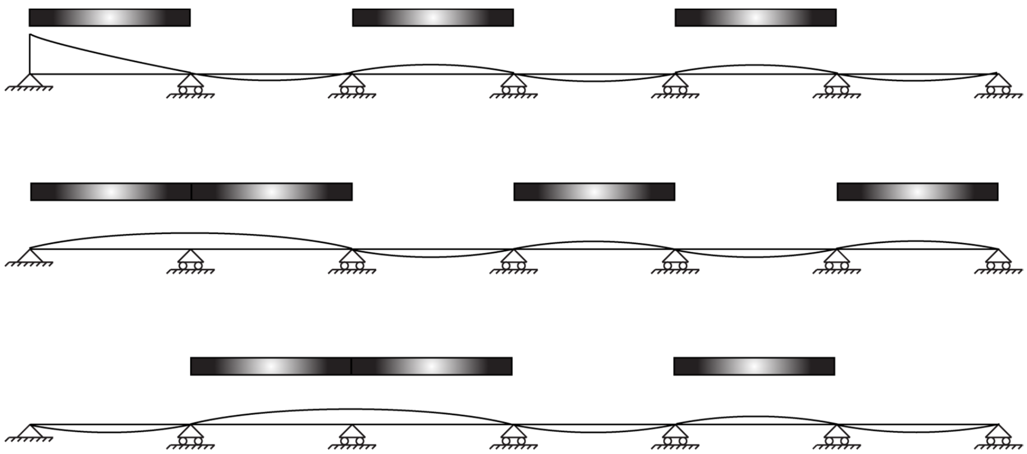
Loading for Maximum Positive Moment



Loading for Maximum Negative Moment

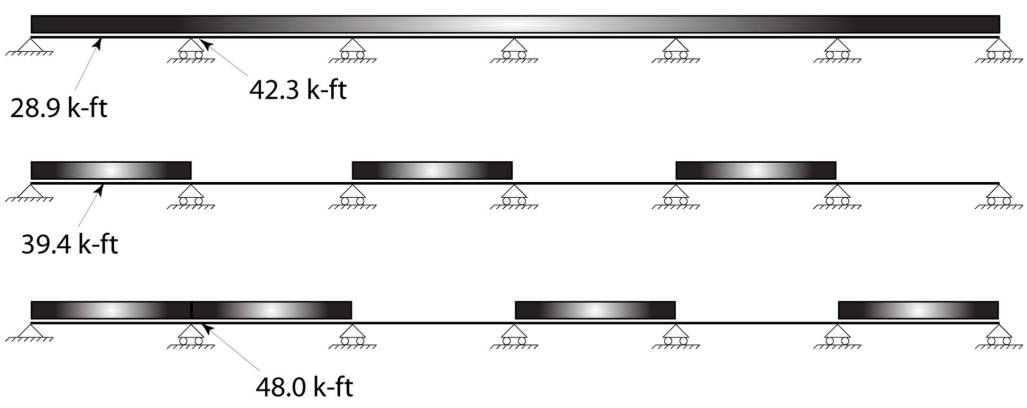


Loading for Maximum Reaction



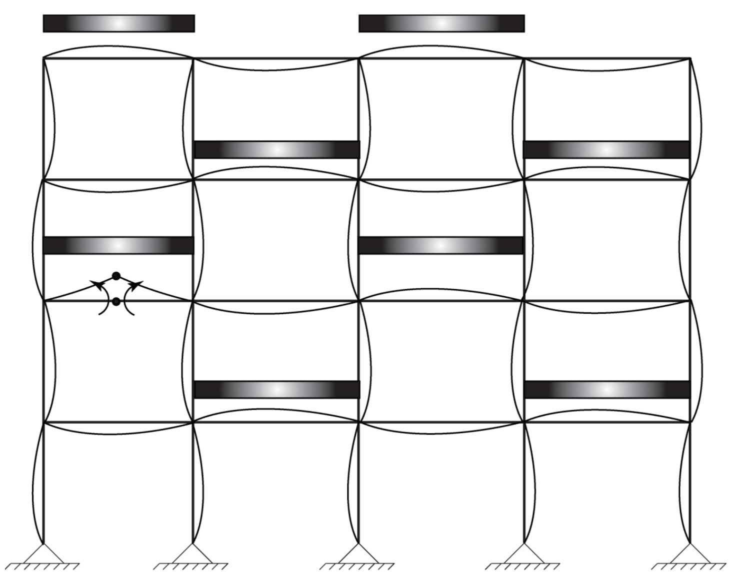
Influence Lines Example 7

All loads 1 k/ft



Influence Lines for Frames

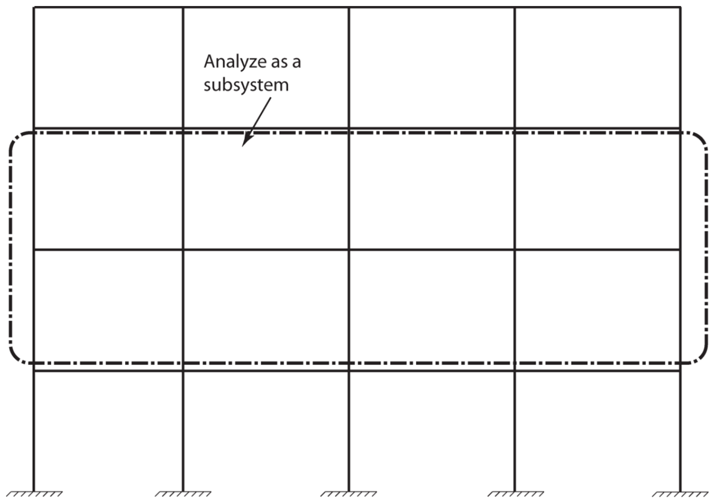
Influence Lines for Rigid Frame



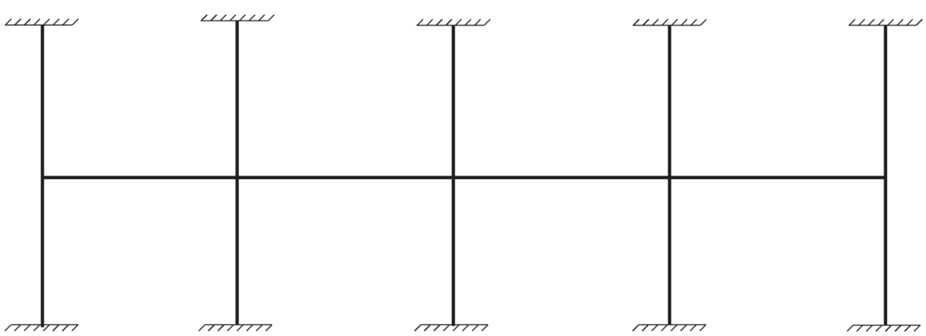
ACI Two-Way Slabs

Equivalent rigid frame method

ACI Section 13.7 - for gravity load only



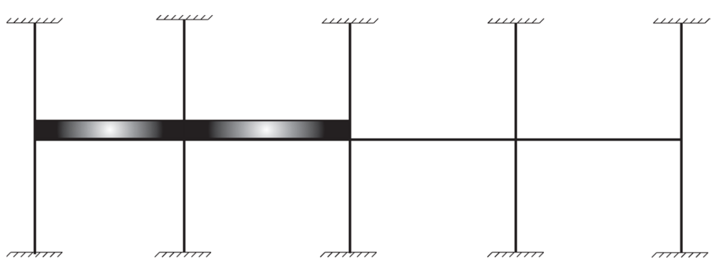
ACI Two-Way Slabs



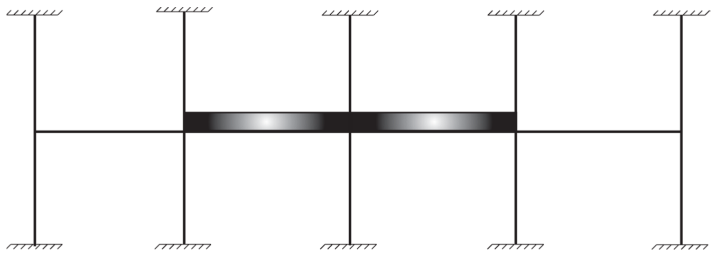
ACI Two-Way Slabs

- When the ratio of the service live load to the service dead load is less than or equal to 0.75, analysis can be made with the factored dead load plus the factored live load on all spans – no pattern loading required.
- When the ratio of the service live load to the service dead load is greater than 0.75, pattern loading required.
  - Only 75% of full-factored live load need be used for the pattern
  - Load two adjacent spans for maximum negative moment
  - For maximum positive moment load alternate spans
  - Design moments may not be less than those occurring with full-factored live load on all spans

