Euler Column Buckling Theory; Effects of Residual Stresses

MORGAN STATE UNIVERSITY
SCHOOL OF ARCHITECTURE AND PLANNING

LECTURE III

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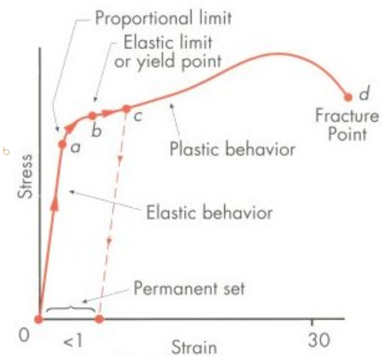
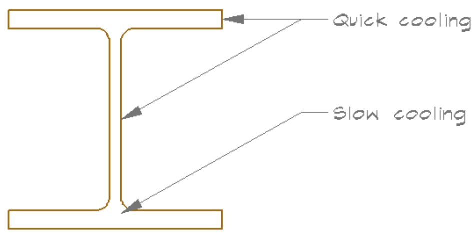
What is Residual Stress?

- Although Steel is considered to be homogenous material, the process of fabrication allows portions of an element to form differently than others.
  - Rolled shapes may go through the rollers hot or cold
  - For cold rolled, it is understood that the steel is exposed to stresses that bring it into its plastic region to have permanent deformation. When an element is stressed to the point that it deforms and it does not return to its original form, portion of the energy that was received remains within it. That is translated to a stress that is carried within the structure of that element. That is residual stress.



What is Residual Stress?

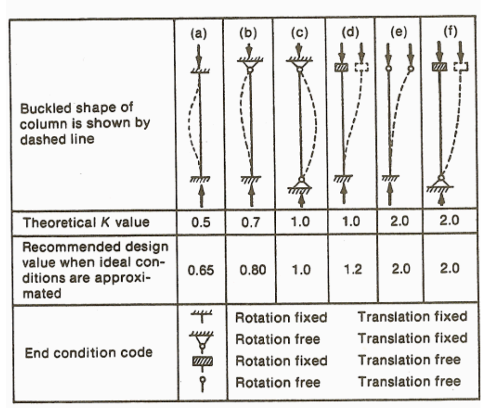
- Continuing on causes of residual stress,
  - For hot rolled elements, the residual stresses are not of the same scale, but they can still be significant, especially on larger elements. Those are developed by the uneven rate of cooling in different areas. Areas that cool quicker, such as the middle of the web or the tips of flanges of a W-Section tend to have residual compressive stress, whilst areas that cool slower, such as the intersections of web and flange, develop residual tensile stress.



What is Residual Stress?

- Residual stress is addressed as drops of the scale of 10-15ksi may have an effect on the reliability of a design. The greatest reductions in strength are noticed in columns that have a "Slenderness ratio" between 70 – 90.
- Slenderness ratio is the result of the division of the effective length "L" over the radius of gyration "r"
  – The former is essentially the length of the element multiplied by the "k" factor
  – The latter is a factor that can be found in the AISC User's Manual.

$$\lambda = \frac{L}{r}$$

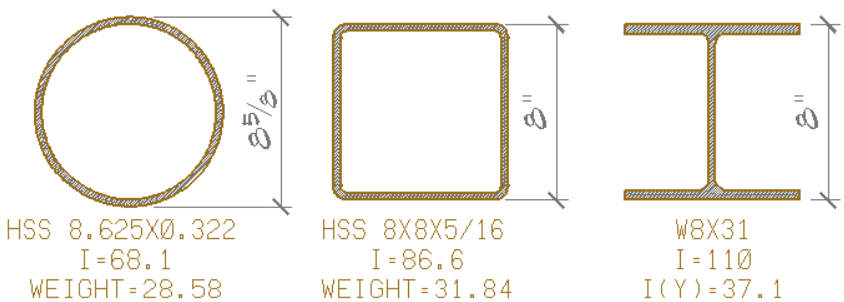


Buckled shape of column is shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.0	1.2	2.0	2.0

End condition code		
	Rotation fixed	Translation fixed
	Rotation free	Translation fixed
	Rotation fixed	Translation free
	Rotation free	Translation free

Qualities of Different Shapes

- Some shapes are more practical to fabricate
- Some shapes have better response to compressive loads
- Some shapes handle bending better



Qualities of Different Shapes

- Other advantages / disadvantages:
  - Round columns have less surface to paint or fireproof
  - Round columns have constanr “r” and “I” values
  - They have better torsional resistance and less resistance to wind loads
  - Square or round columns are more economical and efficient unless moments play an important role, especially in larger structures
  - Hollow columns are easier to keep clean, but also easier to be exposed to corrosion over W, S, or T shapes

Buckling

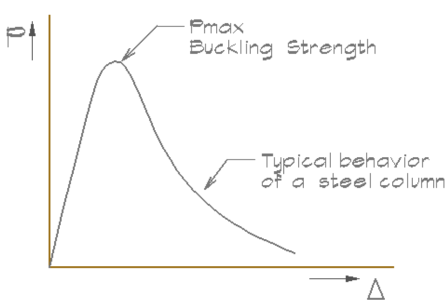
- Main difference of a compressive axially loaded member over a tensile axially loaded member is "buckling."
  - That is the "loss of compressive load carrying capacity resulting from a change in the geometric formation of a member"
  - A slight defect, or a slight eccentricity, may generate the deflection that will lead to a column's failure

