# STRESS vs STRAIN

A lecture assembled for the course on
Statics and Strength of Materials

by Jason E. Charalambides PhD, PE, M.ASCE, AIA, ENV_SP

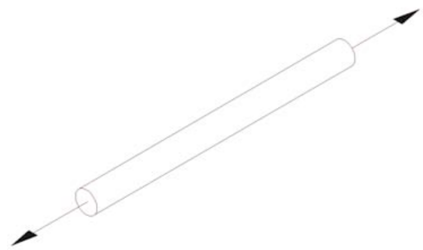


AVANT - GARDE ENGINEERING LLC
CONSULTING ARCHITECTS / ENGINEERS

# Definitions

# STRAIN

- A quantity that is difficult to visualize:
  - As elements are subjected to loading, they tend to change their form. They may
    - Elongate
    - Contract,
    - Deflect,
    - Warp, etc
  - Let's focus on the easiest concept to visualize, that of elongation



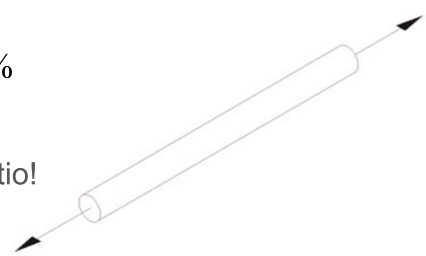


# STRAIN

- A quantity that is difficult to visualize:
  - An element is subjected to tension. It will elongate.
  - Let's assume it has a length $L_0=100^{in}$
  - After it is elongated its length became $L=102^{in}$
  - That allows us to say that it actually experienced strain. The value of strain that it has experienced is the division of the total elongation by the initial length.
  - The total elongation was 2in. That divided by the initial length ives the amount of strain that is symbolized by the greek letter "ε"

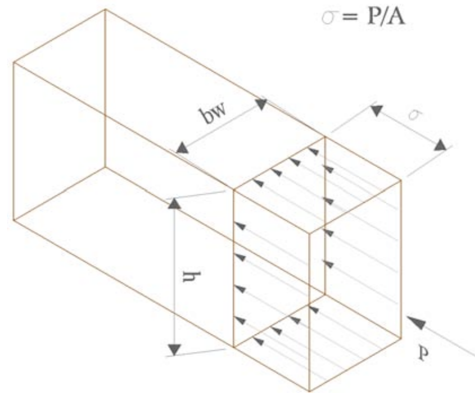
$$\varepsilon = \frac{L_{fin} - L_0}{L_0} \qquad \varepsilon = \frac{102^{in} - 100^{in}}{100^{in}} = 0.02 \quad \text{or} \quad 2\%$$

  - Note that there are no units in strain It is only a ratio!



# Stress

- The unit that is used should be a starting point:
  - If we consider that in the US we use psi or psf as units of stress, that can make everyone think that stress is pressure! However, the unit may deceive us.
  - Pounds (or Kips for that matter) per square foot or per square inch indicate that this is a quantity of load distributed over a surface. That is perfectly fine to consider; that is that we have a sum of uniformly distributed forces on an area, ...and these units are applicable to any stress.
  - The previous statement indicates that there are more than one type of stress
    - We will initially consider the “axial” stress,
    - And then we will address the “flexural” stress.
    - At a later time we will visit the “shear” stress
  - The 1st one, facilitates toward the next step of defining the Modulus of Elasticity “E”.