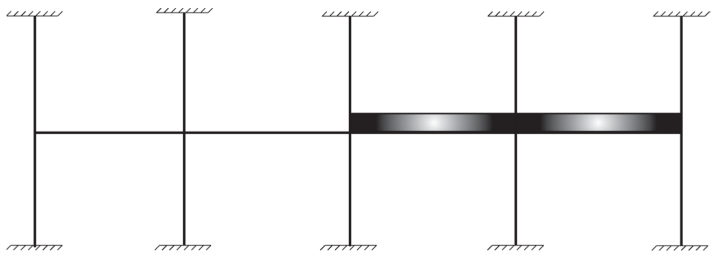
ACI Two-Way Slabs



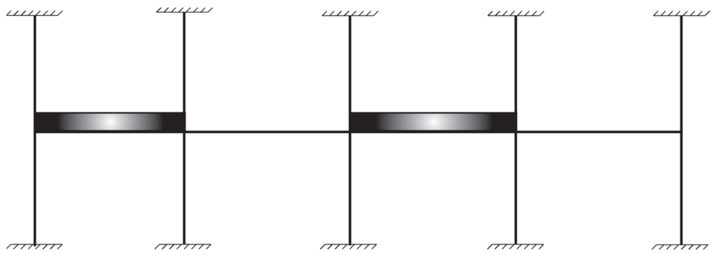
ACI Two-Way Slabs



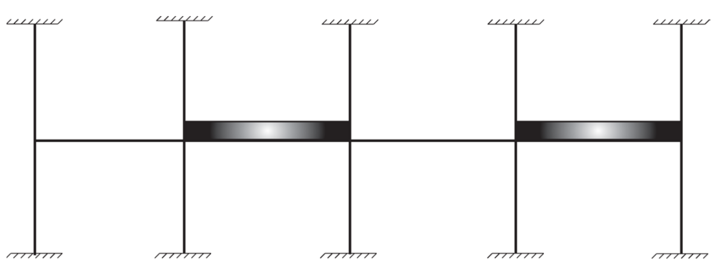
ACI Two-Way Slabs



ACI Two-Way Slabs

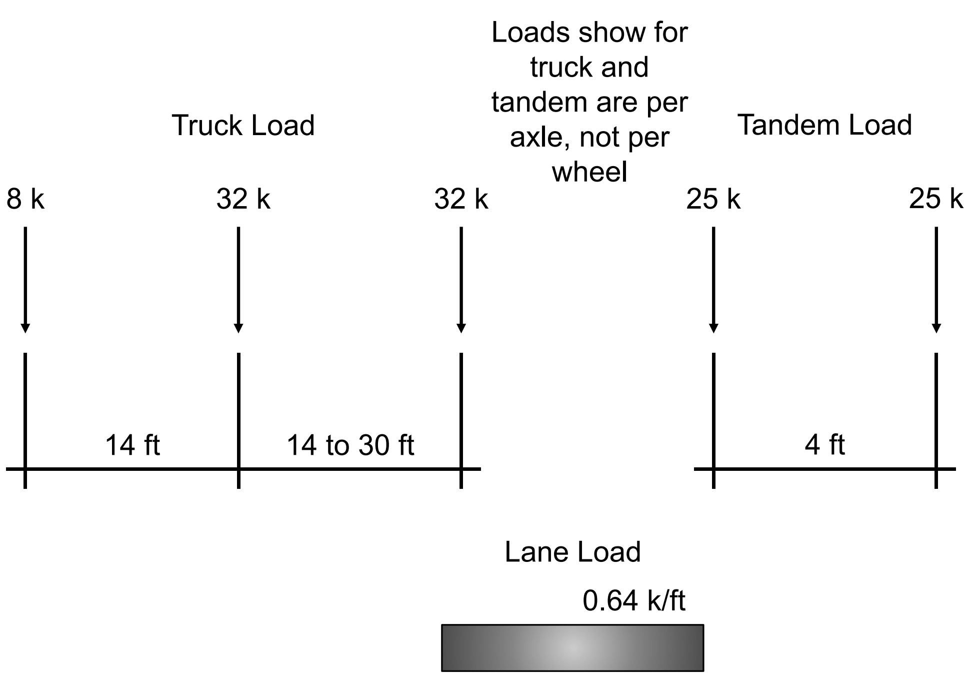


ACI Two-Way Slabs



Moving Loads

AASHTO Vehicle Loads

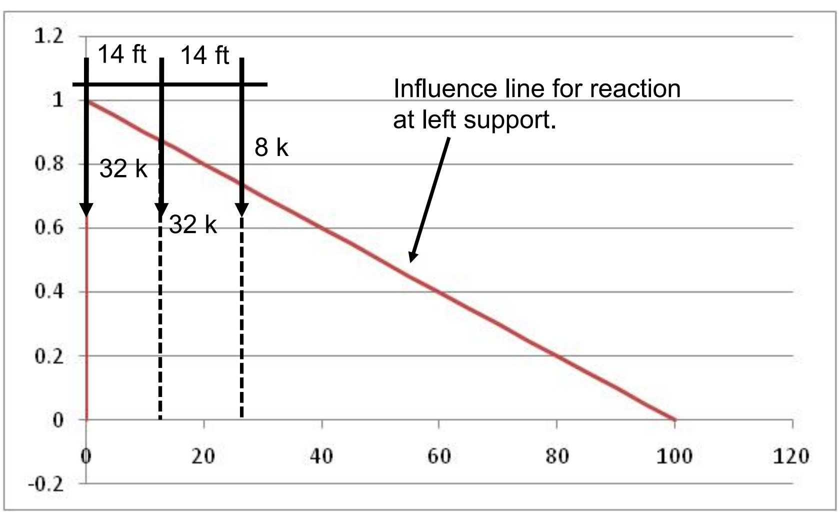


Maximum Reaction (Shear)

Absolute maximum shear due to moving concentrated loads occurs at a support, when one of the loads is at the support. The location that produces the maximum shear can be determine by trial.

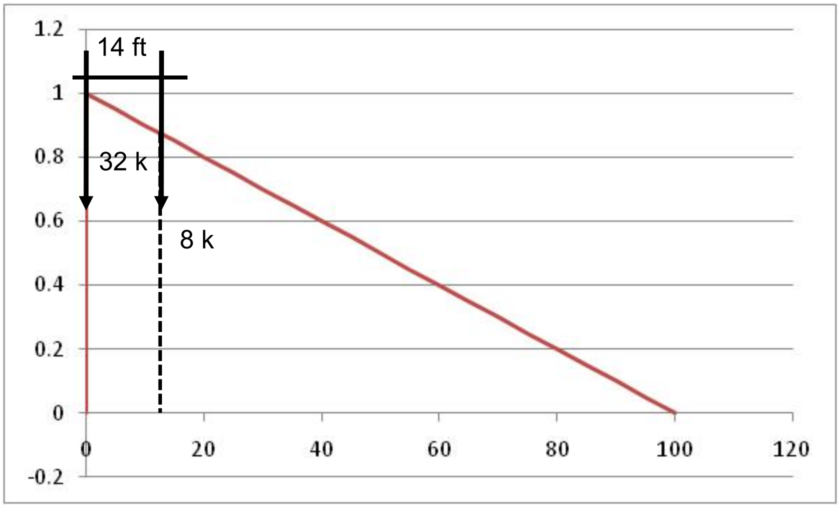
Compute the maximum shear at the left support of a 100-ft-long simply supported beam under the action of the AASHTO truck load.

Maximum Reaction



$$R_L = (32\text{ k})(1) + (32\text{ k})\left(\frac{86}{100}\right) + (8\text{ k})\left(\frac{72}{100}\right) = 65.3\text{ k}$$

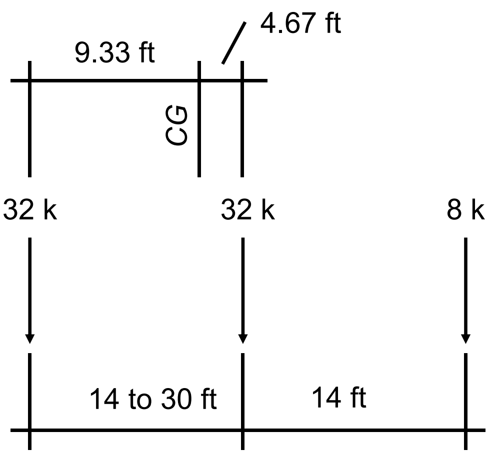
Maximum Reaction



$$R_L = (32\text{ k})(1) + (8\text{ k})\left(\frac{86}{100}\right) = 38.9\text{ k}$$

Absolute Maximum Moment

In a simply supported beam, the absolute maximum moment occurs when the centerline of the span bisects the distance between the center of gravity of the load system and one of the two adjacent loads. The maximum moment usually occurs under the closer load.



Absolute Maximum Moment

Compute the absolute maximum moment for a 100-ft-long simply supported beam under the action of the AASHTO truck load.