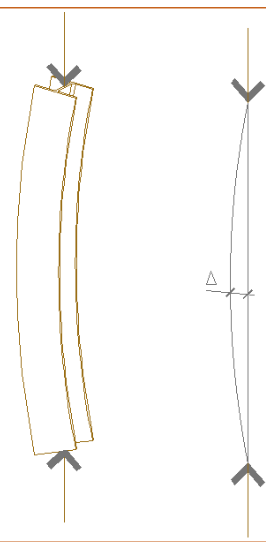
NY City Transit released photos of Cortlandt St station the week of 24 September 2001. Subway columns are buckled from the impact, near the center of the station.All of this is gone.Source: http://www.columbia.edu/~brennan/abandoned/Cort-damage-09.jpg

Buckling

- Design equation:
  - The ultimate axial load is equal or less than the factored nominal strength

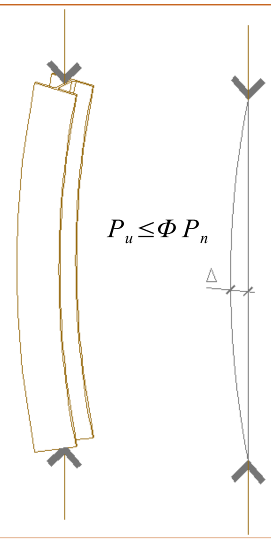
$$P_u \leq \Phi P_n$$





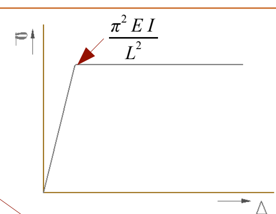
Strength of Isolated Columns

- Euler's solution to theoretical elastic behavior:
  - Based on the following assumptions:
    - The column is pin connected
    - It is perfectly straight
    - Load is perfectly axial
    - Behaves elastically and does not yield
    - No residual stresses
    - Bends and buckles about a principal axis w/out torsion.

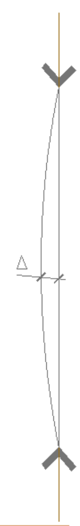


Strength of Isolated Columns

- Euler's elastic buckling:
  - The buckled shape resembles ½ a sinusoidal distribution.
  - The buckling load Pe is proportional to the Moment of Inertia of the element
  - The buckling load is inversely proportional to the square value of the length of the element (L^2)
    - The longer the element the more susceptible to buckling
  - Buckling is proportional to the Young's modulus of elasticity but independent of the yield strength of the material (Fy)

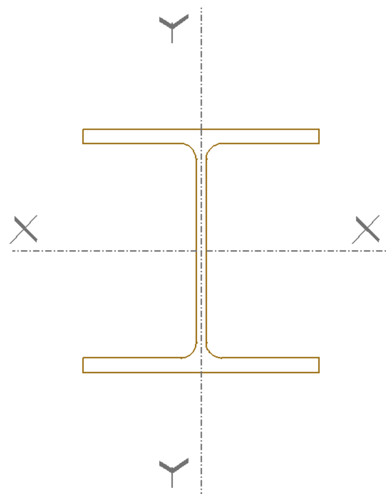


$$P_E = \frac{\pi^2 E I}{L^2}$$



Consider the Effects of Axial Load on a W Shape

- X & Y are Principal Axes
- I_max = Ix (Strong Axis)
  - Higher Moment of Inertia
- I_min = Iy (Weak Axis)
  - Lower Moment of Inertia
- Buckling will be about the (y) weak axis, unless the weak axis is restrained.



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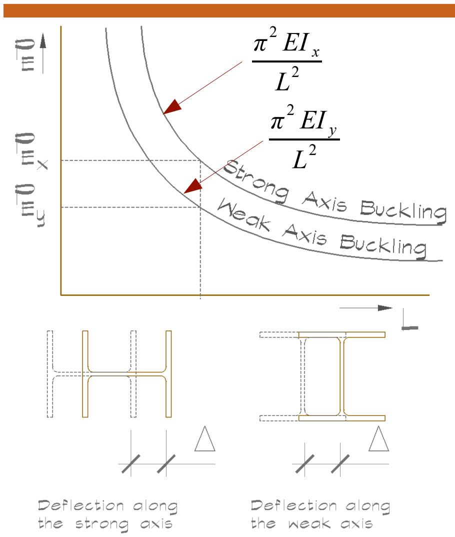


