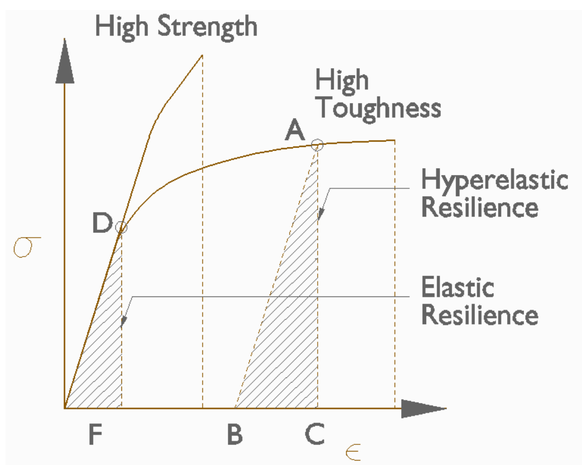
Moduli, Elasticity, Resilience, Toughness, & Stiffness

A lecture assembled for the course on
Statics and Strength of Materials

by Jason E. Charalambides PhD, PE, M.ASCE, AIA, ENV_SP

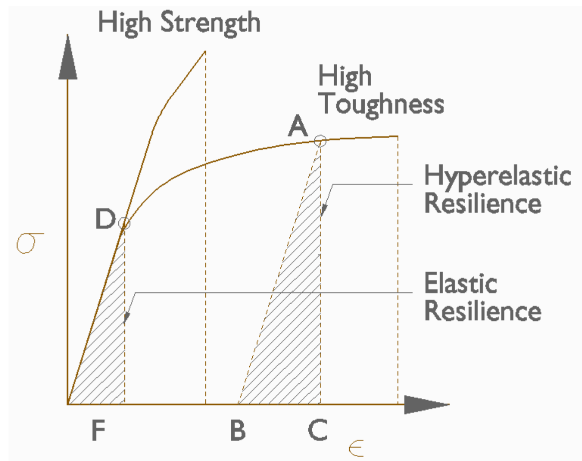
Intro

- On moduli and residual stresses:
  - A study of the behavior of a ductile material as it is loaded and unloaded reveals how permanent deformation actually modifies the behavioral characteristics of an element.
  - In the simplified stress vs. strain figure we can identify the proportional limit with the yield point.



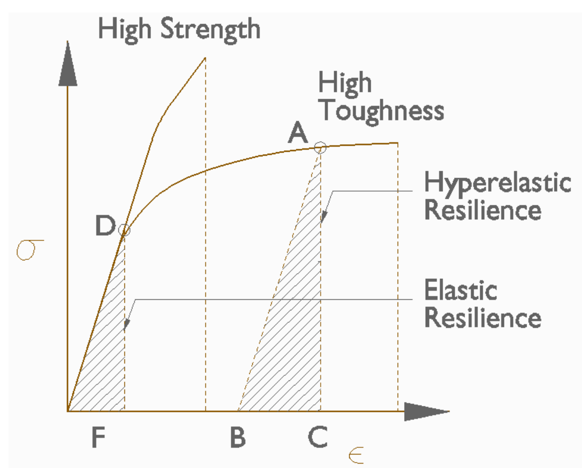
Moduli & Residual Stresses

- On moduli and residual stresses:
  - Let's take point "D" as the yield point. Once the stress applied exceeds the yield point (see point "A" in graph) a level of stress is recorded within the material.
  - Once relieved, the element will no longer return to its original form. Some deformation will be recovered and some deformation will be retained.
  - In the graph that condition would correspond to point "B" where there is no stress applied but a level of strain is retained.



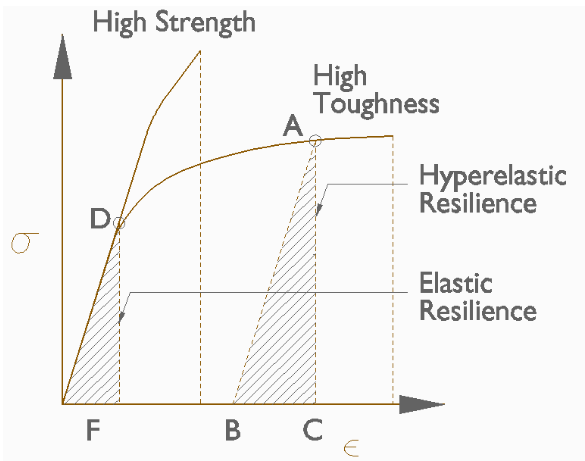
Moduli & Residual Stresses

- On moduli and residual stresses:
  - If the same element is loaded again, it will behave elastically all the way to the point that it reached in the previous loading session.
  - So for example, if a structural steel element of Grade 36 is loaded with 38^{ksi}, surpassing its elastic limit and yield point at 36^{ksi}, that element will behave elastically once loaded again, all the way to 38^{ksi} from which point onward it will continue its plastic deformation.