Center of gravity of the load system

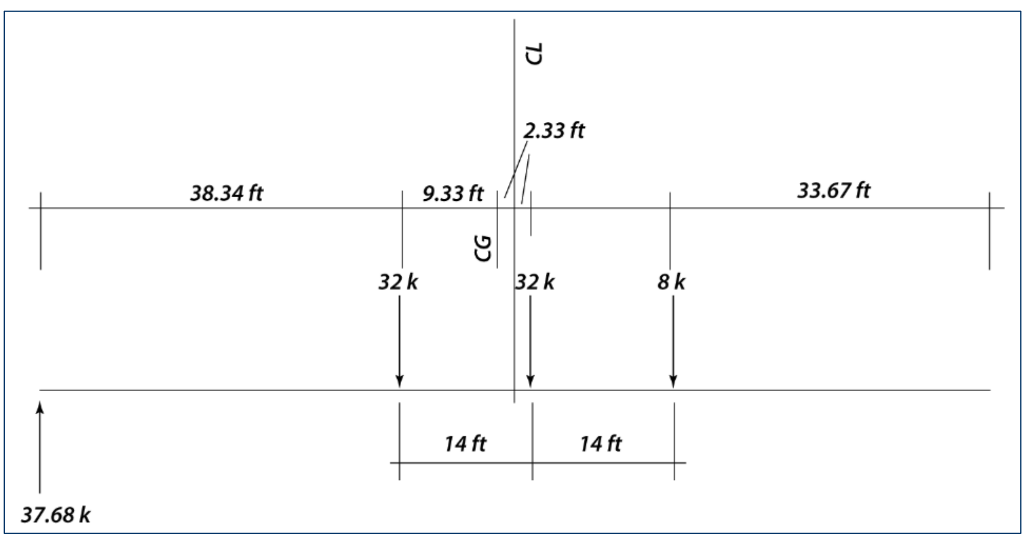
$$CG = \frac{(32\text{ k})(14\text{ ft})+(8\text{ k})(28\text{ ft})}{32\text{ k}+32\text{ k}+8\text{ k}} = 9.33\text{ ft}$$

$$14.0\text{ ft} - 9.33\text{ ft} = 4.67\text{ ft}$$

First case

Position loads so that the centerline of the span bisects the 4.67 ft dimension. Compute the moment under each 32 k load.

Absolute Maximum Moment



Absolute Maximum Moment

Left reaction

$$R_L = \frac{(8\text{ k})(33.67\text{ ft})+(32\text{ k})(47.67\text{ ft})+(32\text{ k})(61.67\text{ ft})}{100\text{ ft}} = 37.68\text{ k}$$

Moment at center 32 k load

$$M = (37.68\text{ k})(52.34\text{ ft}) - (32\text{ k})(14\text{ ft}) = 1{,}524\text{ k}\cdot\text{ft}$$

Absolute Maximum Moment

Note: if the center 32 k load is placed at the centerline of the beam, the moment at that point is $1{,}520\text{ k}\cdot\text{ft}$. There is very little difference and it is not worth the effort to calculate the absolute maximum moment.

Moment at left 32 k load

$$M = (37.68\text{ k})(38.34\text{ ft}) = 1{,}445\text{ k}\cdot\text{ft}$$

Absolute Maximum Moment

Second case

Position loads to bisect the 9.33 ft dimension

Left reaction

$$R_L = \frac{(8\ \text{k})(26.66\ \text{ft})+(32\ \text{k})(40.66\ \text{ft})+(32\ \text{k})(54.66\ \text{ft})}{100\ \text{ft}} = 32.64\ \text{k}$$

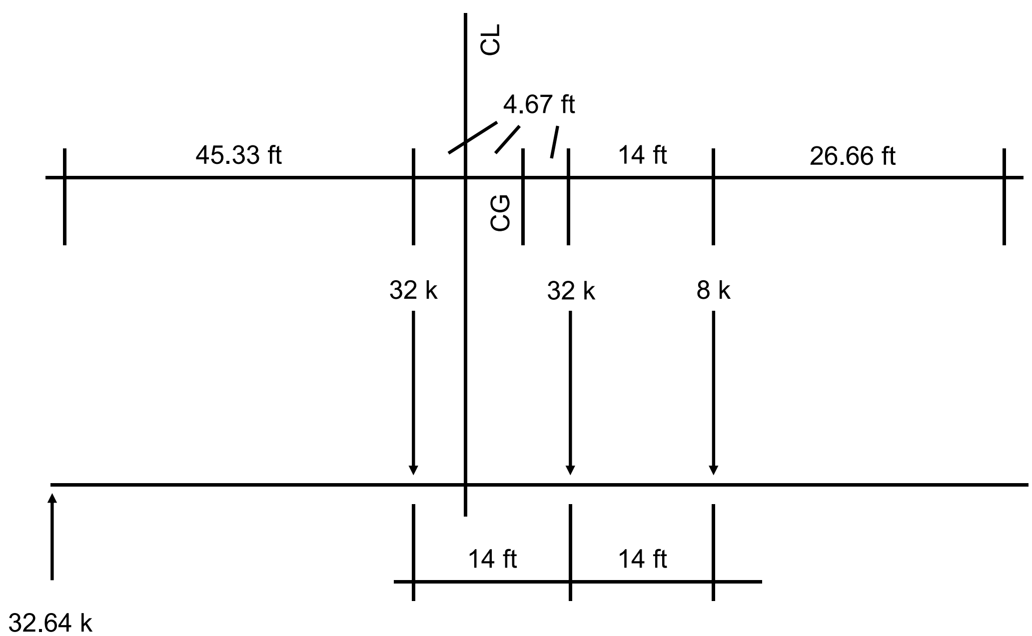
Compute the moment under the left 32 k load

$$M = (32.64\ \text{k})(45.33\ \text{ft}) = 1{,}480\ \text{k} \cdot \text{ft}$$

Compute the moment under the center 32 k load

$$M = (32.64\ \text{k})(59.33\ \text{ft}) - (32\ \text{k})(14\ \text{ft}) = 1{,}490\ \text{k} \cdot \text{ft}$$

Absolute Maximum Moment



Moment Distribution

Moment Distribution

- Sign convention - *Clockwise* moment acting on a *member* is positive. Don't confuse this sign convention with the *beam sign convention*.
- Member rotational stiffness
- Member modified rotational stiffness
- Joint stiffness factor
- Distribution Factor
- Carry-over factor
- Fixed-end moment

Member Rotational Stiffness

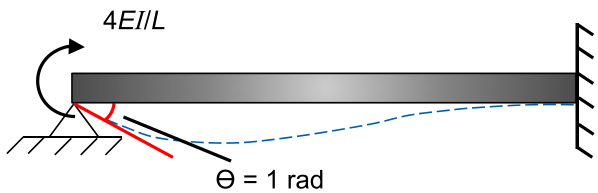
The moment required to rotate the near end of the member one radian, when the far end of the member is fixed.

E is the modulus of elasticity

I is the moment of inertia

L is the length

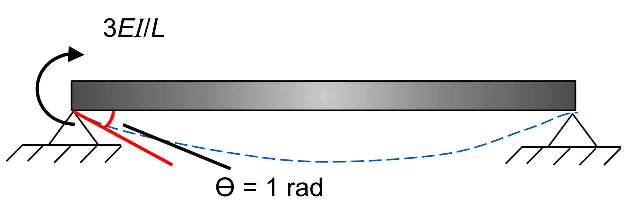
$$K = \frac{4EI}{L}$$



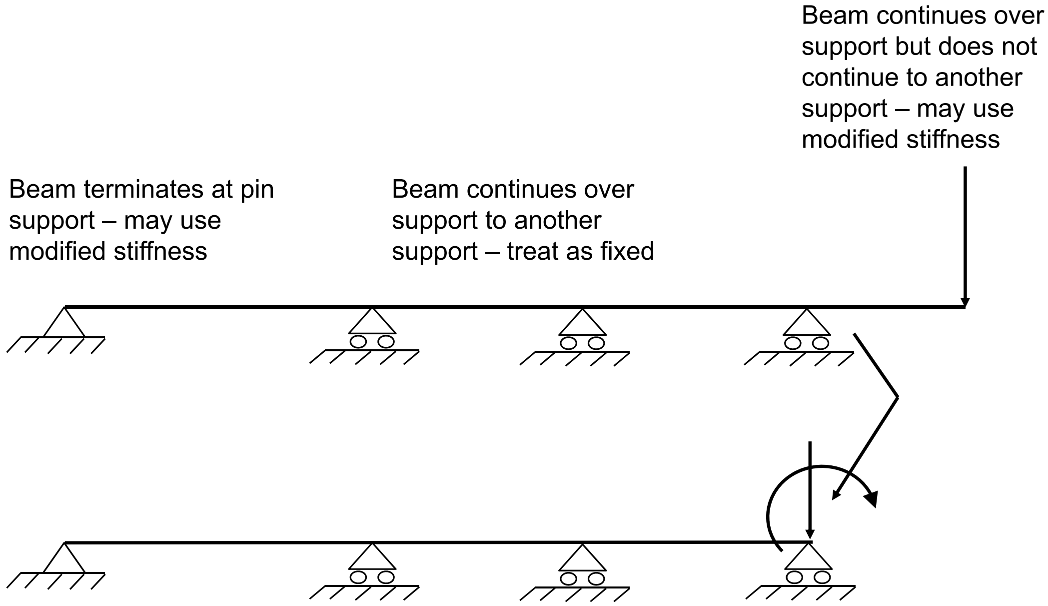
Member Modified Rotational Stiffness

The moment required to rotate the near end of the member one radian, when the far end of the member is pinned.

$$K = \frac{3EI}{L}$$

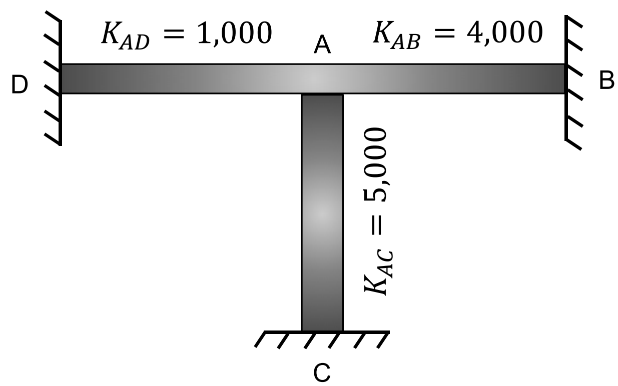


Use of Modified Stiffness



Joint Stiffness Factor

The moment required to rotate a joint through one radian. It is equal to the sum of the rotational stiffnesses of members framing into the joint.