Addressing Euler's Buckling Load w.r.t. Stress

- Stress can be viewed as Load (P) divided by Area (A)
  - If we divide both sides of Euler's equation by Area →

$$\frac{P_E}{A} = \frac{\pi^2 E I}{A L^2}$$

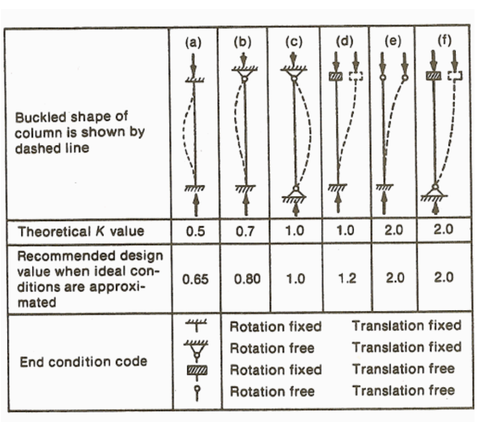
  - But since the radius of gyration (r)is equal to the square root of moment of inertia (I) divided by area (A)...

$$r = \sqrt{\frac{I}{A}} \quad OR \quad r^2 = \frac{I}{A}$$

  - Thus Euler's elastic buckling stress is:

$$F_E = \frac{\pi^2 E r^2}{L^2} \quad OR \quad F_E = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$$

Bringing Back the Factor of Boundary Conditions



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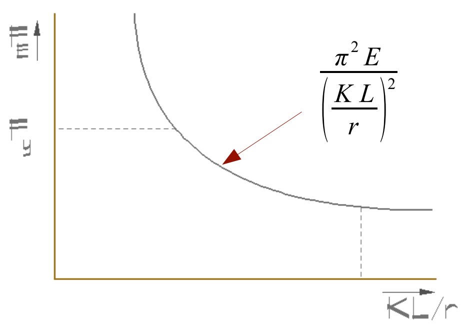
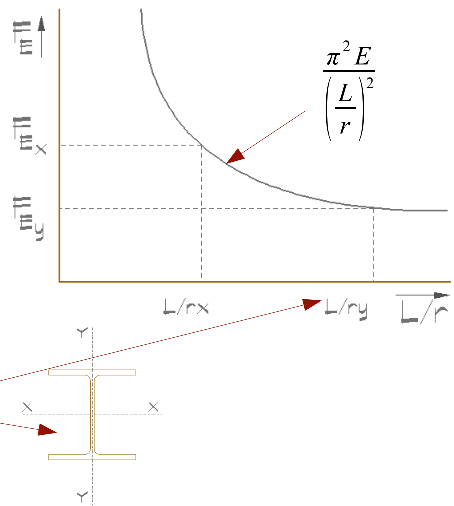


Load Deflection Behavior

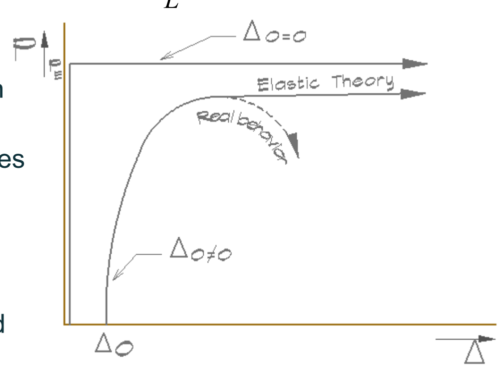
- The Length "L" divided by the radius of gyration "r" is the slenderness ration of a column
- By plotting a graph of the stress in the ordinates and the slenderness ratio in the abscissa…
- r minimum corresponds to I minimum
- (L/r) max corresponds to r min
- Weaker axis of W section (lower I) controls in buckling



Load – Deflection Behavior

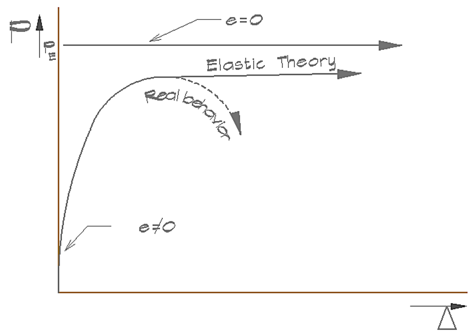
- Effects of deflection
  - The column bends as soon as it is loaded, i.e. buckling is not an instantaneous effect.
  - There is already stress in the column before loading.
  - Based on Elastic theory (material does not yield) P is asymptotic to PE. No loss of strength due to deflection
  - In reality material yields, and the additional bending stress from deflection causes earlier yielding and loss of strength
  - Small deflection → little loss of strength, and vice versa

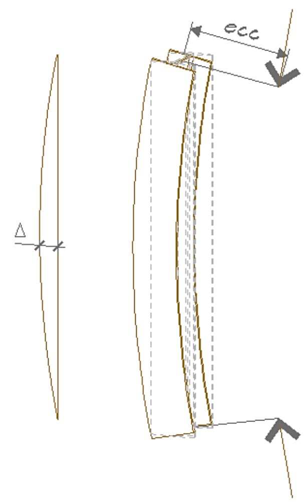
$$P_E = \frac{\pi^2 E\, I}{L^2}$$



Effects of Load Eccentricity

- Effects of eccentricity (eccentrically applied loading) are identical to the effects of Δ0