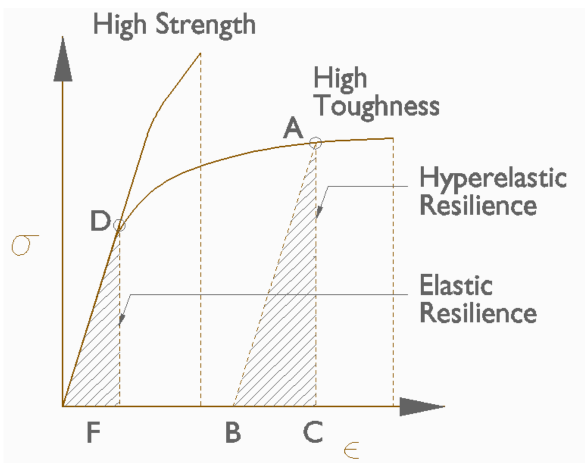
Moduli & Residual Stresses

- On moduli and residual stresses:
  - That initial deformation that is retained is called "plastic strain" while the deformation that will be recovered is the "elastic strain".
  - It must be noted that at a consecutive loading, the elastic behavior will be identical. This is indicated in the figure by the identical slope of the initial loading and of the consecutive loadings (e.g. line AB).



Moduli & Residual Stresses

- On moduli and residual stresses:
  - Integration of the stress strain curve from the point of rest to the point where strain corresponds to yield point, i.e. the area underneath the curve up to that point, gives the "Modulus of Resilience" of the material.
  - That value indicates the amount of energy that it takes per cross sectional area of a material before permanent deformation will be recorded. By observation of the graph one can see that the energy stored for this elastic behavior U_0 would be equal to:



$$U_0 = -\frac{\sigma_x \cdot \varepsilon_x}{2} \quad \text{OR} \quad U_0 = -\frac{\sigma_x^2}{2E}$$

Moduli & Residual Stresses

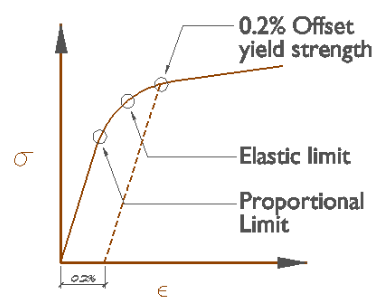
- Let's examine a scenario:
  - By applying the above, we can compare the Modulus of Resilience of A36 type steel with yield point of 36^{ksi} and Modulus of Elasticity of 29,000^{ksi}, to the Modulus of Resilience of Douglas Fir with yield point of 6.5^{ksi} and Modulus of Elasticity of 1,900^{ksi}

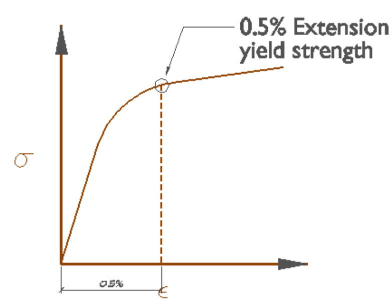
$$U_{steel} = -\frac{\sigma_x^2}{2E} = \frac{36^{ksi2}}{2 \cdot 29{,}000^{ksi}} = 22.35^{lbf \cdot \frac{in}{in^3}}$$

$$U_{df} = -\frac{\sigma_x^2}{2E} = \frac{6.5^{ksi2}}{2 \cdot 1{,}900^{ksi}} = 11.12^{lbf \cdot \frac{in}{in^3}}$$

Methods for Calculating Yield Stress

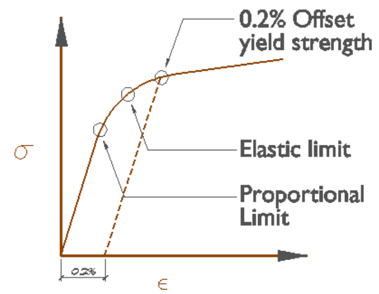
- The Offset Method:
  - Although many standardized structural materials such as steel have a clearly defined yield point, most brittle materials do not have a well defined yield point. There are two simple methods of determining the yield strength of the materials. Granted that strain is a unitless ratio, on the abscissa two specific points are used to follow either of the two methods.
  - The first method is the Offset method where from the point of 0.2% or 0.002 a line parallel to the elastic portion of the stress/strain curve is drawn. The point of intersection of the line to the curve is given as the yield point.

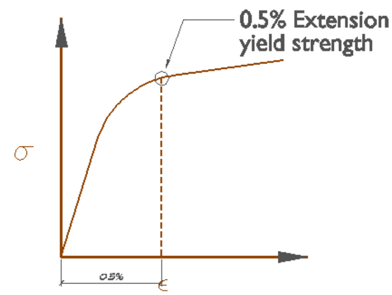




Methods for Calculating Yield Stress

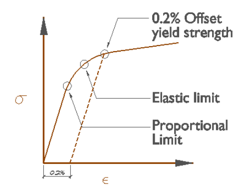
- The Extension Method:
  - The other method is called the Extension method. For that the point of 0.5% or 0.005 is used to draw a line perpendicular to the abscissa.
  - Again, the point of intersection of that line to the stress/strain curve is determined as the yield point of the material.
  - Of the two methods, the Offset method is much preferred