$K_A = \sum K = 4{,}000 + 5{,}000 + 1{,}000 = 10{,}000$

Distribution Factor

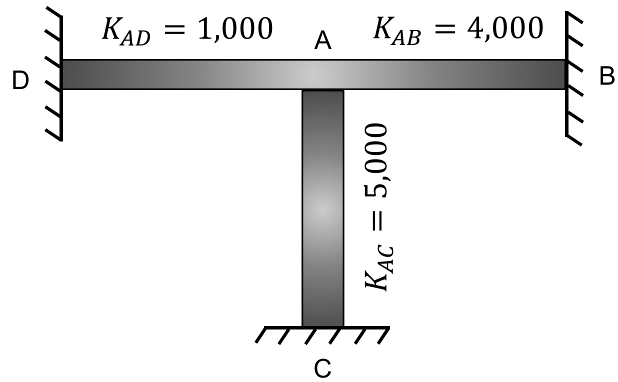
The distribution factor for a member is the ratio of its rotational stiffness factor and the joint stiffness factor for the joint into which it frames.

$$DF = \frac{K}{K_A}$$

$$DF_{AB} = \frac{4{,}000}{10{,}000} = 0.4$$

$$DF_{AC} = \frac{5{,}000}{10{,}000} = 0.5$$

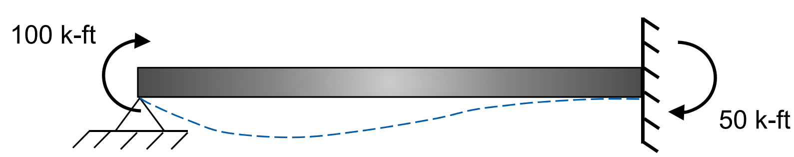
$$DF_{AD} = \frac{1{,}000}{10{,}000} = 0.1$$



Important – the sum of the DFs at a joint equals 1.

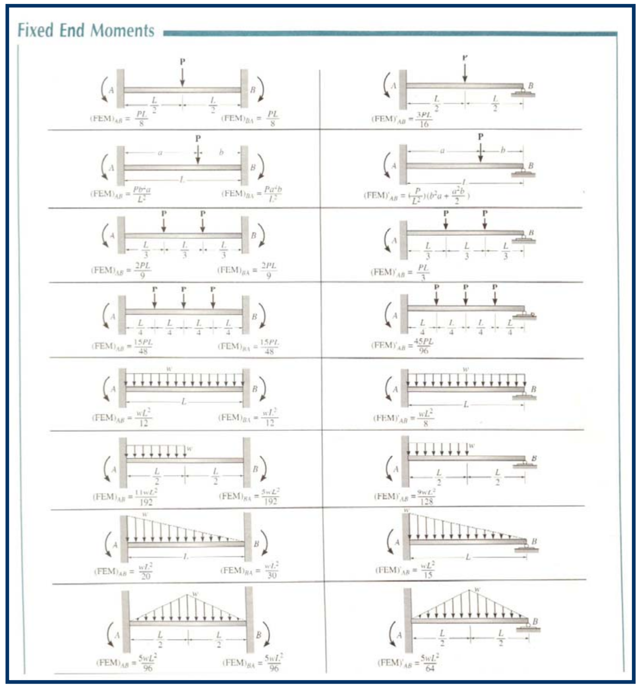
Carry-over Factor

The carry-over factor represents that amount of moment "carried-over" from one end of a member to the other. For a *prismatic member* (*EI = constant*), its value is 0.5.



Fixed-End Moments

Moment(s) developed at the ends of a member assuming that those ends are fixed. Normally determined from entries in tables.



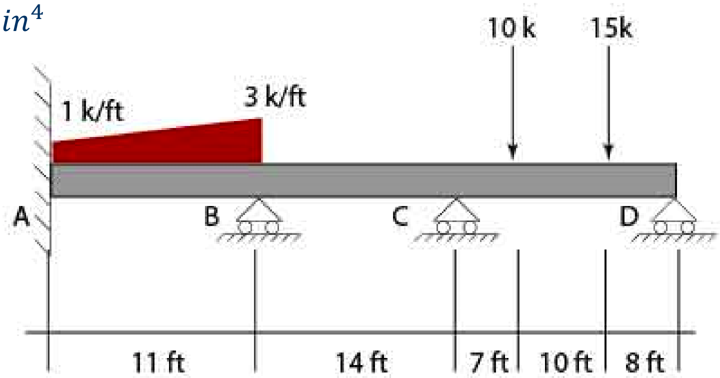
Moment Distribution – Beam Example

Moment Distribution Example

$I_{AB} = 1{,}110\ in^4$

$I_{BC} = 1{,}240\ in^4$

$I_{CD} = 1{,}240\ in^4$



Rotational Stiffness and Distribution Factors

$$K_{BA} = \frac{4EI_{BA}}{L_{BA}} = \frac{4E(1{,}110\ \text{in}^4)}{12(11\ \text{ft})} = 33.64E\ \text{k} \cdot \text{in/rad}$$

$$K_{BC} = K_{CB} = \frac{4EI_{BC}}{L_{BC}} = \frac{4E(1{,}240\ \text{in}^4)}{12(14\ \text{ft})} = 29.52E\ \text{k} \cdot \text{in/rad}$$

This value should be 33.64

$$K_{CD} = \frac{3EI_{CD}}{L_{CD}} = \frac{3E(1{,}240\ \text{in}^4)}{12(25\ \text{ft})} = 12.40E\ \text{k} \cdot \text{in/rad}$$

Note: *E* can be omitted from calculations when it has the same value for all spans

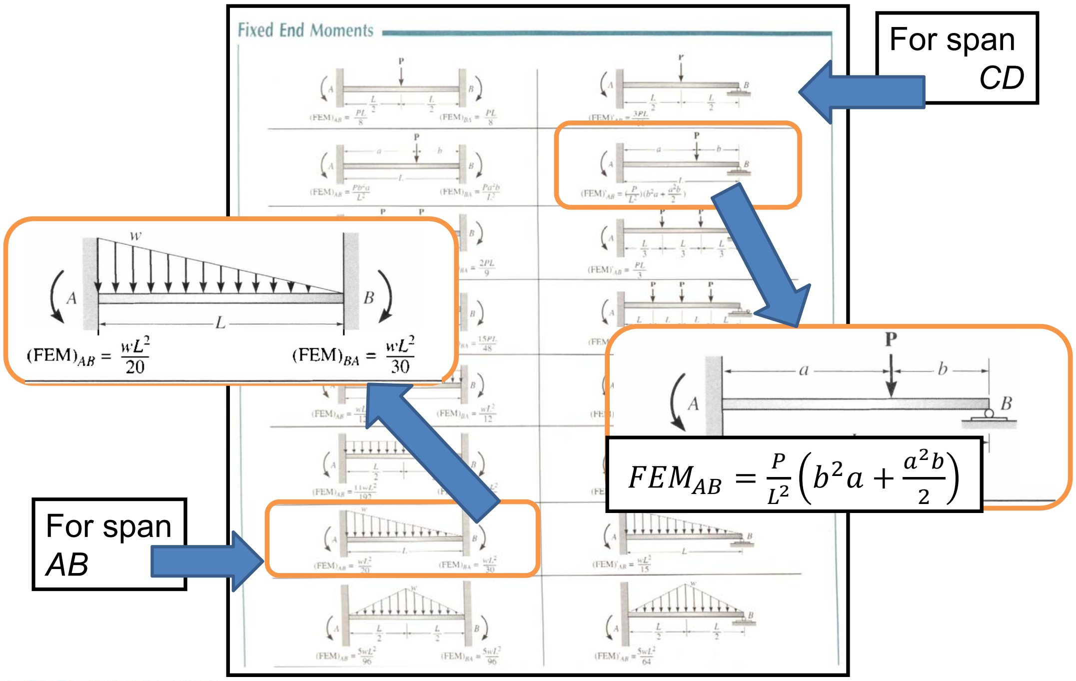
$$DF_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{33.64E\ \text{k·in/rad}}{33.64E\ \text{k·in/rad}+29.52E\ \text{k·in/rad}} = 0.53256$$

$$DF_{BC} = \frac{K_{BC}}{K_{BA}+K_{BC}} = \frac{29.52E\ \text{k·in/rad}}{33.64E\ \text{k·in/rad}+29.52E\ \text{k·in/rad}} = 0.46744$$

$$DF_{CB} = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{29.52E\ \text{k·in/rad}}{29.52E\ \text{k·in/rad}+12.40E\ \text{k·in/rad}} = 0.70423$$