Definition and Effects of Residual Stresses

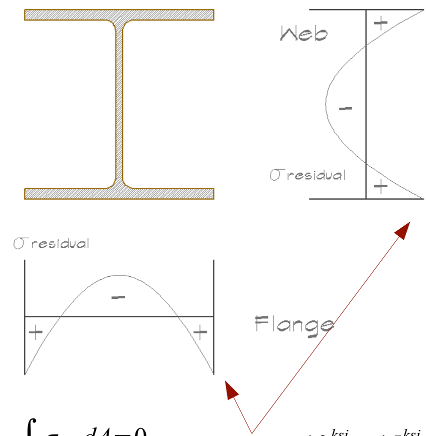
- Residual stress definition
  - They are developed within a member during manufacturing.
  - They are self equilibrating (their sum is zero) as they exist in the absence of any external loading.
  - They are generated by:
    - Uneven cooling of hot rolled elements
    - Uneven cooling of welded built up elements
    - Cold forming or cambering of members
    - Punching, shearing, or cutting
    - Welding at specific points

  (Along entire length: uneven cooling of hot rolled elements; uneven cooling of welded built up elements; cold forming or cambering of members. Localized: punching, shearing, or cutting; welding at specific points.)

- Effect
  - Varying behavior at specific points, areas, or along an axis

Causes for Residual Stress

- Cooling of Rolled Shapes
  - The uneven rate of cooling of the cross section.
  - Member is allowed to coll slowly. Some portions (e.g. flange tips) cool quicker because they have more surface exposed to air
  - Typically residual stresses are:
    - Quick cool → Compressive
    - Slow cool → Tensile
  - Residual stresses are normal, not shear stresses
  - Residual stresses are higher on welded shapes than rolled shapes

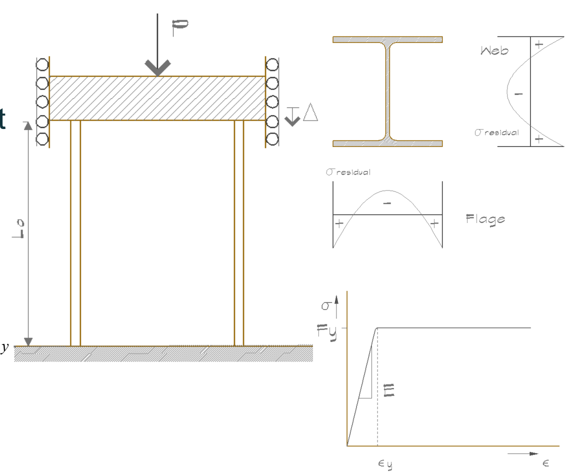


$$\int \sigma_{res}\, dA = 0$$

$$\sigma_{res_{max}} \approx 10^{ksi}\ to\ 15^{ksi}$$

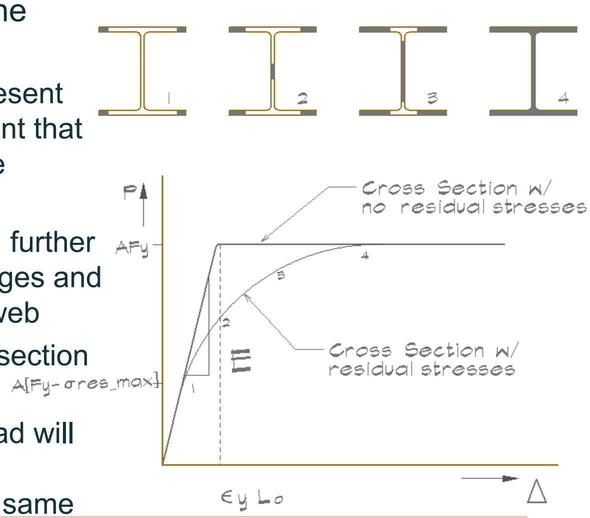
The Stump Column Test

- Residual stresses reduce the stiffness of a member
  - Investigated by testing a “stub column,” i.e. too short to buckle.
  - If there are no residual stresses, all fibers of the cross section yield simultaneously when the applied load reaches $A \times F_y$
    - ...i.e. when the applied stress reaches $\frac{P}{A} = F_y$



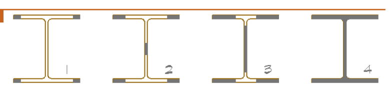
The Stump Column Test

- Residual stresses reduce the stiffness of a member
  - If residual stresses are present the first parts of the element that will yield are the tips of the flanges.
  - Then the effect will extend further beyond the tips of the flanges and the central portion of the web
  - And eventually the whole section will yield
  - Although the maximum load will still be $P = A \times F_y$, the load deflection curve is not the same



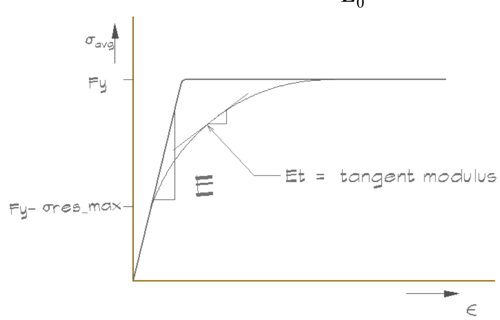
Using the Tangent Modulus

- The P vs Δ curve..
  - Can be replotted in the form of average applied stress vs strain.
  - Et is a measure of the cross section's average stiffness, considering that portions of the cross section are yielded, while others are still elastic