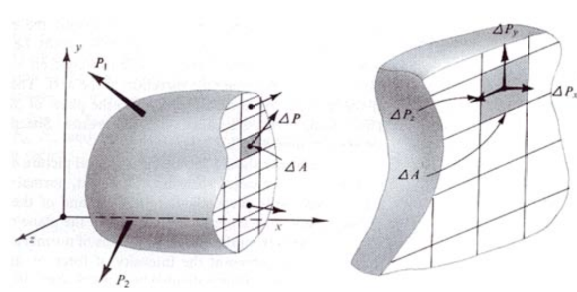



$$\sigma_{axial} = \frac{P}{A}$$



# Definition of Stress (Popov)

- More in Depth
  - In general, the internal forces acting on infinitesimal areas of a cut are of varying magnitudes and directions. These forces are vectorial in nature and they maintain the externally applied forces in equilibrium. In mechanics of solids it is particularly significant to determine the intensity of these forces on the various portions of the cut because resistance to deformation and to forces depends on these intensities.
  - In general, these intensities vary from point to point and are inclined with respect to the plane of the cut. It is customary to resolve these intensities perpendicular and parallel section investigated.
  - As an example, the components of a force ΔP acting on an area ΔA are shown in the figure. In this particular diagram, the cut through the body is perpendicular to the x axis, and the directions of $\Delta P_x$ and of the normal to ΔA coincide.

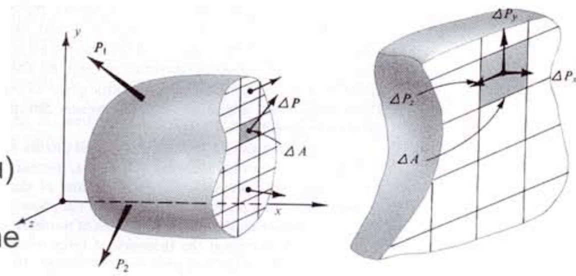


# DEFINITION OF STRESS (POPOV)

- More in Depth
  - Since the components of the intensity of force per unit area hold true only at a point, the mathematical definition of stress is

$$\tau_{xx} = \lim_{\Delta A \to 0} \frac{\Delta P_x}{\Delta A} \qquad \tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta P_y}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta P_z}{\Delta A}$$

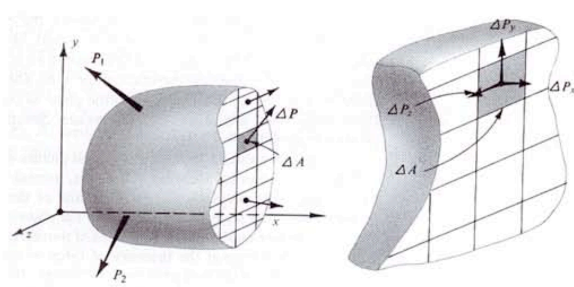
  - … where in all three cases, the first subscript of $\tau$ (tau) indicates that the perpendicular to the x axis is considered, and the second designates direction of the stress component. At a later period all possible combinations of subscripts for stress shall be discussed.
  - The intensity of the force perpendicular to or normal to the section is the normal stress at a point. It is customary to refer to normal stresses that cause traction or tension on the surface of a cut as tensile stresses.





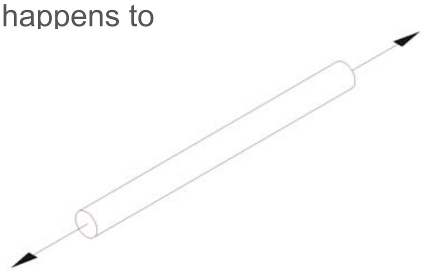

# DEFINITION OF STRESS (POPOV)

- More in Depth
  - Normal stresses will usually be designated by the letter $\sigma$ (sigma). The other components of the intensity of force act parallel to the plane of the elementary area. These components are called shearing stresses. Shearing stresses will be designated by -$\tau$.
  - Form a mental picture of the stresses called "normal" and those called "shearing." Normal stresses result from force components perpendicular to the plane of the cut, and shearing stresses result from components parallel to the plane of the cut.
  - Stresses multiplied by the respective areas on which they act give forces, and it is the sum of these forces at an imaginary cut that keeps a body in equilibrium.