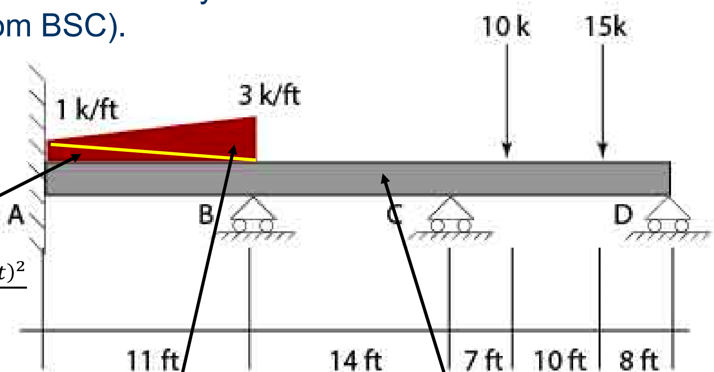
$$DF_{CD} = 1 - DF_{CB} = 1 - 0.70423 = 0.29577$$

Table of Fixed-end Moments



Fixed-end Moments

Split the trapezoidal load into two triangular loads, one with intensity of 1 k/ft at *A* and the other with intensity of 3 k/ft at *B*. Clockwise moment is positive (different from BSC).



$$FEM_{AB} = -\frac{(1k/ft)(11\,ft)^2}{20} - \frac{(3k/ft)(11\,ft)^2}{30} = -18.150\; k \cdot ft$$

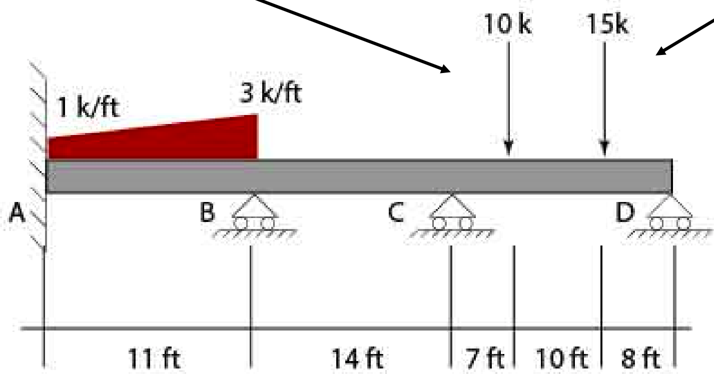
$$FEM_{BC} = FEM_{CB} = 0$$

$$FEM_{BA} = \frac{(1\,k/ft)(11\,ft)^2}{30} + \frac{(3\,k/ft)(11\,ft)^2}{20} = 22.183\; k \cdot ft$$

Fixed-end Moments

$$-\left(\frac{10\ \text{k}}{(25\ \text{ft})^2}\right)\left((18\ \text{ft})^2(7\ \text{ft}) + \frac{(7\ \text{ft})^2(18\ \text{ft})}{2}\right) = -43.34\ \text{k}\cdot\text{ft}$$

$$-\left(\frac{15\ \text{k}}{(25\ \text{ft})^2}\right)\left((8\ \text{ft})^2(17\ \text{ft}) + \frac{(17\ \text{ft})^2(8\ \text{ft})}{2}\right) = -53.86\ \text{k}\cdot\text{ft}$$



$$FEM_{CD} = -43.34\ \text{k}\cdot\text{ft} - 53.86\ \text{k}\cdot\text{ft} = -97.20\ \text{k}\cdot\text{ft}$$

Moment Distribution

		Joint A		Joint B			Joint C			Joint D
Member		AB		BA	BC		CB	CD		DC
DF		0		0.53256	0.46744		0.70423	0.29577		1
FEM		-18.150		22.183	0		0	-97.200		0
Dist				-11.814	-10.369		68.451	28.749		
CO		-5.9069			34.225		-5.185			
Dist				-18.227	-15.998		3.651	1.534		
CO		-9.1135			1.8256		-7.9992			
Dist				-0.9722	-0.85337		5.633	2.366		
CO		-0.4861			2.8166		-0.4267			
Dist				-1.500	-1.3166		0.30048	0.126		
CO		-0.7500			0.1502		-0.65831			
S		-33.657		-10.330	10.330		64.425	-64.425		0

Initial Distributions

Joint *B* is out of balance by -22.183 k-ft. Apply moments at that joint in proportion to the DFs for that joint.

$$0.53256(-22.183\ \mathrm{k \cdot ft}) = -11.814\ \mathrm{k \cdot ft}$$

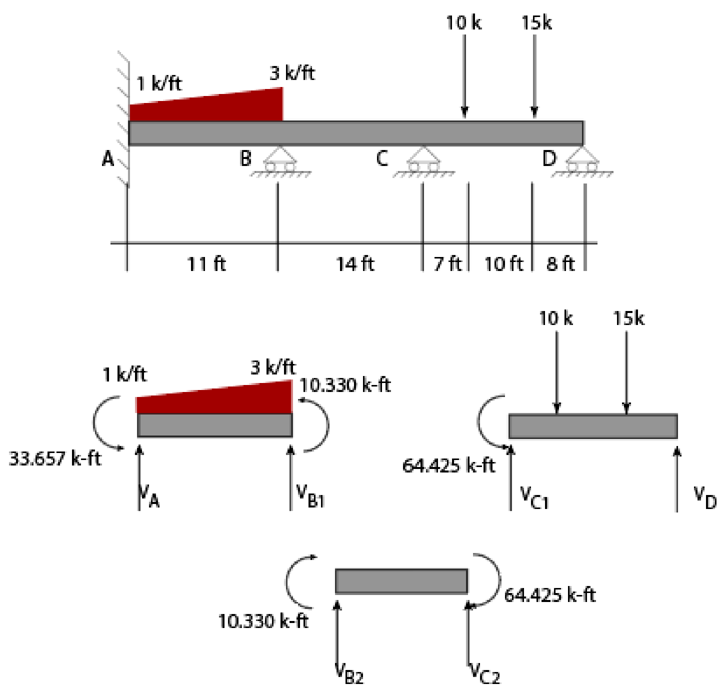
$$0.46744(-22.183\ \mathrm{k \cdot ft}) = -10.369\ \mathrm{k \cdot ft}$$

Joint *C* is out of balance by 97.2 k-ft. Apply moments at that joint in proportion to the DFs for that joint.

$$0.70423(97.2\ \mathrm{k \cdot ft}) = 68.451\ \mathrm{k \cdot ft}$$

$$0.29577(97.2\ \mathrm{k \cdot ft}) = 28.749\ \mathrm{k \cdot ft}$$

Reactions



Reactions

$$\sum M_B = 0$$

$$V_A = -\frac{(-33.657\ \text{k·ft} - 10.330\ \text{k·ft})}{11\ \text{ft}} + \frac{1}{2}(1\ \text{k/ft})(11\ \text{ft})\left(\frac{2}{3}\right) +$$

$$\frac{1}{2}(3\ \text{k/ft})(11\ \text{ft})\left(\frac{1}{3}\right) = 13.165\ \text{k}\uparrow$$

$$\sum M_A = 0$$

$$V_{B1} = \frac{(-33.657\ \text{k·ft} - 10.330\ \text{k·ft})}{11\ \text{ft}} + \frac{1}{2}(1\ \text{k})(11\ \text{ft})\left(\frac{1}{3}\right) +$$

$$\frac{1}{2}(3\ \text{k})(11\ \text{ft})\left(\frac{2}{3}\right) = 8.835\ \text{k}\uparrow$$

$$\sum M_C = 0$$

$$V_{B2} = -\frac{(10.330\ k\cdot ft + 64.425\ k\cdot ft)}{14\ ft} = 5.340\ k\downarrow$$

$$V_B = 8.835\ \text{k}\uparrow + 5.340\ \text{k}\downarrow = 3.495\ \text{k}\uparrow$$

Reactions

$$7 + 10 + 8 = 25\ ft$$

$$\sum M_D = 0$$

$$V_{C1} = -\frac{(-64.425\ \text{k·ft})}{25\ \text{ft}} + \frac{(10\ \text{k})(10\ \text{ft} + 8\ \text{ft})}{25\ \text{ft}} + \frac{(15\ \text{k})(8\ \text{ft})}{25\ \text{ft}} = 14.577\ \text{k}\uparrow$$

$$\sum M_B = 0$$

$$V_{C2} = \frac{(10.330\ \text{k·ft} + 64.425\ \text{k·ft})}{14\ \text{ft}} = 5.340\ \text{k}\uparrow$$

$$V_C = 5.340\ \text{k}\uparrow + 14.577\ \text{k}\uparrow = 19.917\ \text{k}\uparrow$$

$$\sum M_C = 0$$

$$V_D = \frac{(-64.425\ \text{k·ft})}{25\ \text{ft}} + \frac{(10\ \text{k})(7\ \text{ft})}{25\ \text{ft}} + \frac{(15\ \text{k})(7\ \text{ft} + 10\ \text{ft})}{25\ \text{ft}} = 10.423\ \text{k}\uparrow$$

Shear and Moment Diagrams

Shear and Moment Diagrams

Relation between shear and load –

1) The slope of the shear diagram at any point is equal to the intensity of the loading diagram at the same point (upward acting load is positive).