$$\sigma_{avg} = \frac{P}{A} = average\ applied\ stress$$

$$\varepsilon = \frac{\Delta}{L_0} = applied\ strain$$

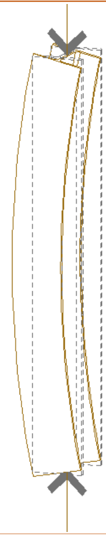


Effect of Residual Stresses on Column Strength

- Consider a column that is initially perfectly straight
  - The buckling load can be obtained using the “tangent modulus theory” that was just discussed.
  - The buckling load can be computed using Euler's equation, but replacing E with Et.
  - The resulting buckling load is referred to as “Tangent Modulus buckling load”
  - Similarly we can define the “Tangent Modulus buckling stress”
  - This leads to two classes of buckling:
    - Elastic, and
    - Inelastic

$$P_t = \frac{\pi^2 E_t I}{(K \times L)^2}$$

$$F_t = \frac{P_t}{A} = \frac{\pi^2 E_t}{\left(\frac{K \times L}{r}\right)^2}$$



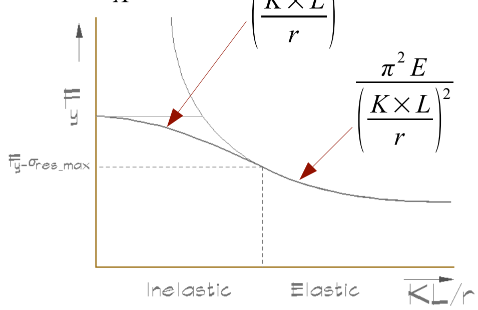
Elastic / Inelastic Buckling

- Elastic
  - No yielding of the cross section occurs prior to buckling and Et=E at buckling
  - $F_E = \frac{\pi^2 E}{\left(\frac{K \times L}{r}\right)^2}$ predicts buckling
- Inelastic
  - Yielding occurs on portions of the cross section prior to buckling and there is loss of stiffness.
  - $F_T = \frac{\pi^2 E_T}{\left(\frac{K \times L}{r}\right)^2}$ predicts buckling

$$F_t = \frac{P_t}{A}$$

$$\frac{\pi^2 E_t}{\left(\frac{K \times L}{r}\right)^2}$$

$$\frac{\pi^2 E}{\left(\frac{K \times L}{r}\right)^2}$$



Strength of Columns

- The discussion that was held until now indicates that the strength of a column is dependent upon the following:
  - Slenderness $\lambda = \frac{L}{r}$
  - End restraint $K\ factor$
  - Eccentricity (loading or form)
  - Yielding and Residual stresses

  Must consider variability in these factors

- All of the above factors need to be addressed in order to determine the strength of a real column but there are two approaches to do that:
  - Experiments (*we shall not engage in this!*)
  - Numerical Analysis

Numerical Method of Analysis

- The AISC provides a series of equations that allow us to compute the column strength:
  - Nominal compressive strength $P_n = A_g \times F_{cr}$

    where Ag is Area gross, and Fcr is the critical or buckling stress
  - Design compressive strength $\Phi P_n = \Phi \times A_g \times F_{cr}$

    where Φ is the factor of safety and it is equal to 0.9
  - Criterion for design $P_u \leq \Phi \times P_n$

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  - Criterion for design $P_u \leq \Phi \times P_n$

Computing the Nominal Compressive Strength

- The definition is: $F_E = \dfrac{\pi^2 E}{\left(\dfrac{KL}{r}\right)^2}$ for Euler's buckling Stress
  - When $\dfrac{KL}{r} \leq 4.71\sqrt{\dfrac{E}{F_y}}$ OR $\dfrac{F_y}{F_e} \leq 2.25$ → Inelastic Buckling
    - $F_{cr} = \left[0.658^{\left(\frac{F_y}{F_E}\right)}\right] F_y$ (E3-2)
  - When $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{F_y}}$ OR $\dfrac{F_y}{F_e} > 2.25$ → Elastic Buckling
    - $F_{cr} = 0.877 F_e$ (E3-3)

    For elastic buckling we adopt 0.877 times the Euler's formula, accounting for geometric imperfections.
    - Note that Fcr is independent of Fy

The Red Line for Elastic / Inelastic

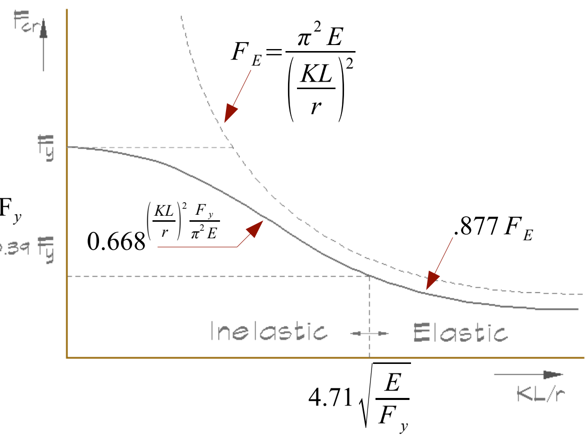
- Inelastic Buckling
  - $$F_{cr} = \left[0.658^{\left(\frac{F_y}{F_E}\right)}\right] F_y = \left[0.658^{\left(\frac{F_y}{\frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}}\right)}\right] F_y$$
  - Note: As KL/r→ 0 Fcr→Fy
- Elastic Buckling
  - $$F_{cr} = 0.877 F_E = 0.877 \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