## Modulus of Elasticity

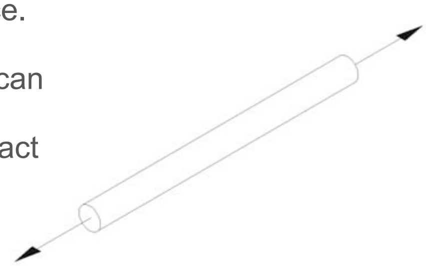
- After Thomas Young:
  - In a linear element which is axially loaded the main deformation will be viewed on that axis. To make an obvious example it is recommended that a steel rod of similar qualities of a standard #8 type reinforcement bar is used.
  - A steel rod of diameter of one inch and length of one foot is subjected to tension. That tension will elongate that rod. To keep numbers simple let's say that the elongation happens to be ¼".





## Modulus of Elasticity

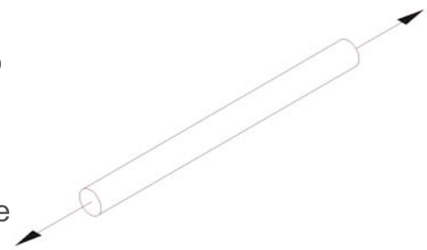
- Analyzing the effects:
  - That elongation divided by the original length signifies the Strain that the rod experienced as it was discussed earlier. In the case of tension, such as in the figure, it will be anticipated that the element will elongate along the direction of the force.
  - In case of compression however, the rod will bend toward a direction perpendicular to that of the force. But beyond that, let's consider the volume of an element. By elongating a one foot rod by ¼", we can not anticipate that we shall end up with an extra volume of ¼" by the area of the cross section! In fact the rod will become slightly thinner in order to compromise the gain in length so that the total volume will be maintained.



# MODULUS OF ELASTICITY

- Analyzing the effects:
  - Given now the concepts of Stress and Strain, one can refer to the strength of a material. The strain that the rod experienced (elongation/length) was caused by the stress that was applied (load/area).
  - It is easy to visualize that a material that was not as strong as steel, would have experienced a larger elongation if it was subjected to the same stress. Therefore it can be understood that this relation of Stress vs Strain should be characteristic to each material. That is a the Modulus of Elasticity of each material.
    - where E is the symbol of the elastic modulus, σ is the symbol of stress, and ε is the symbol of strain ...

$$E = \frac{\frac{Load}{Area}}{\frac{Elongation}{Original\ Length}} \quad or \quad E = \frac{\sigma}{\varepsilon}$$

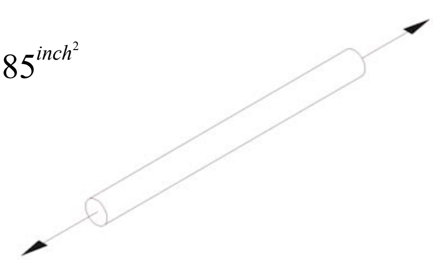




# MODULUS OF ELASTICITY

- Analyzing the effects:
  - Every material used in the construction industry has its own known Elastic Modulus, otherwise known as Young's modulus of elasticity after Thomas Young, the 19th century British scientist.
  - Steel has an elastic modulus of 29,000$^{ksi}$. Given that value, it is easy to deduce the following: For a diameter of 1" the area of the steel rod was approximately 0.785$^{sq_in}$.

$$A = \pi \cdot r^2 \quad thus \quad A = 3.14 \cdot \left(\frac{1}{2}^{in}\right)^2 = 0.785^{inch^2}$$



# Modulus of Elasticity

- Analyzing the effects:
  - For an elongation of ¼” on an initial length of 12”, the Strain experienced was about 0.021.
    - where "ε" is the symbol of strain

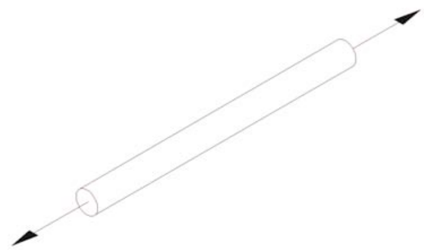
$$\varepsilon = \frac{l_{fin} - l_0}{l_0} \quad thus \quad \frac{0.25}{12} = 0.021$$

  - Having an elastic modulus of $29{,}000^{ksi}$ it can be deduced that the load that was applied in tension to that rod was equal to approximately $478^{k}$ or $478{,}065^{lbf}$.
  - By using the formula:

$$E = \frac{\frac{Load}{Area}}{\frac{Elongation}{Original\ Length}} \quad or \quad E = \frac{Stress}{Strain}, E = \frac{\sigma}{\varepsilon}$$

    - where “E” is the symbol of the elastic modulus, “σ” is the symbol of stress, and “ε” is the symbol of strain ...