2) The change in shear between two points on the shear diagram is equal to the area under the loading diagram between the same two points.

Shear and Moment Diagrams

Relation between moment and shear –

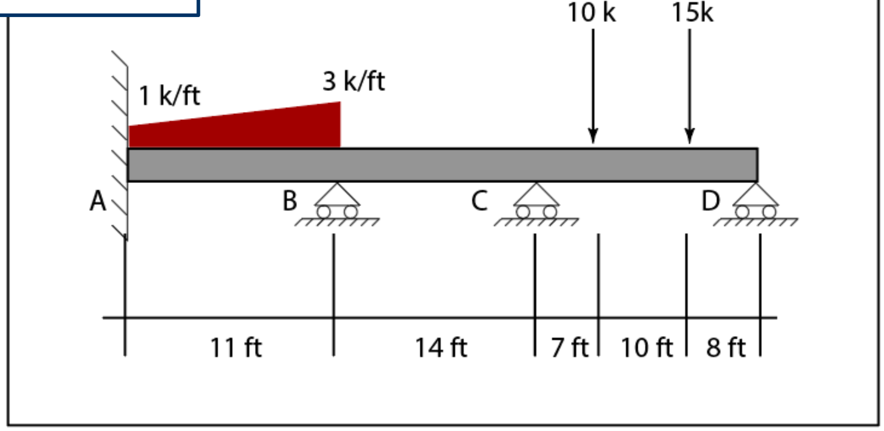
1) The slope of the moment diagram at any point is equal to the intensity of the shear diagram at the same point.
2) The change in moment between two points on the moment diagram is equal to the area under the shear diagram between the same two points.

Shear and Moment Diagram Example

$I_{AB} = 1{,}110\ in^4$

$I_{BC} = 1{,}240\ in^4$

$I_{CD} = 1{,}240\ in^4$



Shear and Moment Diagram Example

$V = 13.165\text{ k} - (1\text{ k/ft})x - \frac{1}{2}x\frac{2x}{11\text{ ft}} = 13.165\text{ k} - \left(x + \frac{x^2}{11}\right)\text{k}$

$V = 0 = 13.165\text{ k} - \left(x + \frac{x^2}{11}\right)\text{k}$

$x = 7.731\ ft$

$A = \int\left(13.165\text{ k} - \left(x + \frac{x^2}{11}\right)\text{k}\right)dx$

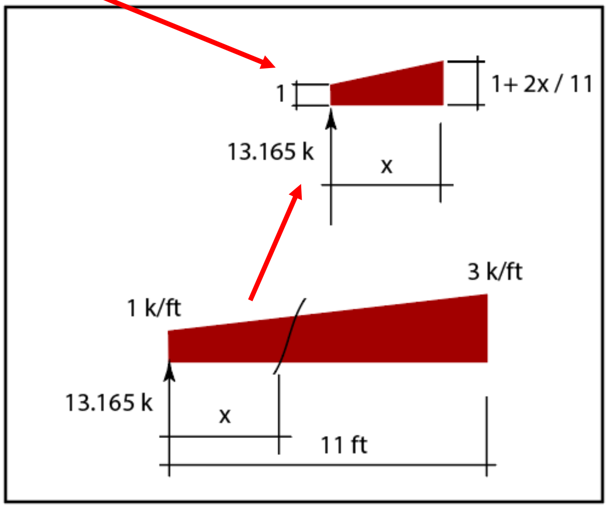
$= 13.165x - \left(\frac{x^2}{2} + \frac{x^3}{33}\right)$