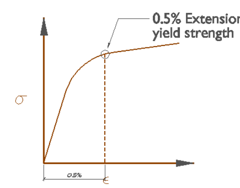




Methods for Calculating Yield Stress

- In Class Example:
  - An elastoplastic material of 20' length is subjected to 82^{ksi} stress. Its modulus of elasticity is 29000ksi and its yield strength is 65^{ksi} . The slope of the strain hardening segment of the stress/strain curve has a slope of 32000^{ksi}.
  - Determine the strain that corresponds to the 82ksi stress, the permanent strain once the 82^{ksi} is removed, 366 and the final length:
  - The first thing to do is to determine the strain (ε) that this element is subjected to. That will be the sum of the strains before it reaches the yield point and after it exceeds it:





$$\varepsilon_x = \frac{Yield\ strength}{E} + \frac{Stress - Yield\ strength}{Slope\ inside\ plastic\ region}$$

Methods for Calculating Yield Stress

- In Class Example:
  - Applying the formula's values:

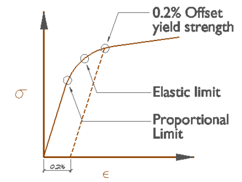
$$\varepsilon_x = \frac{65^{ksi}}{29{,}000^{ksi}} + \frac{82^{ksi} - 65^{ksi}}{3{,}200^{ksi}} = 0.0076 \frac{in}{in}$$

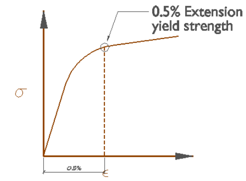
  - The next step is to subtract the permanent strain by subtracting the strain that will be recovered in order to determine how much strain will be retained:

$$\varepsilon_{perm} = \varepsilon_x - \frac{Stress}{Elastic\ Modulus} \quad \rightarrow \quad \varepsilon_{perm} = 0.0076 - \frac{82^{ksi}}{29{,}000^{ksi}} = 0.0047 \frac{in}{in}$$

  - Finally, by applying this value to the initial length we can determine that the final length would be ...

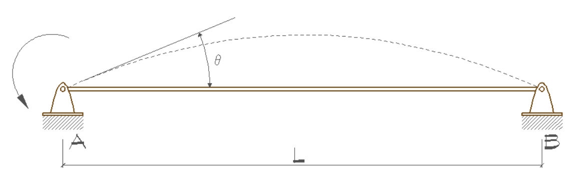
$$L_{fin} = (\varepsilon_{perm} \cdot L_0) + L_0 = (0.0047 \cdot 240^{in}) + 240^{\text{in}} = 241.13^{in}\ or\ \ 20^{ft}\text{ - }1.13^{in}$$



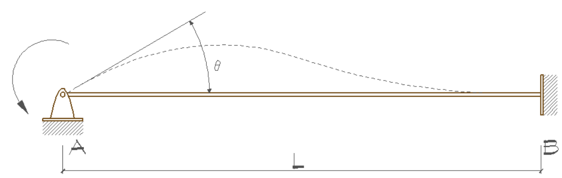


Stiffness

- Defining:
  - Stiffness (K) of a structural member can be defined as the quality of an element to resist bending. There are two reasons for which this term is presented here. After all the other such terms like toughness, strength, elasticity etc. this very important quality may also be covered.

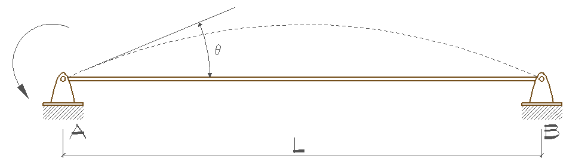


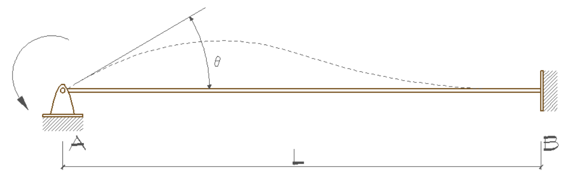
  - The specific difference of the quality we address here is that stiffness is not only a material quality but it is also related to the form and to the end conditions of the specific element. It may be unnecessarily complicated to analyze and prove the formula of what constitutes the stiffness, but it can be granted that for two specific scenarios stiffness is given by the following formulas accordingly:



STIFFNESS

- Defining:
  - if far end member is hinged $K = 3\frac{E \cdot I}{L}$
  - if far end member is fixed $K = 4\frac{E \cdot I}{L}$
  - What is of most importance at this point is the relation of the Young's Modulus of Elasticity “E”, to the Moment of Inertia “I”, and to the Length of the element “L” determining the value of stiffness.





STIFFNESS

- Defining:
  - This is used extensively in structures for design of columns, to determining the deflection of beams, and to more advanced structural analysis methods such as the Moment Distribution method and the Stiffness or Matrix analysis method, that allow the analysis of indeterminate structures.
  - In metallurgy the term "Hardness" is used in a number of ways, such as the resistance of a material to permanent surface indentations, or scratches.
  - Although related through involvement of the Modulus of Elasticity, the specific quality is not to be confused with this concept of stiffness.