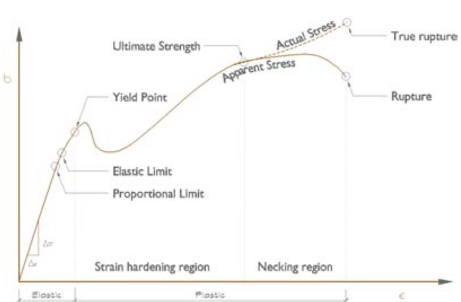
$$29000^{ksi} = \frac{\frac{Load}{0.785^{in^2}}}{\frac{0.25^{in}}{12^{in}}} \rightarrow Load = 474.5^{kip}$$





# Modulus of Elasticity

- Material behavioral conditions:
  - Special behavioral conditions of materials however are necessary to be studied. As slight stress can be applied on an element it deforms. Once the load is removed, the material can return to its original state. That is what is called elastic behavior.
  - There is however the chance that the material may deform permanently. It may not necessarily break but it can keep that deformation. That behavior is called plastic.
  - Each material carries different properties and behaves differently. The graph depicts how steel of approximately 0.2% carbon behaves under stress. The abscissa defines the scale of strain and the ordinates define the scale of stress.

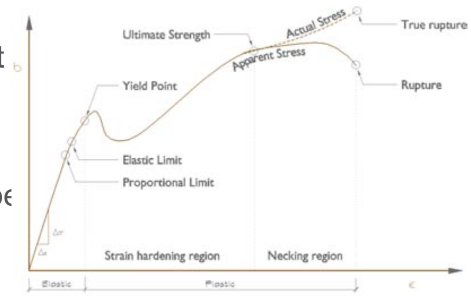





Also hyperlink to a Youtube video. This is from the British High School standard Advanced Level Physics for, but it is still good reference.https://www.youtube.com/watch?v=qvcpeA5pBH0

# MODULUS OF ELASTICITY

- Material behavioral conditions:
  - This stress vs strain relationship is key to understanding and predicting the behavior of any solid material.
  - As a visual aid let's consider the same rebar that was used at the beginning of this lecture and visualize how it behaves under tensile stress following this stress/strain curve. It is evident that as stress is applied on the material it will deform. Initially this relation of stress and deformation (or strain) will be perfectly proportional, i.e. y stress will cause x strain, 2(x) stress will cause 2(y) strain.

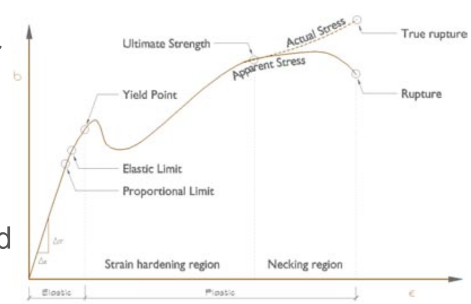





See also hyperlink to a Youtube videohttps://www.youtube.com/watch?v=CrJH3S7Bd00

# MODULUS OF ELASTICITY

- Material behavioral conditions:
  - At a certain point this perfect mathematical proportion will fail, and the material will start elongating slightly more. That will be the "Proportional limit" of this graph, and it will be followed by the "Elastic limit" which signifies the point where the behavior is no longer elastic but deformations start becoming permanent.
  - The "Yield Point" is the location in the graph at which a material elongates with minimal to negligible amount of added stress.
  - Beyond these points not only the deformation is permanent but inner stresses are generated within the material.