$A_0^{7.731} = 57.892$

$A_0^{11} = 43.982$

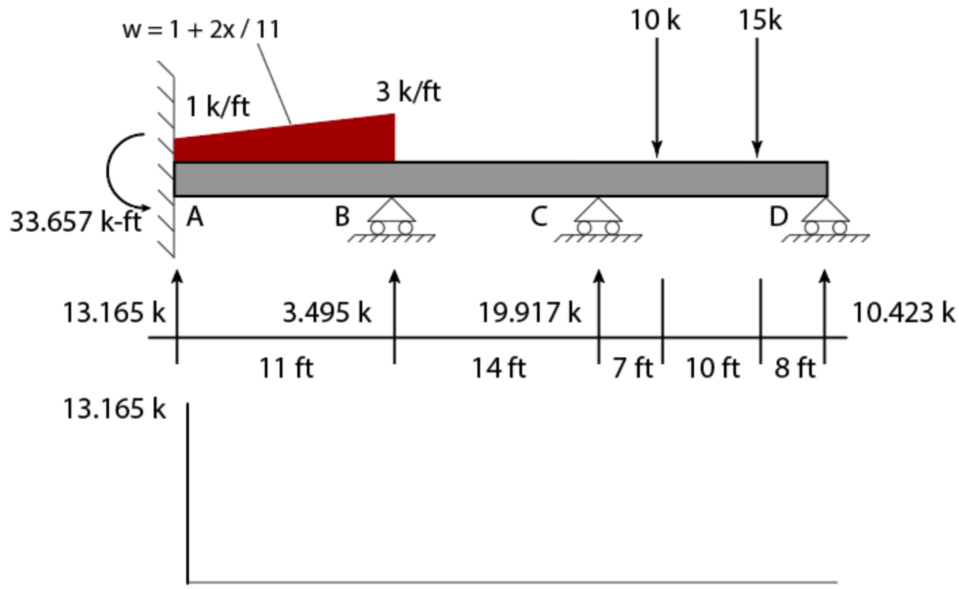
$43.982 - 57.892 = -13.910$

$\frac{(1\text{ k/ft}+3\text{ k/ft})(11\text{ ft})}{2} = 22\text{ k}$

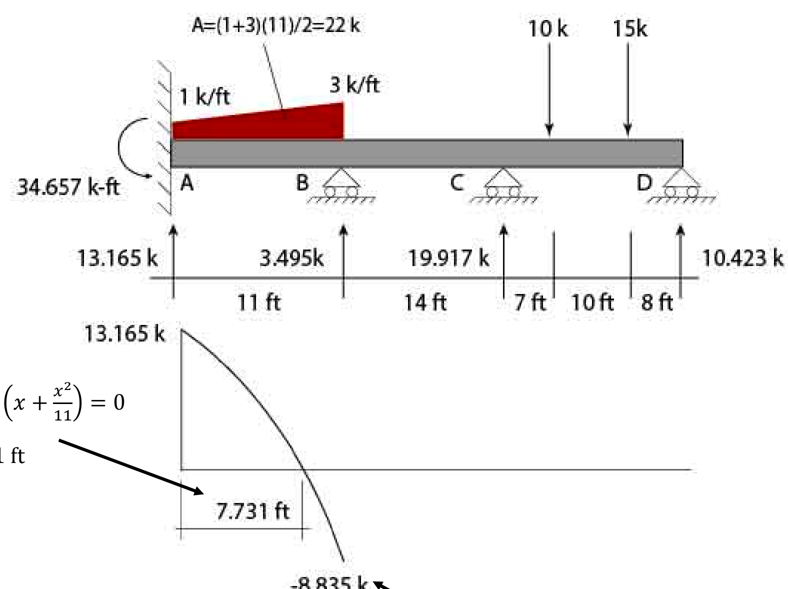


Shear Diagram Example – Continuous Beam

Beam Shear Diagram



Beam Shear Diagram

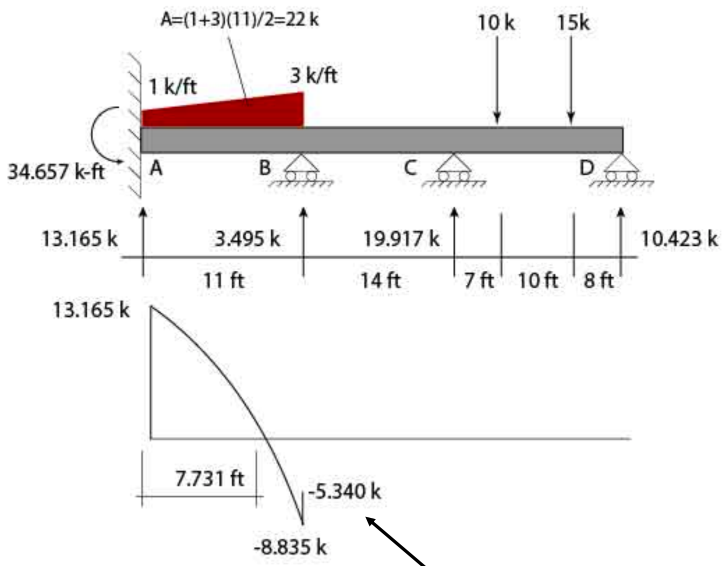


$$13.165 - \left(x + \frac{x^2}{11}\right) = 0$$

$x = 7.731$ ft

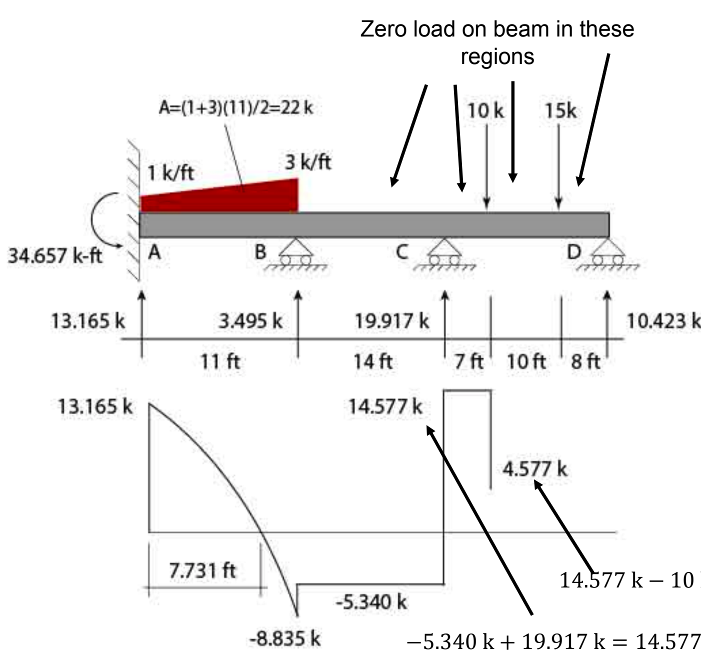
$13.165\ \text{k} - 22\ \text{k} = -8.835\ \text{k}$

Beam Shear Diagram



$-8.835 + 3.495\ \text{k} = -5.340\ \text{k}$

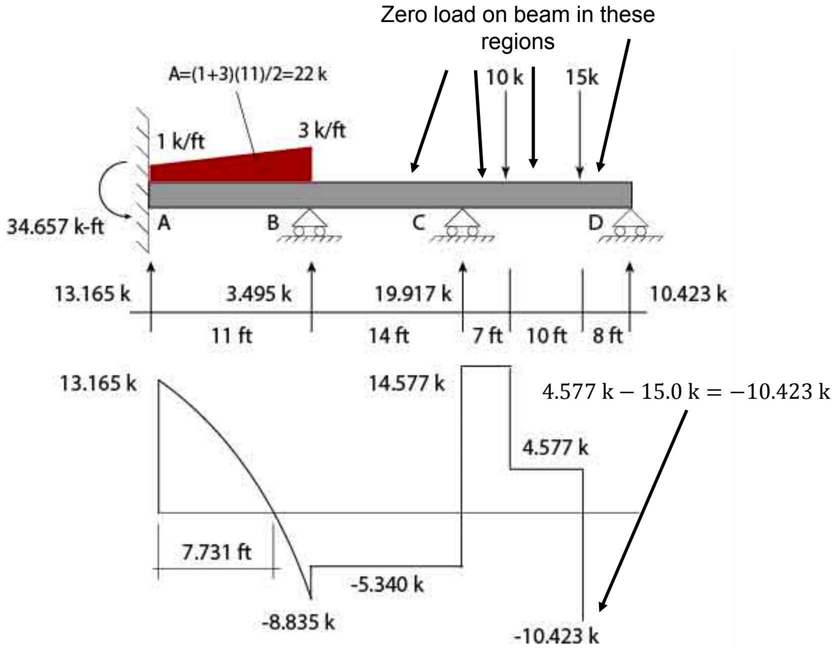
Beam Shear Diagram



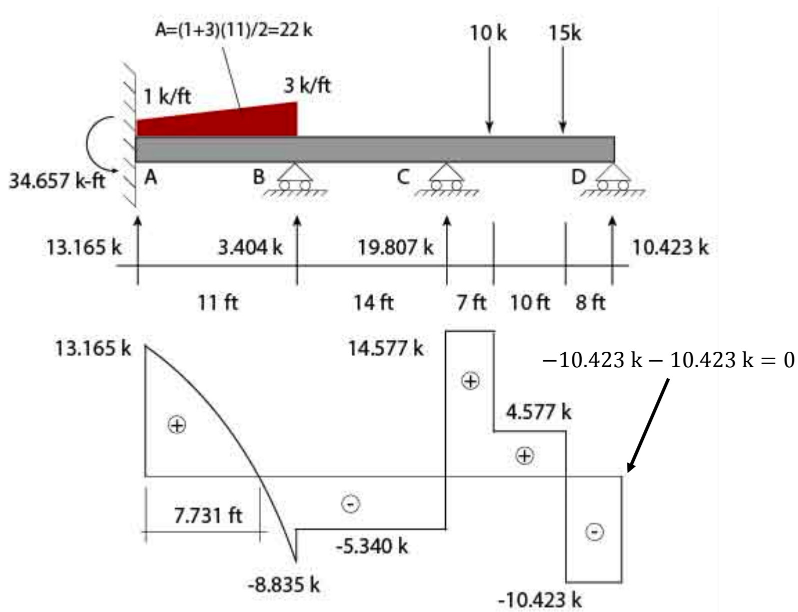
$14.577\ \text{k} - 10\ \text{k} = 4.577\ \text{k}$