The Red Line for Elastic / Inelastic

- Taking the limit: $\frac{KL}{r} = 4.71\sqrt{\frac{E}{F_y}}$ for for 36 and 50 grade steel
  - $4.71\sqrt{\frac{29{,}000^{ksi}}{36^{ksi}}} = 133.681$ → 36 ksi gives 133.7
  - $4.71\sqrt{\frac{29{,}000^{ksi}}{50^{ksi}}} = 113.432$ → 50 ksi gives 113
    - Take a typical column e.g W12x53, of Lu=12' and r=2.48”. With K of 1.0 this W section will give KL/r=58. In either of the grades of steel this column will buckle in the inelastic range

Relation of Critical Stress and Slenderness Ratio

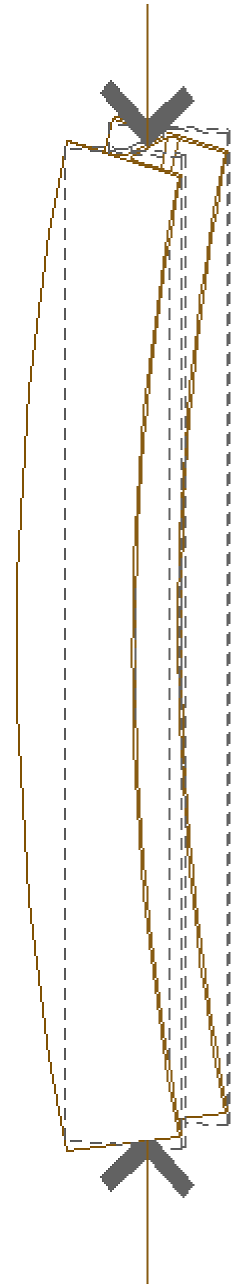
- At $\frac{KL}{r} = 4.71\sqrt{\frac{E}{F_y}}$
- $F_E = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 E F_y}{4.71^2 E} = 0.44F_y$
- Thus $\frac{F_y}{F_E} = 2.25$
- $F_{cr} = 0.877F_E = 0.877 * .44F_y = 0.39F_y$
- The transition from elastic to inelastic occurs at an applied axial compression stress of 0.39Fy



Basic Procedures for Analysis

- Given the shape, the K factor, the Length and the type of steel can we determine the Φpn?
  - Well, ….. $\Phi P_n = \Phi \times A_g \times F_{cr}$
  - Fcr depends uponKL/r and Fy,
  - The r we chose is the weaker one →(KL/r) max controls
    - Compute (KL/r)x and (KL/r)y, and larger will govern

Wide Flange Shape subjected to axial loading



Problem Statement:

Determine the capacity in axial loading of the given W shape. The element is pinned at top and bottom with no intermediate bracing, therefore having an unbraced length of 15ft in both directions.Use A992 steel

Area	$A_g := 15.8in^2$	Young's Modulus of Elasticity	$E := 29000ksi$
Length:	$L_u := 15ft$	Bolt diameter	$d_b := 0.875in$
radius of gyration y	$r_y := 2.56in$	Yield Stress:	$F_y := 50ksi$
radius of gyration x	$r_x := 4.37in$	Ultimate Strength:	$F_u := 65ksi$
K factor	$K := 1$	Factor of Safety phi	$\phi := 0.9$

Solution Method 1: Using Chapter E Equations:

1) Determining the governing **slenderness ratio**

$$\lambda_x := \frac{K \cdot L_u}{r_x} \qquad \frac{\left(15ft \cdot 12\frac{in}{ft}\right)}{4.37in} \qquad \lambda_x = 41.19$$

$$\lambda_y := \frac{K \cdot L_u}{r_y} \qquad \frac{\left(15ft \cdot 12\frac{in}{ft}\right)}{2.56in} \qquad \lambda_y = 70.313$$

$$r := \min(r_x, r_y) \qquad r = 2.56 \cdot in$$ governing radius of gyration

The above was already obvious but it was carried on just to "academicallly" justify the numbers

2) Calculating **Euler's Buckling Stress**

$$F_E := \frac{\pi^2 \cdot E}{\left(\frac{K \cdot L_u}{r}\right)^2} \qquad \frac{3.14^2 \cdot 29000ksi}{\left(\frac{15ft \cdot 12\frac{in}{ft}}{2.56in}\right)^2} \qquad F_E = 57.894 \cdot ksi$$

3) Determining if the buckling will be elastic or inelastic.