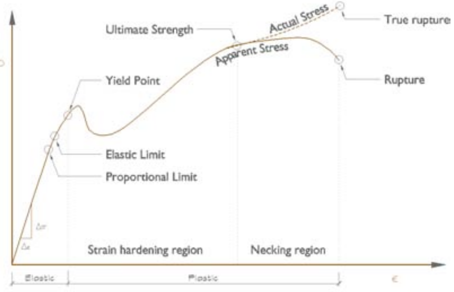





See also hyperlink to a Youtube video https://www.youtube.com/watch?v=RJAhRCxhg6M

# Modulus of Elasticity

- Material behavioral conditions:
  - Nevertheless designing structures where elements tend to reach within the plastic region is appropriate, and can certainly prove economical, as long as deformations do not surpass defined limits.
  - As indicated in the graph, the area between the Yield point and the Ultimate stress, is called "Strain hardening region" and that is because between those points, the strain that will be applied to the material will produce those inner stresses and make it harder.
  - The "Necking region" is the area where the material starts generating weaker points.






See also hyperlink to a Youtube video https://www.youtube.com/watch?v=RJAhRCxhg6M

# Modulus of Elasticity

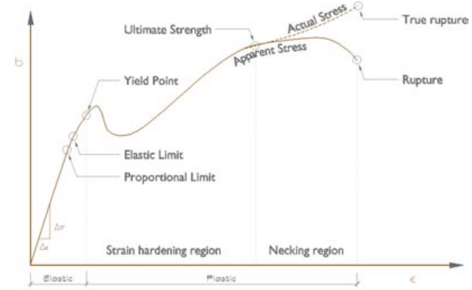
- Material behavioral conditions:
  - In the case of a rod it is anticipated that there will be a point that will produce particularly shorter circumference which will cause the final rupture.
  - It should also be noted that once a weaker point is established, probably due to the slightest imperfection in geometry, that weakness will be demonstrated and that weak point will rapidly develop the necking and the fracture.
    - *It is not very common to capture these important nuances on "youtube" videos. Some may incorporate effects such as necking and some may simply give a more summarized description.*



See also hyperlink to a Youtube video https://www.youtube.com/watch?v=2icfdyelx5M&feature=youtu.be

# Modulus of Elasticity

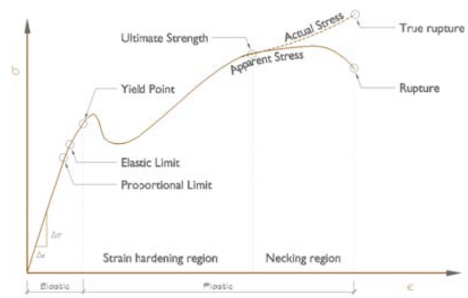
- Material behavioral conditions:
  - This particular relation of stress vs strain - as depicted in graph and the photographs of the rod under tension - is indicative of a "ductile" material. That is a material that has a very long plastic region which actually incorporates the strain hardening and the necking regions.
  - The great advantage to materials that are ductile is that they can deform under extreme stresses and not suffer a brittle rupture; a great advantage in structural design.





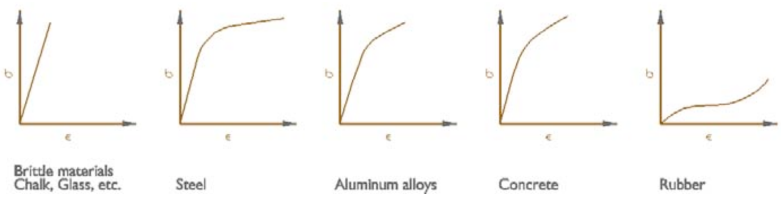

# Modulus of Elasticity

- Material behavioral conditions:
  - Note the split of “actual stress” and “apparent stress” as indicated in the right side of the graph. The term “apparent” is used because designers will consider the original cross section of an element.
  - However, if one considers the effect of “necking” as seen in the photograph, the cross sectional area is no longer the same, and thus the actual stress applies to that portion of the curve where the load is actually distributed in a smaller area.