$-5.340\ \text{k} + 19.917\ \text{k} = 14.577\ \text{k}$

Beam Shear Diagram

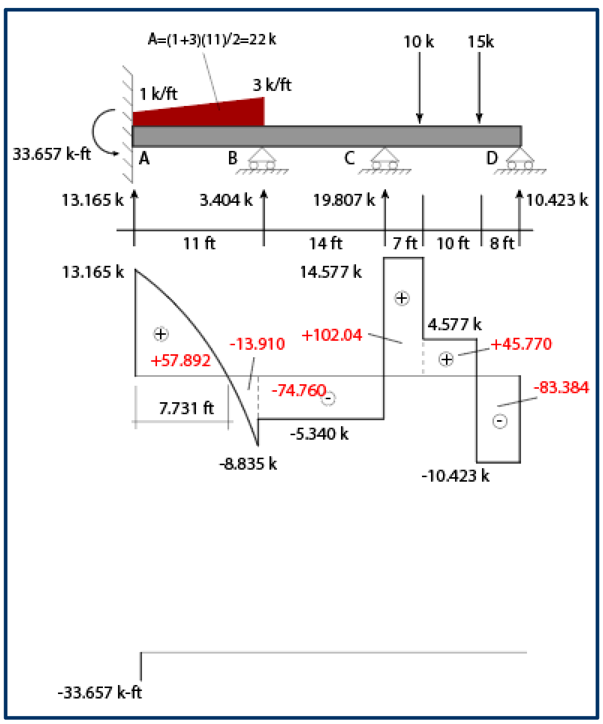


Beam Shear Diagram

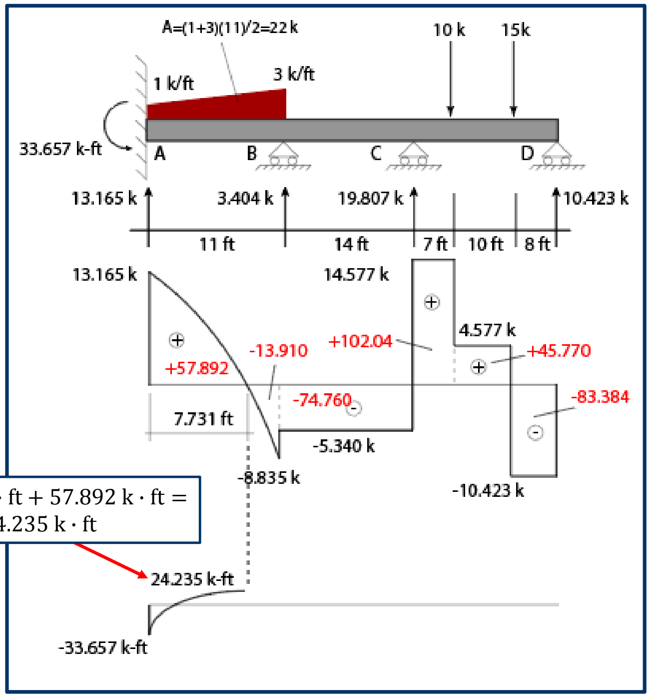


Moment Diagram Example – Continuous Beam

Beam Moment Diagram

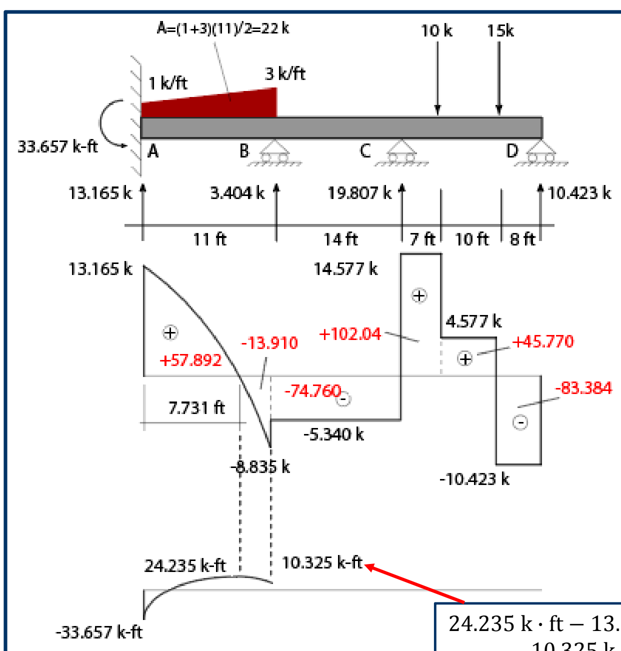


Beam Moment Diagram



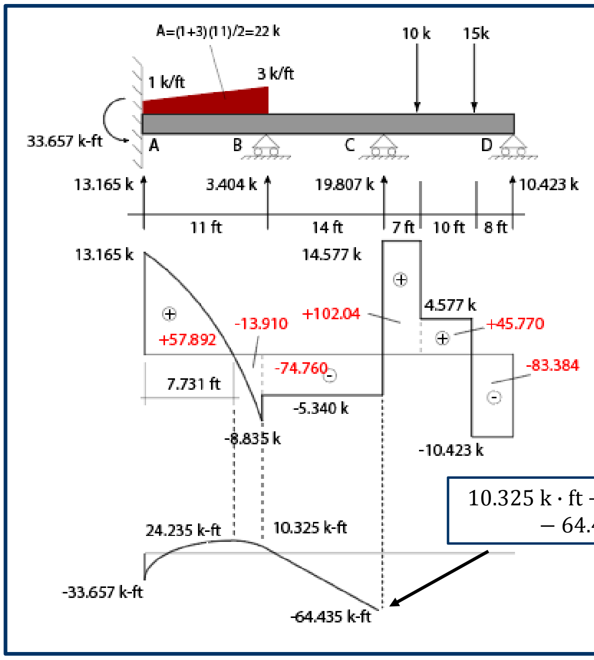
$$-33.657\ \mathrm{k \cdot ft} + 57.892\ \mathrm{k \cdot ft} = 24.235\ \mathrm{k \cdot ft}$$

Beam Moment Diagram



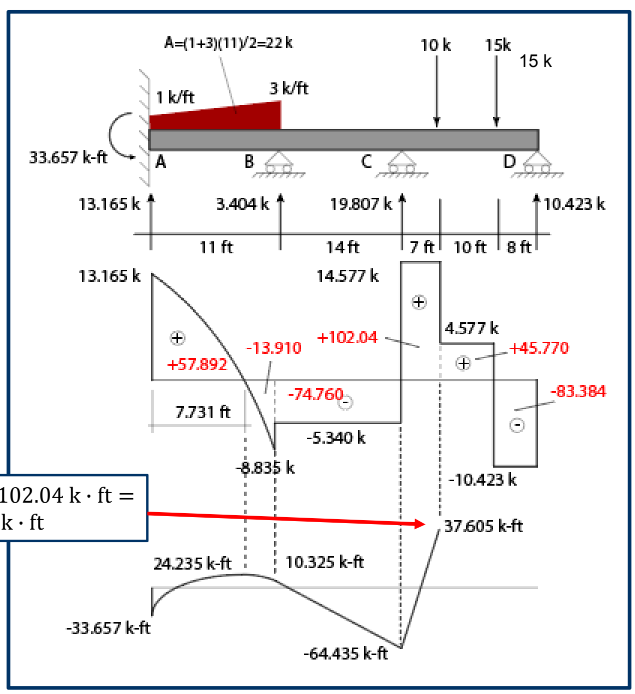
$$24.235\ \mathrm{k \cdot ft} - 13.910\ \mathrm{k \cdot ft} = 10.325\ \mathrm{k \cdot ft}$$

Beam Moment Diagram



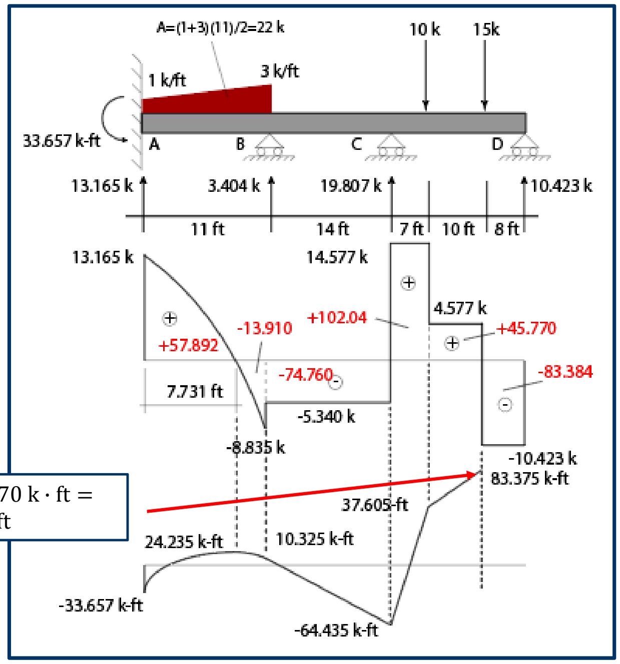
$$10.325\ \text{k} \cdot \text{ft} - 74.760\ \text{k} \cdot \text{ft} = -64.435\ \text{k} \cdot \text{ft}$$

Beam Moment Diagram



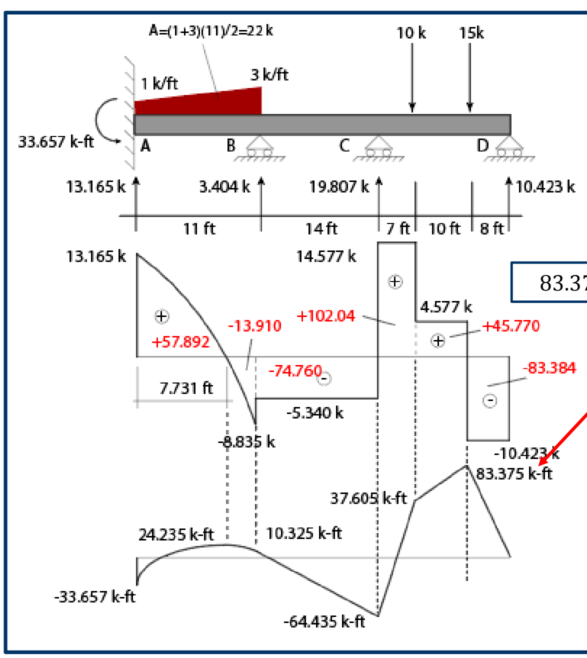
$$-64.435\ \text{k} \cdot \text{ft} - 102.04\ \text{k} \cdot \text{ft} = 37.605\ \text{k} \cdot \text{ft}$$

Beam Moment Diagram



37.605 k · ft + 45.770 k · ft =
83.375 k · ft

Beam Moment Diagram

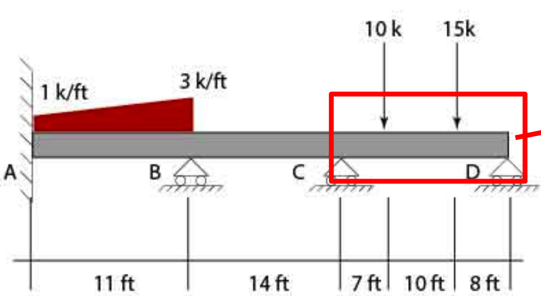


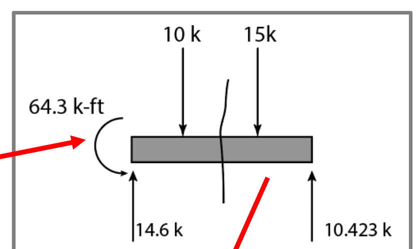
$83.375 \text{ k} \cdot \text{ft} - 83.384 \text{ k} \cdot \text{ft} \approx 0$

Moment and Shear by Section

Moment and Shear by Section

Determine the moment and shear midway between the 10 k and 15 k loads.





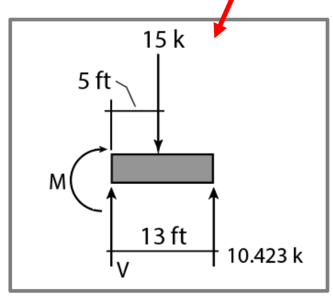
$V = 10.423\text{ k} - 15\text{ k} = 4.577\text{ k} \uparrow$

(positive shear by the BCS)

$M = (10.423\text{ k})(13\text{ ft}) - (15\text{ k})(5\text{ ft})$

$= 60.499\text{ k} - \text{ft CW}$

(positive moment by the BSC)



Questions

Thank you!

Questions?