When $\frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}}$ OR $\frac{F_y}{F_e} \le 2.25$ → Inelastic Buckling

$$Buckling := \text{if}\left[\left(\frac{K \cdot L_u}{r}\right) \le 4.71\sqrt{\frac{E}{F_y}}, \text{"Inelastic"}, \text{"Elastic"}\right] \qquad Buckling = \text{"Inelastic"}$$

Alternatively we can also follow the process below:

$$\frac{F_y}{F_E} = 0.864 \qquad Buckling := \text{if}\left[\left(\frac{F_y}{F_E}\right) \le 2.25, \text{"Inelastic"}, \text{"Elastic"}\right] \qquad Buckling = \text{"Inelastic"}$$

4) Calculating the Buckling Stress (Fcr) and the load capacity of the section:

$$F_{cr} := \left[0.658^{\left(\frac{F_y}{F_E}\right)}\right] \cdot F_y \qquad \left[0.658^{\left(\frac{50ksi}{57.89ksi}\right)}\right] \cdot 50ksi \qquad F_{cr} = 34.832 \cdot ksi$$

$$\Phi P_n := \phi \cdot A_g \cdot F_{cr} \qquad 0.9 \cdot 15.8in^2 \cdot 34.832ksi \qquad \Phi P_n = 495.314 \cdot kip$$

Solution Method 2: Using Table 4-22:

1) Determining the governing **slenderness ratio**

$$\lambda_y := \frac{K \cdot L_u}{r_y} \qquad \frac{\left(16ft \cdot 12 \frac{in}{ft}\right)}{2.56in} \qquad \lambda_y = 70.313$$

2) Using table we locate the KL/r value corresponding to the Fy used for factrized critical stress:

The value indicated would be between 31.1 and 31.4. Let's take 31.3

$$\Phi F_{cr} := 31.3ksi$$

Note: From our previous calculations:

$$F_{cr} = 34.832 \cdot ksi$$

Therefore:

$$\phi \cdot F_{cr} = 31.349 \cdot ksi$$

3) Calculating the capacity of the element:

$$\Phi P_n := \phi \cdot A_g \cdot F_{cr}$$

$$0.9 \cdot 15.8in^2 \cdot 34.832ksi$$

$$\Phi P_n = 495.314 \cdot kip$$

Table 4-22 (continued)

Available Critical Stress for Compression Members

$F_y = 35$ ksi			$F_y = 36$ ksi			$F_y = 42$ ksi			$F_y = 46$ ksi			$F_y = 50$ ksi		
$\frac{KL}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{KL}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{KL}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{KL}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{KL}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD
41	19.2	28.9	41	19.7	29.7	41	22.7	34.1	41	24.6	37.0	41	26.5	39.8
42	19.2	28.8	42	19.6	29.5	42	22.6	33.9	42	24.5	36.8	42	26.3	39.5
43	19.1	28.7	43	19.6	29.4	43	22.5	33.7	43	24.3	36.6	43	26.2	39.3
44	19.0	28.5	44	19.5	29.3	44	22.3	33.6	44	24.2	36.3	44	26.0	39.1
45	18.9	28.4	45	19.4	29.1	45	22.2	33.4	45	24.0	36.1	45	25.8	38.8
46	18.8	28.3	46	19.3	29.0	46	22.1	33.2	46	23.9	35.9	46	25.6	38.5
47	18.7	28.1	47	19.2	28.9	47	22.0	33.0	47	23.8	35.7	47	25.5	38.3
48	18.6	28.0	48	19.1	28.7	48	21.8	32.8	48	23.6	35.4	48	25.3	38.0
49	18.5	27.9	49	19.0	28.5	49	21.7	32.6	49	23.4	35.2	49	25.1	37.7
50	18.4	27.7	50	18.9	28.4	50	21.6	32.4	50	23.3	35.0	50	24.9	37.5
51	18.3	27.6	51	18.8	28.3	51	21.4	32.2	51	23.1	34.8	51	24.8	37.2
52	18.3	27.4	52	18.7	28.1	52	21.3	32.0	52	23.0	34.5	52	24.6	36.9
53	18.2	27.3	53	18.6	28.0	53	21.2	31.8	53	22.8	34.3	53	24.4	36.7
54	18.1	27.1	54	18.5	27.8	54	21.0	31.6	54	22.6	34.0	54	24.2	36.4
55	18.0	27.0	55	18.4	27.6	55	20.9	31.4	55	22.5	33.8	55	24.0	36.1
56	17.9	26.8	56	18.3	27.5	56	20.7	31.2	56	22.3	33.5	56	23.8	35.8
57	17.7	26.7	57	18.2	27.3	57	20.6	31.0	57	22.1	33.3	57	23.6	35.5
58	17.6	26.5	58	18.1	27.1	58	20.5	30.7	58	22.0	33.0	58	23.4	35.2
59	17.5	26.4	59	17.9	27.0	59	20.3	30.5	59	21.8	32.8	59	23.2	34.9
60	17.4	26.2	60	17.8	26.8	60	20.2	30.3	60	21.6	32.5	60	23.0	34.6
61	17.3	26.0	61	17.7	26.6	61	20.0	30.1	61	21.4	32.2	61	22.8	34.3
62	17.2	25.9	62	17.6	26.5	62	19.9	29.9	62	21.3	32.0	62	22.6	34.0
63	17.1	25.7	63	17.5	26.3	63	19.7	29.6	63	21.1	31.7	63	22.4	33.7
64	17.0	25.5	64	17.4	26.1	64	19.6	29.4	64	20.9	31.4	64	22.2	33.4
65	16.9	25.4	65	17.3	25.9	65	19.4	29.2	65	20.7	31.2	65	22.0	33.0
66	16.8	25.2	66	17.1	25.8	66	19.2	28.9	66	20.5	30.9	66	21.8	32.7
67	16.7	25.0	67	17.0	25.6	67	19.1	28.7	67	20.4	30.6	67	21.6	32.4
68	16.5	24.9	68	16.9	25.4	68	18.9	28.5	68	20.2	30.3	68	21.4	32.1
69	16.4	24.7	69	16.8	25.2	69	18.8	28.2	69	20.0	30.1	69	21.1	31.8
70	16.3	24.5	70	16.7	25.0	70	18.6	28.0	70	19.8	29.8	70	20.9	31.4
71	16.2	24.3	71	16.5	24.8	71	18.5	27.7	71	19.6	29.5	71	20.7	31.1
72	16.1	24.2	72	16.4	24.7	72	18.3	27.5	72	19.4	29.2	72	20.5	30.8
73	16.0	24.0	73	16.3	24.5	73	18.1	27.2	73	19.2	28.9	73	20.3	30.5
74	15.8	23.8	74	16.2	24.3	74	18.0	27.0	74	19.1	28.6	74	20.1	30.2
75	15.7	23.6	75	16.0	24.1	75	17.8	26.8	75	18.9	28.4	75	19.8	29.8
76	15.6	23.4	76	15.9	23.9	76	17.6	26.5	76	18.7	28.1	76	19.6	29.5
77	15.5	23.3	77	15.8	23.7	77	17.5	26.3	77	18.5	27.8	77	19.4	29.2
78	15.4	23.1	78	15.6	23.5	78	17.3	26.0	78	18.3	27.5	78	19.2	28.8
79	15.2	22.9	79	15.5	23.3	79	17.1	25.8	79	18.1	27.2	79	19.0	28.5
80	15.1	22.7	80	15.4	23.1	80	17.0	25.5	80	17.9	26.9	80	18.8	28.2