# Modulus of Elasticity

- Material behavioral conditions:
  - The following figures indicate a more simplified set of uniaxial stress/strain relationships for a series of materials. As seen in the figure, a brittle material, such as glass can perform elastically but has no ductile qualities (not to scale).




  - Materials such as rubber may behave in a manner that would not be fitting for building structures. Aluminum alloys would be ideal for their structural qualities but particularly expensive. Structural steel and concrete reinforced with steel constitute the most popular materials for the construction industry on any discipline other than small scale residential buildings. Steel is very elastic and very ductile. Although concrete is a brittle substance, when reinforced with steel, the coupling of the two can produce a very successful mechanism.



# Modulus of Elasticity

- Typical values of E for Construction Materials:

**Typical values of Yield (Fy) and Ultimate (Fu) strength and Modulus of Elasticity (E) of Materials**

| | Metals | Yield Strength (Ksi) | Ultimate Strength (Ksi) | | E (Ksi) |
|---|---|---|---|---|---|
| Specified Values | Structural Steel, A 36 | 36* | 58 | | 29000 |
| | Structural Steel, A 572 (for ready shapes) | 50 | 65 | | 29000 |
| | Structural Steel, A 572 (for plates)** | 42 | 60 | | 29000 |
| | Structural Steel, A 992 | 50 | 65 | | 29000 |
| | Steel for concrete reinforcing, A 615 | 40-60 (most common) | 70-90 | | 29000 |
| | Steel for concrete reinforcing, A 617 | 40-60 (most common) | 70-89 | | 29000 |
| | Steel Strand, A886 (Seven-Wire for Prestressed Conc.) | 237.5 (Given as 0.95 Fu) | 250 | | 29000 |
| Variable/Approximate Values | Copper, annealed | 10 | 32 | | 16000 – 18000 |
| | Brass | 29 | 80 | | 14000 – 16000 |
| | Titanium | 135 | 150 | | 16000 |
| | **Brittle Materials** | **Tens Ult Strength (Ksi)** | **Comp Ult Strength (Ksi)** | | **E (Ksi)** |
| | Concrete | N/A | 2 – 8+ | | 2500 – 5000+ [1] |
| | Glass | N/A | 5 | | 7000 – 12000 |
| | Clay Masonry (brick) | N/A | 1.7 – 13.2 | | 1200 – 9250 [2] |
| | Concrete Masonry Unit (CMU) | N/A | 1.25 – 5.25 | | 1125 – 4725 [2] |
| | Carbon fiber | 580 | 232 | | 21750 |
| | Marble | N/A | 210 | | 6000 – 14000 |
| | **Wood** | **Tens Ult Strength (Ksi)** | **Comp Ult Strength (Ksi)** | **Comp Ult Strength (Ksi)** | **E (Ksi)** |
| | Douglas Fir | 1.2 – 2.2 (‖ to grain) | 0.1 – 1.8 (‖ to grain) | 0.3 – 0.45 (⊥ to grain) | 1700 |
| | Southern Pine | 1.2 – 2.8 (‖ to grain) | 0.9 – 2.2 (‖ to grain) | 0.26 – 0.45 (⊥ to grain) | 1600 |
| | Glued Laminated timber | 1.6 – 2.6 (‖ to grain) | 1.5 (‖ to grain) | 0.38 – 0.45 (⊥ to grain) | 1800 |

* 32 Ksi if plate is thicker than 8", ** For up to 8" thickness max

1. Depends on the ultimate strength of the concrete. Formula used is Ec=57000√f'c where f'c is the value of ultimate strength of concrete in psi.
2. Depends on the ultimate strength of the clay. Formula used is Em=700f'm or 900f'm for clay brick and concrete masonry units (cmu) respectively, where f'm is the value of ultimate strength of masonry in psi.