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

Solution Method 3:
Using Table 4-1 for W shapes pp 4-12 to 4-23:

Oh you will love this one! All you need is the unbraced length and the shape:

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

W10

F_y = 50 ksi

Shape	W10×									
lb/ft	54		49		45		39		33	
Design	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y										
0	473	711	431	648	398	598	344	517	291	437
6	446	671	407	611	363	545	313	470	263	395
7	437	657	398	598	350	527	302	454	253	381
8	427	642	388	584	337	507	290	436	243	365
9	415	624	378	568	322	485	277	416	232	348
10	403	605	366	550	307	461	263	396	220	330
11	389	585	354	532	291	437	249	374	207	311
12	375	564	341	512	274	411	234	352	194	292
13	361	542	327	492	256	385	219	329	181	272
14	345	519	313	471	239	359	203	306	168	253
15	330	495	299	449	222	333	188	283	155	233
16	314	471	284	427	204	307	173	260	142	214
17	297	447	269	404	188	282	158	238	130	195
18	281	422	254	382	171	257	144	217	117	177
19	265	398	239	360	155	234	130	196	106	159
20	249	374	224	337	140	211	118	177	95.4	143
22	217	327	196	294	116	174	97.2	146	78.8	118
24	188	282	168	253	97.4	146	81.7	123	66.2	99.5
26	160	240	143	216	83.0	125	69.6	105	56.4	84.8
28	138	207	124	186	71.5	108	60.0	90.2	48.7	73.1
30	120	180	108	162	62.3	93.7	52.3	78.6	42.4	63.7
32	106	159	94.7	142	54.8	82.3	46.0	69.1	37.3	56.0
34	93.5	141	83.9	126						
36	83.4	125	74.8	112						
38	74.8	112	67.2	101						
40	67.6	102	60.6	91.1						

Properties

Property	54		49		45		39		33	
P_{wo}, kips	69.1	104	60.1	90.1	65.3	98.0	54.1	81.1	45.2	67.8
P_{wi}, kips/in.	12.3	18.5	11.3	17.0	11.7	17.5	10.5	15.8	9.67	14.5
P_{wb}, kips	112	168	86.6	130	94.2	142	68.7	103	53.7	80.7
P_{fb}, kips	70.8	106	58.7	88.2	71.9	108	52.6	79.0	35.4	53.2
L_p, ft	9.04		8.97		7.10		6.99		6.85	
L_r, ft	33.6		31.6		26.9		24.2		21.8	
A_g, in.^{2}	15.8		14.4		13.3		11.5		9.71	
I_x, in.^{4}	303		272		248		209		171	
I_y, in.^{4}	103		93.4		53.4		45.0		36.6	
r_y, in.	2.56		2.54		2.01		1.98		1.94	
r_x/r_y	1.71		1.71		2.15		2.16		2.16	
$P_{ex}(KL)^2/10^4$, k-in.^{2}	8670		7790		7100		5980		4890	
$P_{ey}(KL)^2/10^4$, k-in.^{2}	2950		2670		1530		1290		1050	

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

Note: Heavy line indicates KL/r_y equal to or greater than 200.